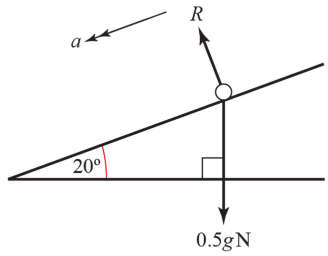
Applications of forces 7E

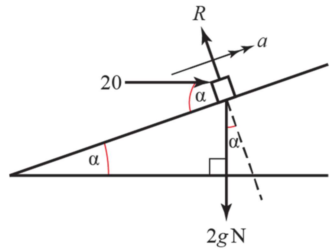
1 $R(\swarrow)$



$$F = ma$$
$$0.5g \sin 20° = 0.5a$$
$$a = 3.35 \text{ (3 s.f.)}$$

2 $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$

a $R(\nwarrow)$



$$R - 20\sin\alpha - 2g\cos\alpha = 0$$
$$R = 20\sin\alpha + 19.6\cos\alpha$$
$$= 12 + 15.68$$
$$= 27.7 \text{ N}$$

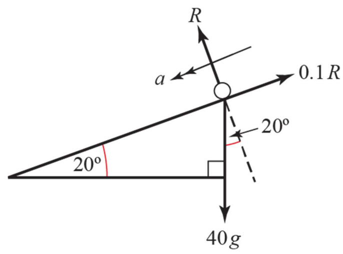
The normal reaction is 27.7 N (3 s.f.).

b $R(\nearrow)$

$$F = ma$$
$$20\cos\alpha - 2g\sin\alpha = 2a$$
$$2a = 20 \times \frac{4}{5} - 2 \times 9.8 \times \frac{3}{5}$$
$$a = 2.12 \text{ m s}^{-2}$$

The acceleration of the box is 2.12 m s^{-2}

3 a $R(\nwarrow)$



$$R - 40g\cos 20° = 0$$
$$R = 368.36$$

$R(\swarrow)$

$$40g\sin 20° - 0.1R = 40a$$
$$392\cos 70° - 36.836 = 40a$$
$$a = 2.43 \text{ (3 s.f.)}$$

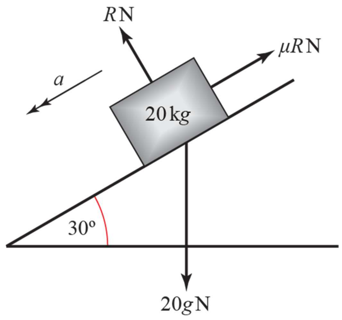
The acceleration of the boy is 2.43 m s^{-2} (3 s.f.).

b $u = 0, \quad a = 2.43, \quad s = 5, \quad v = ?$

$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2 \times 2.43 \times 5 = 24.3$$
$$v = 4.93 \text{ m s}^{-1} \text{ (3 s.f.)}$$

The speed of the boy is 4.93 m s^{-1} (3 s.f.).

4 $u = 0\text{ ms}^{-1}$, $v = 21\text{ ms}^{-1}$, $t = 6\text{ s}$, $a = ?$



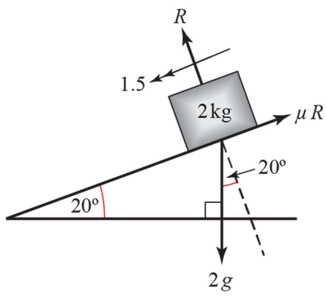
$$v = u + at$$
$$21 = 0 + 6a$$
$$a = \frac{21}{6} = \frac{7}{2}$$

$R(\nwarrow)$:
$$R = 20g\cos 30^\circ$$

$R(\swarrow)$
$$F = ma$$
$$20g\sin 30^\circ - \mu R = 20 \times \frac{7}{2}$$
$$20g\sin 30^\circ - (\mu \times 20g\cos 30^\circ) = 20 \times \frac{7}{2}$$
$$\frac{1}{2}g - \frac{\sqrt{3}}{2}\mu g = \frac{7}{2}$$
$$g - \sqrt{3}\mu g = 7$$
$$\mu = \frac{9.8 - 7}{\sqrt{3} \times 9.8}$$
$$= 0.16495...$$

The coefficient of friction is 0.165 (3s.f.).

5 $R(\nwarrow)$

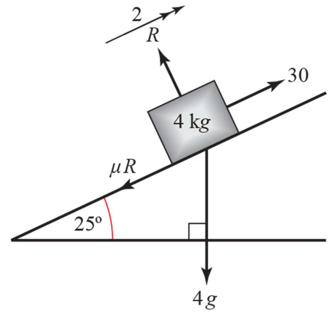


$$R - 2g\cos 20° = 0$$
$$R = 2g\cos 20°$$

$R(\swarrow)$
$$F = ma$$
$$2g\sin 20° - \mu R = 2 \times 1.5$$
$$2g\sin 20° - \mu \times 2g\cos 20° = 3$$
$$\mu = \frac{2g\sin 20° - 3}{2g\cos 20°} = 0.201\ (3\text{ s.f.})$$

The coefficient of friction is 0.20 (2 s.f.).

6 $R(\nwarrow)$



$R - 4g\cos 25° = 0$

$R = 4g\cos 25°$

$R(\nearrow)$

$$F = ma$$

$$30 - 4g\sin 25° - \mu R = 4 \times 2$$

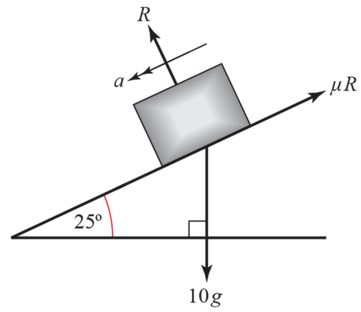
$$30 - 4g\sin 25° - \mu 4g\cos 25° = 8$$

$$\frac{22 - 4g\sin 25°}{4g\cos 25°} = \mu$$

$$0.15 \text{ (2 s.f.)} = \mu$$

The coefficient of friction is 0.15 (2 s.f.).

7 a $R(\nwarrow)$



$R - 10g\cos 25° = 0$

$R = 98\cos 25°$

$= 88.8\,\text{N}$ (3 s.f.)

The normal reaction is $88.8\,\text{N}$ (3 s.f.).

b $u = 0,\ \ s = 4,\ \ t = 2,\ \ a = ?$

$s = ut + \frac{1}{2}at^2$

$4 = 0 + \frac{1}{2}a \times 2^2$

$a = 2\,\text{m s}^{-2}$

$R(\swarrow)$

$$F = ma$$

$$10g\sin 25° - \mu R = 10 \times 2$$

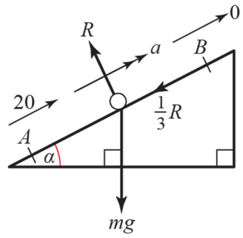
$$\mu \times 98\cos 25° = 10g\sin 25° - 20$$

$$\mu = \frac{98\sin 25° - 20}{98\cos 25°}$$

$$= 0.241 \text{ (3 s.f.)}$$

The coefficient of friction is 0.24 (2 s.f.).

8 a Let mass of particle be m.



$R(\nwarrow)$

$R - mg\cos\alpha = 0$

$$R = \frac{4mg}{5}$$

$R(\nearrow)$

$$F = ma$$

$$-mg\sin\alpha - \frac{1}{3}R = ma$$

$$-\frac{3mg}{5} - \frac{1}{3} \times \frac{4mg}{5} = ma$$

$$-\frac{13g}{15} = a$$

The deceleration is $\frac{13g}{15}$.

b $u = 20, \quad v = 0, \quad a = -\frac{13g}{15}, \quad s = ?$

$$v^2 = u^2 + 2as$$

$$0 = 20^2 - \frac{26g}{15}s$$

$$s = \frac{6000}{26g} = 23.5\text{m (3 s.f.)}$$

$AB = 23.5\,\text{m (3 s.f.)}$

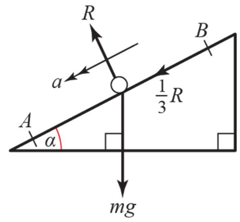
c $u = 20, \quad v = 0, \quad a = -\frac{13g}{15}, \quad t = ?$

$$v = u + at$$

$$0 = 20 - \frac{13gt}{15}$$

$$t = \frac{300}{13g} = 2.35\text{s (3 s.f.)}$$

8 d As the particle begins to decelerate downwards from B, friction now acts up the slope.



$R(\nwarrow)$

$R = \frac{4mg}{5}$, as before

$R(\swarrow)$

$$F = ma$$

$$mg\sin\alpha - \frac{1}{3}R = ma$$

$$\frac{3mg}{5} - \frac{1}{3} \times \frac{4mg}{5} = ma$$

$$\frac{g}{3} = a$$

Now use equations of motion for constant acceleration:

$u = 0,\quad a = \frac{g}{3},\quad s = \frac{6000}{26g},\quad v = ?$

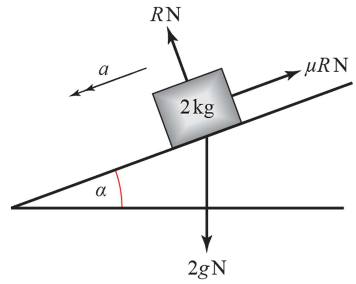
$v^2 = u^2 + 2as$

$v^2 = 0 + \frac{2g}{3} \times \frac{6000}{26g}$

$= \frac{4000}{26}$

$v = 12.4\,\text{m s}^{-1}$ (3 s.f.)

The speed of the particle as it passes A on the way down is $12.4\,\text{m s}^{-1}$ (3 s.f.).



9 $\tan\alpha = \frac{2}{5} \Rightarrow \sin\alpha = \frac{2}{\sqrt{29}}$ and $\cos\alpha = \frac{5}{\sqrt{29}}$

$u = 0 \text{ ms}^{-1}$, $v = 6 \text{ ms}^{-1}$, $t = 3$ s, $a =$?

$$v = u + at$$
$$6 = 0 + 3a$$
$$a = \frac{6}{3} = 2 \text{ ms}^{-2}$$

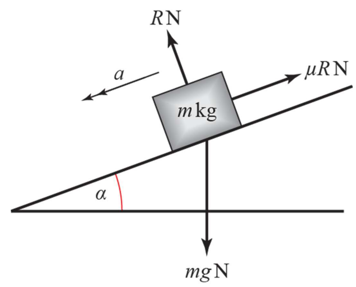
$R(\nwarrow)$

$$R = 2g\cos\alpha$$

$R(\swarrow)$

$$F = ma$$
$$2g\sin\alpha - \mu R = 2\times 2$$
$$2g\sin\alpha - (\mu\times 2g\cos\alpha) = 4$$
$$g\sin\alpha - \mu g\cos\alpha = 2$$
$$\mu = \frac{g\sin\alpha - 2}{g\cos\alpha}$$
$$\mu = \frac{\left(9.8\times\frac{2}{\sqrt{29}}\right) - 2}{9.8\times\frac{5}{\sqrt{29}}}$$
$$\mu = \frac{(9.8\times 2) - 2\sqrt{29}}{9.8\times 5}$$
$$= 0.18019...$$

The coefficient of friction is 0.180 (3s.f.).



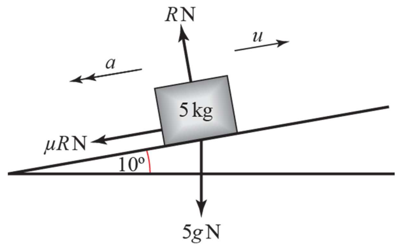
10 $R(\nwarrow)$

$$R = mg\cos\alpha$$

$R(\swarrow)$

$$F = ma$$
$$mg\sin\alpha - \mu R = ma$$
$$mg\sin\alpha - (\mu\times mg\cos\alpha) = ma$$
$$g\sin\alpha - \mu g\cos\alpha = a$$

Since this expression does not contain m, the acceleration is independent of the mass.



11 a $u = 16\text{ ms}^{-1}$, $v = 0\text{ ms}^{-1}$, $t = 5$ s, a =?

$v = u + at$

$0 = 16 + 5a$

$a = -\frac{16}{5}$

$R(\nwarrow)$

$R = 5g\cos 10^\circ$

$R(\swarrow)$

$$F = ma$$

$$5g\sin 10^\circ + \mu R = ma$$

$$5g\sin 10^\circ + (\mu \times 5g\cos 10^\circ) = 5 \times \frac{16}{5}$$

$$5g\sin 10^\circ + 5\mu g\cos 10^\circ = 16$$

$$\mu = \frac{16 - 5g\sin 10^\circ}{5g\cos 10^\circ}$$

$$= 0.15524...$$

The coefficient of friction is 0.155 (3s.f.).

b The particle will move back down the slope if the component of its weight acting down the slope is greater than the frictional force acting up the slope, i.e. if

$$5g\sin 10^\circ > 5\mu g\cos 10^\circ$$

$$\sin 10^\circ > 0.155 \times \cos 10^\circ$$

$$0.17364... > 0.15288...$$

Since this inequality is true (i.e. 0.174 > 0.153), the particle will move back down the slope.