Further kinematics 8A

1 a Using $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, $\mathbf{r} = (2\mathbf{i}) + (\mathbf{i} + 3\mathbf{j}) \times 4 = 2\mathbf{i} + 4\mathbf{i} + 12\mathbf{j} = 6\mathbf{i} + 12\mathbf{j}$

b Using $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, $\mathbf{r} = (3\mathbf{i} - \mathbf{j}) + (-2\mathbf{i} + \mathbf{j}) \times 5 = 3\mathbf{i} - \mathbf{j} - 10\mathbf{i} + 5\mathbf{j} = -7\mathbf{i} + 4\mathbf{j}$

c Using $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, $(4\mathbf{i} + 3\mathbf{j}) = \mathbf{r}_0 + (2\mathbf{i} - \mathbf{j}) \times 3$, $\mathbf{r}_0 = (4\mathbf{i} + 3\mathbf{j}) - (6\mathbf{i} - 3\mathbf{j}) = 4\mathbf{i} + 3\mathbf{j} - 6\mathbf{i} + 3\mathbf{j} = -2\mathbf{i} + 6\mathbf{j}$

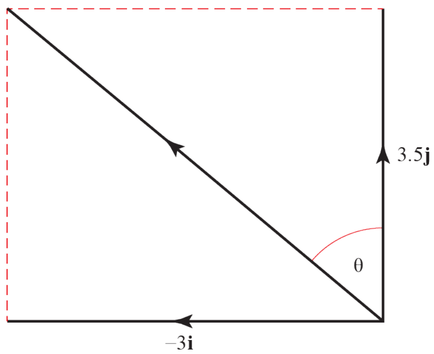
d Using $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, $(-2\mathbf{i} + 5\mathbf{j}) = \mathbf{r}_0 + (-2\mathbf{i} + 3\mathbf{j}) \times 6$, $\mathbf{r}_0 = (-2\mathbf{i} + 5\mathbf{j}) - (-12\mathbf{i} + 18\mathbf{j})$
$= -2\mathbf{i} + 5\mathbf{j} + 12\mathbf{i} - 18\mathbf{j} = 10\mathbf{i} - 13\mathbf{j}$

e Using $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, $(8\mathbf{i} - 7\mathbf{j}) = (2\mathbf{i} + 2\mathbf{j}) + \mathbf{v} \times 3$, $3\mathbf{v} = (8\mathbf{i} - 7\mathbf{j}) - (2\mathbf{i} + 2\mathbf{j}) = 6\mathbf{i} - 9\mathbf{j}$
$\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

f Using $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, $(12\mathbf{i} - 11\mathbf{j}) = (4\mathbf{i} + \mathbf{j}) + (2\mathbf{i} - 3\mathbf{j}) \times t$
$12 = 4 + 2t \Rightarrow t = 4$ s

2 $\mathbf{r}_0 = (10\mathbf{i} - 5\mathbf{j})$ m, $\mathbf{r} = (-2\mathbf{i} + 9\mathbf{j})$ m, $t = 4$ s, $\mathbf{v} = ?$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$$
$$-2\mathbf{i} + 9\mathbf{j} = (10\mathbf{i} - 5\mathbf{j}) + 4\mathbf{v}$$
$$4\mathbf{v} = -2\mathbf{i} + 9\mathbf{j} - (10\mathbf{i} - 5\mathbf{j})$$
$$\mathbf{v} = -3\mathbf{i} + \frac{7}{2}\mathbf{j}$$



$$\text{Speed} = \sqrt{3^2 + \left(\frac{7}{2}\right)^2} = \frac{\sqrt{85}}{2}$$

$$\text{Bearing} = 360^\circ - \theta \text{ where } \tan\theta = \frac{3}{3.5}$$
$$\theta = 40.601...^\circ$$

The boat travels at a speed of $\frac{\sqrt{85}}{2}$ ms^{-1} at a bearing of 319° (3s.f.).

3 $\mathbf{r}_0 = (-2\mathbf{i} + 3\mathbf{j})$ m, $\mathbf{r} = (6\mathbf{i} - 3\mathbf{j})$ m, $t = ?$, $v = 4$ ms^{-1}
Change in position $= (6\mathbf{i} - 3\mathbf{j}) - (-2\mathbf{i} + 3\mathbf{j}) = (8\mathbf{i} - 6\mathbf{j})$

Distance travelled $= \sqrt{8^2 + 6^2} = 10$

$$v = \frac{s}{t}$$
$$4 = \frac{10}{t}$$
$$t = 2.5$$

The mouse takes 2.5 s to travel to the new position.

4 a $\mathbf{r_0} = \begin{pmatrix}120\\-10\end{pmatrix}\text{m}, \mathbf{v} = \begin{pmatrix}-30\\40\end{pmatrix}\text{ms}^{-1}, \; t = t, \; \mathbf{r} = ?$

$\mathbf{r} = \mathbf{r_0} + \mathbf{v}t$

$\mathbf{r} = \begin{pmatrix}120\\-10\end{pmatrix} + \begin{pmatrix}-30\\40\end{pmatrix}t$

$\mathbf{r} = \begin{pmatrix}120-30t\\-10+40t\end{pmatrix}$

b When the helicopter is due north of the origin, the **i** component of its position vector is 0.

$120 - 30t = 0$

$t = \frac{120}{30} = 4$

The helicopter is due north of the origin after 4 s.

5 Using $\mathbf{r} = \mathbf{r_0} + \mathbf{v}t$ for P

$\mathbf{r} = (4\mathbf{i}) + (\mathbf{i} + \mathbf{j}) \times 8$

$= 4\mathbf{i} + 8\mathbf{i} + 8\mathbf{j}$

$= 12\mathbf{i} + 8\mathbf{j}$

Using $\mathbf{r} = \mathbf{r_0} + \mathbf{v}t$ for Q

$\mathbf{r} = (-3\mathbf{j}) + \mathbf{v} \times 8$

At $t = 8$ s, position vectors of P and Q are equal:

$12\mathbf{i} + 8\mathbf{j} = (-3\mathbf{j}) + \mathbf{v} \times 8$

$8\mathbf{v} = 12\mathbf{i} + 8\mathbf{j} + 3\mathbf{j}$

$= 12\mathbf{i} + 11\mathbf{j}$

$\mathbf{v} = \frac{1}{8}(12\mathbf{i} + 11\mathbf{j})$

$= 1.5\mathbf{i} + 1.375\mathbf{j}$

speed $= |\mathbf{v}|$

$= \sqrt{1.5^2 + 1.375^2}$

$= \sqrt{2.25 + 1.890625}$

$\approx 2.03 \text{ m s}^{-1}$

6 a Using $\mathbf{r} = \mathbf{r_0} + \mathbf{v}t$ for F

$\mathbf{r} = 400\mathbf{j} + (7\mathbf{i} + 7\mathbf{j}) \times t$

$= 400\mathbf{j} + 7t\,\mathbf{i} + 7t\,\mathbf{j}$

$= 7t\,\mathbf{i} + (400 + 7t)\mathbf{j}$

Using $\mathbf{r} = \mathbf{r_0} + \mathbf{v}t$ for S

$\mathbf{r} = 500\mathbf{i} + (-3\mathbf{i} + 15\mathbf{j}) \times t$

$= 500\mathbf{i} - 3t\,\mathbf{i} + 15t\,\mathbf{j}$

$= (500 - 3t)\mathbf{i} + 15t\,\mathbf{j}$

6 b For F and S to collide, $7t\mathbf{i}+(400+7t)\mathbf{j}=(500-3t)\mathbf{i}+15t\mathbf{j}$

$$\begin{aligned}\mathbf{i}\text{ components equal: } 7t&=500-3t\\ 10t&=500\\ t&=50\end{aligned}$$

$$\begin{aligned}\mathbf{j}\text{ components equal: } 400+7t&=15t\\ 400&=8t\\ t&=50\end{aligned}$$

Both conditions give the same value of t, so the two position vectors are equal when $t=50$, i.e. F and S collide at $\mathbf{r}=7\times50\mathbf{i}+(400+7\times50)\mathbf{j}=350\mathbf{i}+750\mathbf{j}$.

7 a $\mathbf{u}=\begin{pmatrix}0\\0\end{pmatrix}\text{ms}^{-1}$, $\mathbf{v}=\begin{pmatrix}3\\4\end{pmatrix}\text{ms}^{-1}$, $t=5\,\text{s}$, $\mathbf{a}=?$

Using $\mathbf{v}=\mathbf{u}+\mathbf{a}t$

$$\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}+5\mathbf{a}$$

$$\mathbf{a}=\begin{pmatrix}\frac{3}{5}\\\frac{4}{5}\end{pmatrix}$$

The acceleration of the particle is $\begin{pmatrix}\frac{3}{5}\\\frac{4}{5}\end{pmatrix}\text{ms}^{-2}$

b Using

$$\mathbf{s}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s}=\begin{pmatrix}0\\0\end{pmatrix}+\frac{1}{2}\begin{pmatrix}\frac{3}{5}\\\frac{4}{5}\end{pmatrix}5^2$$

$$\mathbf{s}=\frac{1}{2}\begin{pmatrix}15\\20\end{pmatrix}$$

After 5 s, the displacement vector of the particle is $\begin{pmatrix}\frac{15}{2}\\10\end{pmatrix}$ m.

8 a $\mathbf{u}=(15\mathbf{i}+4\mathbf{j})\text{ ms}^{-1}$, $\mathbf{v}=(5\mathbf{i}-3\mathbf{j})\text{ ms}^{-1}$, $t=4$ s, $\mathbf{a}=?$

Using $\mathbf{v}=\mathbf{u}+\mathbf{a}t$

$$5\mathbf{i}-3\mathbf{j}=15\mathbf{i}+4\mathbf{j}+4\mathbf{a}$$

$$5\mathbf{i}-3\mathbf{j}-(15\mathbf{i}+4\mathbf{j})=4\mathbf{a}$$

$$\mathbf{a}=-\frac{5}{2}\mathbf{i}-\frac{7}{4}\mathbf{j}$$

The acceleration of the particle is $\left(-\frac{5}{2}\mathbf{i}-\frac{7}{4}\mathbf{j}\right)\text{ ms}^{-2}$

8 b $\mathbf{r} = \mathbf{r_o} + \mathbf{s}$ where $\mathbf{r_o} = (10\mathbf{i} - 8\mathbf{j})$ m

Using

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{r} = (10\mathbf{i} - 8\mathbf{j}) + (15\mathbf{i} + 4\mathbf{j})t + \frac{1}{2}\left(-\frac{5}{2}\mathbf{i} - \frac{7}{4}\mathbf{j}\right)t^2$$

$$\mathbf{r} = \left(10 + 15t - \frac{5}{4}t^2\right)\mathbf{i} + \left(-8 + 4t - \frac{7}{8}t^2\right)\mathbf{j}$$

The position vector of the particle after t s is $\left(10 + 15t - \frac{5}{4}t^2\right)\mathbf{i} + \left(-8 + 4t - \frac{7}{8}t^2\right)\mathbf{j}$ m.

9 a $\mathbf{a} = \begin{pmatrix} -1 \\ 1.5 \end{pmatrix}$ ms^{-2}, $\mathbf{u} = \begin{pmatrix} 70 \\ -30 \end{pmatrix}$ ms^{-1}, $\mathbf{v} = ?$, $t = 10$ s,

Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$$\mathbf{v} = \begin{pmatrix} 70 \\ -30 \end{pmatrix} + 10\begin{pmatrix} -1 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 60 \\ -15 \end{pmatrix}$$

After 10s, the velocity of the plane is $\begin{pmatrix} 60 \\ -15 \end{pmatrix}$ ms^{-1}

b Using

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = 10\begin{pmatrix} 70 \\ -30 \end{pmatrix} + \frac{10^2}{2}\begin{pmatrix} -1 \\ 1.5 \end{pmatrix}$$

$$\mathbf{s} = \begin{pmatrix} 650 \\ -225 \end{pmatrix}$$

Distance travelled $= \sqrt{650^2 + 225^2} = 687.84...$

The plane is 688 m (3s.f.) from its starting point after 10 s.

10 $\mathbf{v} = (4\mathbf{i} + 3\mathbf{j})$ ms^{-1}, $t = 20$ s, $\mathbf{a} = (0.2\mathbf{i} + 0.6\mathbf{j})$ ms^{-2}, $\mathbf{s} = ?$

Using

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = 20 \times (4\mathbf{i} + 3\mathbf{j}) - \frac{20^2}{2}(0.2\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{s} = 80\mathbf{i} + 60\mathbf{j} - 40\mathbf{i} - 120\mathbf{j}$$

$$\mathbf{s} = 40\mathbf{i} - 60\mathbf{j}$$

After 20 s, the displacement vector of the boat from its starting position is $(40\mathbf{i} - 60\mathbf{j})$ m.

11 a Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ and $t = 3$

For A:

$\mathbf{v} = (-\mathbf{i} + \mathbf{j}) + (2\mathbf{i} - 4\mathbf{j}) \times 3$

$= (-1 + 6)\mathbf{i} + (1 - 12)\mathbf{j}$

$= 5\mathbf{i} - 11\mathbf{j}$

Speed $= \sqrt{5^2 + 11^2} = \sqrt{25 + 121}$

$= \sqrt{146} = 12.1\,\text{m s}^{-1}$ (3 s.f.)

For B:

$v = \mathbf{i} + 2\mathbf{j} \times 3$

$= \mathbf{i} + 6\mathbf{j}$

Speed $= \sqrt{1^2 + 6^2} = \sqrt{1 + 36}$

$= \sqrt{37} = 6.08\,\text{m s}^{-1}$ (3 s.f.)

b Using $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ for A,

$\mathbf{s} = (-\mathbf{i} + \mathbf{j}) \times 3 + \frac{1}{2} \times (2\mathbf{i} - 4\mathbf{j}) \times 9$

$= -3\mathbf{i} + 3\mathbf{j} + 9\mathbf{i} - 18\mathbf{j}$

$= 6\mathbf{i} - 15\mathbf{j}$

So at the instant of the collision, A is at the point with position vector

$\mathbf{r} = \mathbf{r}_0 + \mathbf{s}$

$\mathbf{r} = (12\mathbf{i} + 12\mathbf{j}) + (6\mathbf{i} - 15\mathbf{j})$

$= 18\mathbf{i} - 3\mathbf{j}$

c First find the displacement through which B travels during the motion:

Using $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ for B,

$\mathbf{s} = (\mathbf{i}) \times 3 + \frac{1}{2} \times (2\mathbf{j}) \times 9$

$= 3\mathbf{i} + 9\mathbf{j}$

So B's starting point is given by:

$\mathbf{r}_0 = (\text{final position}) - (\text{displacement through which B travels})$

$\mathbf{r}_0 = \mathbf{r} - \mathbf{s}$

$\mathbf{r}_0 = (18\mathbf{i} - 3\mathbf{j}) - (3\mathbf{i} + 9\mathbf{j}) = 15\mathbf{i} - 12\mathbf{j}$

12 a $\mathbf{u} = (-4\mathbf{i} + 8\mathbf{j})$ kmh^{-1}, $\mathbf{v} = (-2\mathbf{i} - 6\mathbf{j})$ kmh^{-1}, $t = 2$ h, $\mathbf{a} = ?$
Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$$-2\mathbf{i} - 6\mathbf{j} = -4\mathbf{i} + 8\mathbf{j} + 2\mathbf{a}$$
$$2\mathbf{a} = 2\mathbf{i} - 14\mathbf{j}$$
$$\mathbf{a} = \mathbf{i} - 7\mathbf{j}$$

The acceleration of the ship is $(\mathbf{i} - 7\mathbf{j})$ kmh^{-2}

b Using

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$
$$\mathbf{s} = (-4\mathbf{i} + 8\mathbf{j})t + \frac{1}{2}(\mathbf{i} - 7\mathbf{j})t^2$$
$$\mathbf{s} = (-4t + 0.5t^2)\mathbf{i} + (8t - 3.5t^2)\mathbf{j}$$

After t h, the ship's displacement vector from O is $(-4t + 0.5t^2)\mathbf{i} + (8t - 3.5t^2)\mathbf{j}$ km.

c When the ship is SW of O, then the coefficients of **i** and **j** are equal (and negative) so:

$$-4t + 0.5t^2 = 8t - 3.5t^2$$
$$4t^2 = 12t$$
$$t = 3$$

(The solution t = 0 can be ignored as at this time both coefficients are zero, ship is at O.)

The ship is SW of O 3 h after 12:00, i.e. at 15:00.

d When the two ships meet **r** = **s**. Since **r** has no **i** component, the **i** component of **s** must also be 0.

$$-4t + 0.5t^2 = 0$$
$$0.5t^2 = 4t$$
$$t = 8 \quad \text{(solution } t = 0 \text{ can again be ignored)}$$
$$\mathbf{r} = (40 - 25t)\mathbf{j}$$
$$\mathbf{r} = (40 - 25 \times 8)\mathbf{j}$$
$$\mathbf{r} = -160\mathbf{j}$$

The two ships meet at position vector $-160\mathbf{j}$ km (i.e. 160 km S of O).

13 a For the particle to be NE of O, the coefficients of **i** and **j** are equal so:

$$2t^2 - 3 = 7 - 4t$$
$$2t^2 + 4t - 10 = 0$$
$$t^2 + 2t - 5 = 0 \quad \text{as required.}$$

b Using formula for the roots of a quadratic equation:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-2 \pm \sqrt{4 + 20}}{2}$$
$$t = \sqrt{6} - 1$$

Negative root can be ignored as equation only applies for $t \geq 0$.

13 b Since the two coefficients are equal, we need only calculate one of them:

$$7-4t=7-4\times\left(\sqrt{6}-1\right)$$
$$=11-4\sqrt{6}$$

$$\text{Distance}=\sqrt{\left(11-4\sqrt{6}\right)^2+\left(11-4\sqrt{6}\right)^2}$$
$$=1.6999...$$

The particle is 1.70 m from O when it is NE of O.

c $\mathbf{u} = (5\mathbf{i} + 6\mathbf{j})\ \text{ms}^{-1}$, $\mathbf{v} = (b\mathbf{i} + 2b\mathbf{j})\ \text{ms}^{-1}$, $t = 2$ s, $\mathbf{a} = (3a\mathbf{i} - 2a\mathbf{j})$

Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$$\begin{pmatrix} b \\ 2b \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} + 2\begin{pmatrix} 3a \\ -2a \end{pmatrix}$$

Considering coefficients of **i**:

$b = 5 + 6a$ **(1)**

Considering coefficients of **j**:

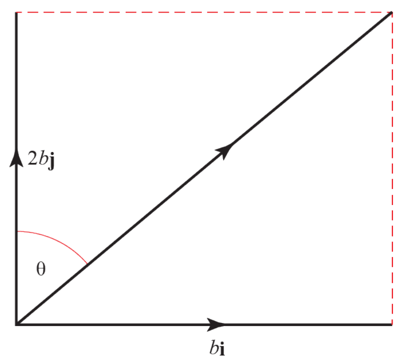
$2b = 6 - 4a$ **(2)**

Substituting $b = 5 + 6a$ from (**1**) into (**2**):

$$2(5+6a)=6-4a$$
$$5+6a=3-2a$$
$$8a=-2$$
$$a=-0.25$$

Substituting $a = -0.25$ into (**1**):

$b = 5 - 1.5 = 3.5$



Therefore at $t = 2$ s, $\mathbf{v} = (3.5\mathbf{i} + 7\mathbf{j})\ \text{ms}^{-1}$

Speed $= \sqrt{3.5^2+7^2} = 7.8262...$

Bearing $= \theta$ where

$$\tan\theta=\frac{b}{2b}=0.5$$
$$\theta=26.565...$$

The particle is travelling at speed of 7.83 ms^{-1} at a bearing of 026.6° (both to 3s.f.).

13 d At $t = 2$ s, for the first particle:

$\mathbf{r}_A = (2\times4-3)\mathbf{i} + (7-4\times2)\mathbf{j}$

$\mathbf{r}_A = 5\mathbf{i} - \mathbf{j}$

For the second particle, the displacement since $t = 0$ is given by:

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = 2\begin{pmatrix}5\\6\end{pmatrix} + \frac{2^2}{2}\begin{pmatrix}-3\times0.25\\2\times0.25\end{pmatrix}$

$\mathbf{s} = \begin{pmatrix}10-1.5\\12+1\end{pmatrix} = 8.5\mathbf{i} + 13\mathbf{j}$

Displacement of second particle from O,

$\mathbf{r}_B = \mathbf{r}_\mathbf{o} + \mathbf{s}$ where $\mathbf{r}_\mathbf{o} = 5\mathbf{j}$

$\mathbf{r}_B = 5\mathbf{j} + 8.5\mathbf{i} + 13\mathbf{j} = 8.5\mathbf{i} + 18\mathbf{j}$

Relative displacement of the two particles:

$\mathbf{r}_B - \mathbf{r}_A = \begin{pmatrix}8.5\\18\end{pmatrix} - \begin{pmatrix}5\\-1\end{pmatrix}$

$= \begin{pmatrix}3.5\\19\end{pmatrix}$

Distance $= |\mathbf{r}_B - \mathbf{r}_A|$

$= \sqrt{3.5^2 + 19^2}$

$= 19.319...$

The distance between the two particles is 19.3 m (3s.f.).

Challenge

The planes cross at **r** relative to the control tower after a time T after the first plane passes it.
For the first plane:

$$\mathbf{u} = \begin{pmatrix} 20 \\ -100 \end{pmatrix} \text{ms}^{-1}, \ \mathbf{a} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \text{ms}^{-2}, \ \mathbf{s} = \mathbf{r}, \ t = T$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{r} = \begin{pmatrix} 20 \\ -100 \end{pmatrix} T + \frac{T^2}{2} \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 20T \\ -100T + 3T^2 \end{pmatrix}$$

For the second plane:

$$\mathbf{u} = \begin{pmatrix} 70 \\ 40 \end{pmatrix} \text{ms}^{-1}, \ \mathbf{a} = \begin{pmatrix} 0 \\ -8 \end{pmatrix} \text{ms}^{-2}, \ \mathbf{s} = \mathbf{r}, \ t = (T - t)$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{r} = \begin{pmatrix} 70 \\ 40 \end{pmatrix} (T - t) + \frac{(T - t)^2}{2} \begin{pmatrix} 0 \\ -8 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 70T - 70t \\ 40T - 40t - 4T^2 - 4t^2 + 8Tt \end{pmatrix}$$

Equating **i** components:

$$\begin{aligned} 20T &= 70T - 70t \\ 70T - 20T &= 70t \\ T &= \frac{7}{5}t \end{aligned}$$

Equating **j** components and substituting in this value of T:

$$\begin{aligned} -100T + 3T^2 &= 40T - 40t - 4T^2 - 4t^2 + 8Tt \\ 7T^2 + 4t^2 - 8Tt &= 140T - 40t \\ \frac{7 \times 7^2}{5^2}t^2 + 4t^2 - \frac{8 \times 7}{5}t^2 &= \frac{140 \times 7}{5}t - 40t \\ \left(\frac{343}{25} + 4 - \frac{56}{5}\right)t^2 &= (196 - 40)t \\ \frac{163}{25}t^2 &= 156t \\ t &= 23.926... \end{aligned}$$

The second plane passes over the control tower 24 s after the first plane (2s.f.).