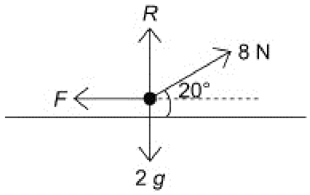
Applications of forces 7C

1 Let the normal reaction be R N, the friction force be F N and the coefficient of friction be μ.



Resolve horizontally to find F, vertically to find R and use $F = \mu R$ to find μ:

$R(\rightarrow)$

$8\cos 20° - F = 0$

$\therefore F = 8\cos 20°$

$R(\uparrow)$

$R + 8\sin 20° - 2g = 0$

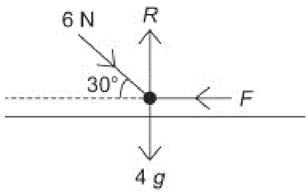
$\therefore R = 2g - 8\sin 20°$

As the book is on the point of slipping the friction is limiting:

$$F = \mu R$$

$$\therefore \mu = \frac{F}{R} = \frac{8\cos 20°}{2g - 8\sin 20°} = \frac{7.518}{16.86} = 0.446 \text{ (3 s.f.)}$$

2 Let the normal reaction be R N, the friction force be F N and the coefficient of friction be μ.



$R(\rightarrow)$: $6\cos 30° - F = 0$

$$F = 6\cos 30° = 3\sqrt{3} = 5.20 \text{ (3 s.f.)}$$

$R(\uparrow)$: $R - 6\sin 30° - 4g = 0$

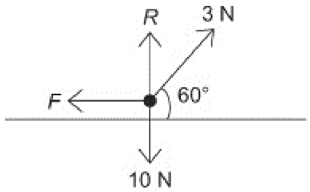
$$R = 6\sin 30° + 4g = 3 + 4 \times 9.8 = 42.2$$

As the block is on the point of slipping

$$F = \mu R$$

$$\therefore \mu = \frac{F}{R} = 0.123 \text{ (3 s.f.)}$$

3 Let the normal reaction force be R and the friction force be F.



a Resolve horizontally to find the magnitude of the friction force necessary to maintain equilibrium:

$R(\rightarrow)$

$3\cos 60° - F = 0$

$\therefore F = 3\cos 60°$

$F = 1.5\,\text{N}$

b Resolve vertically to calculate R and hence μR :

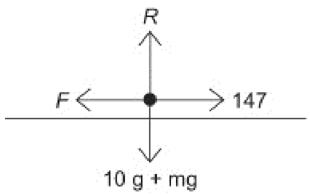
$R(\uparrow)$

$R + 3\sin 60° - 10 = 0$

$$\begin{aligned}\therefore R &= 10 - 3\sin 60° \\ &= 10 - \frac{3\sqrt{3}}{2} \\ &= 7.40 \text{ (3 s.f.)} \\ \therefore \mu R &= 0.3 \times 7.40 \\ &= 2.22 \text{ (3 s.f.)}\end{aligned}$$

Since $F = 1.5\,\text{N} < 2.2\,\text{N} = \mu R$, the friction required to maintain equilibrium is not limiting friction.

4 **a** Let the normal reaction be R N and the friction force required to maintain equilibrium be F N.
Let the mass of the books be m kg.



$R(\rightarrow)$

$147 - F = 0$

$\therefore F = 147\,\text{N}$

$R(\uparrow)$

$R - 10g - mg = 0$

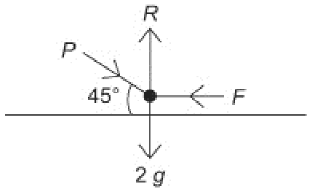
$\therefore R = 10g + mg$

As the equilibrium is limiting, $F = \mu R$

$$\begin{aligned}147 &= 0.3(10g + mg) \\ 147 &= 3g + 0.3mg \\ \therefore m &= \frac{147 - 3g}{0.3g} \\ &= 40 \text{ kg}\end{aligned}$$

b The assumption is that the crate and books may be modelled as a particle.

5 a Let R be the normal reaction and F be the force of friction when P acts downwards.



$R(\rightarrow)$
$P\cos 45° - F = 0$
$\therefore F = P\cos 45°$

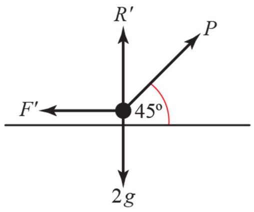
$R(\uparrow)$
$R - P\sin 45° - 2g = 0$
$\therefore R = P\sin 45° + 2g$

Resolve horizontally and vertically to find F and R, then use the condition for limiting friction.

As the equilibrium is limiting, $F = \mu R$

$$\therefore P\cos 45° = 0.3(P\sin 45° + 2g)$$
$$P(\cos 45° - 0.3\sin 45°) = 0.6g$$
$$\therefore P = \frac{0.6g}{\cos 45° - 0.3\sin 45°}$$
$$= \frac{6g\sqrt{2}}{7}$$
$$= 11.9\text{ N (3 s.f.)}$$

b Let R' be the normal reaction and F' be the force of friction when P acts upwards.



$R(\rightarrow)$
$P\cos 45° - F' = 0$
$\therefore F' = P\cos 45°$

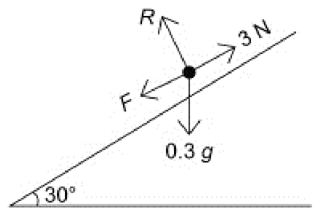
$R(\uparrow)$
$R' + P\sin 45° - 2g = 0$
$\therefore R' = 2g - P\sin 45°$

As the equilibrium is limiting, $F = \mu R$

$$\therefore P\cos 45° = 0.3(2g - P\sin 45°)$$
$$P(\cos 45° + 0.3\sin 45°) = 0.6g$$
$$\therefore P = \frac{6g\sqrt{2}}{13}$$
$$= 6.40\text{ N (3 s.f.)}$$

6 Let R be the normal reaction and F be the force of friction required to maintain equilibrium.

Since the particle is on the point of slipping up the plane, the force of friction acts down the slope.



$R(\nearrow)$

$$3 - F - 0.3g \sin 30° = 0$$
$$\therefore F = 3 - 0.3g \sin 30°$$
$$= 1.53 \text{ N}$$

$R(\nwarrow)$

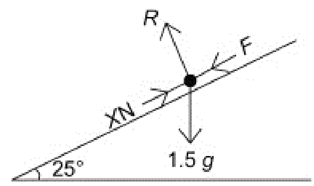
$$R - 0.3g \cos 30° = 0$$
$$\therefore R = 0.3g \cos 30°$$
$$= 2.546 \text{ N}$$

As the particle is on the point of slipping, $F = \mu R$

$$\therefore 1.53 = \mu \times 2.546$$
$$\therefore \mu = \frac{1.53}{2.546}$$
$$= 0.601 \text{ (3 s.f.) (accept 0.6)}$$

7 Let R be the normal reaction and F be the force of friction required to maintain equilibrium.

Since the particle is on the point of slipping up the plane, the force of friction acts down the slope.



a $R(\nwarrow)$

$$R - 1.5g \cos 25° = 0$$
$$R - 1.5g \cos 25° = 0$$
$$\therefore R = 1.5g \cos 25°$$
$$= 13.3 \text{ N (3 s.f.)}$$

b $R(\nearrow)$

$$X - F - 1.5g \sin 25° = 0$$
$$X = F + 1.5g \sin 25° \qquad \textbf{(1)}$$

The particle is in limiting equilibrium, so $F = \mu R$

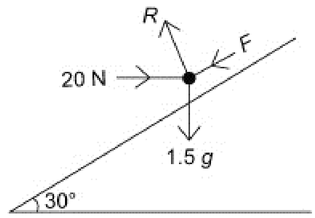
$\therefore F = 0.25 \times 13.3227$ (using $R = 13.3$ N from **a**)

$= 3.3306...$

Sub $F = 3.33$ N into (**1**):

$$X = 3.33 + 1.5g \sin 25°$$
$$= 9.54 \text{ N (3 s.f.)}$$

8 Let the normal reaction be R and the friction force be F acting down the plane.



a $R(\nwarrow)$

$R - 20\sin 30° - 1.5g\cos 30° = 0$

$\therefore R = 20\sin 30° + 1.5g\cos 30°$

$= 22.7$ (3 s.f.)

The normal reaction has magnitude 22.7 N or 23 N (2 s.f.).

If you are told the particle is in equilibrium, but not told which way the particle is about to slip, then draw a diagram showing all the forces acting on the particle, with friction acting down the plane.

Resolve forces parallel to the plane. If $F > 0$ then you have chosen the correct direction. If $F < 0$ then you know friction acts up the plane.

b $R(\nearrow)$

$20\cos 30° - F - 1.5g\sin 30° = 0$

$\therefore F = 20\cos 30° - 1.5g\sin 30°$

$= 9.97$ (3 s.f.)

The friction force has magnitude 9.97 N and acts down the plane.

c Minimum possible value of μ occurs when frictional force required to maintain equilibrium is μR :

$F = \mu R$

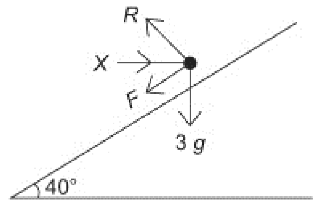
$9.9705... = \mu \times 22.730...$ (using F from **b** and R from **a**)

$\mu = \dfrac{22.730...}{9.9705...}$

$= 0.43863...$

The coefficient of friction must be at least 0.439 (3s.f.) to prevent the block sliding.

9 Let the normal reaction be R and the friction force be F acting down the plane.



a

$R(\nwarrow)$

$R - X\sin 40° - 3g\cos 40° = 0$

$$R = X\sin 40° + 3g\cos 40° \quad \textbf{(1)}$$

$R(\nearrow)$

$X\cos 40° - F - 3g\sin 40° = 0$

$$F = X\cos 40° - 3g\sin 40° \quad \textbf{(2)}$$

As the friction is limiting, $F = \mu R$
Using F from (**2**) and R from (**1**) gives:

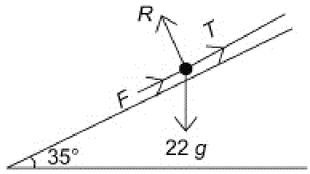
$$X\cos 40° - 3g\sin 40° = 0.3(X\sin 40° + 3g\cos 40°)$$
$$X\cos 40° - 0.3X\sin 40° = 0.9g\cos 40° + 3g\sin 40°$$
$$X(\cos 40° - 0.3\sin 40°) = 0.9g\cos 40° + 3g\sin 40°$$
$$X = \frac{0.9g\cos 40° + 3g\sin 40°}{\cos 40° - 0.3\sin 40°}$$
$$= \frac{25.65}{0.5732}$$
$$X = 44.8 \text{ N (3 s.f.)}$$

b Substituting $X = 44.8$ N into equation (**1**) gives

$$R = 44.8 \times \sin 40° + 3g\cos 40°$$
$$= 51.3 \text{ N (3 s.f.)}$$

10 Let the normal reaction be R and the friction force be F acting up the plane.

The friction acts up the plane, as the sledge is on the point of slipping down the plane.



$R(\nearrow)$

$T + F - 22g\sin 35° = 0$ **(1)**

$R(\nwarrow)$

$R - 22g\cos 35° = 0$

$\therefore R = 22g\cos 35°$

$R = 176.6$ N

As the friction is limiting, $F = \mu R$

$\therefore F = 0.125 \times 176.6$

$= 22.1$ N (3 s.f.)

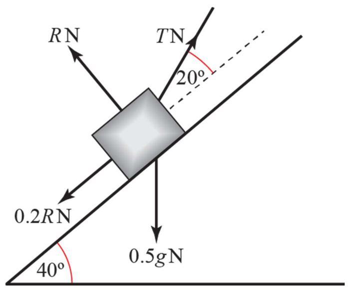
Substituting $T = 22.1$ N into equation (**1**) gives:

$T = 22g\sin 35° - 22.1$

$= 101.6$

$= 102$ N (3 s.f.)

11 $R(\nwarrow)$



$R - 0.5g\cos 40^\circ + T\sin 20^\circ = 0$

$R = 0.5g\cos 40^\circ - T\sin 20^\circ$

T_{MAX} occurs when the particle is on the point of moving up the plane.
At this point, limiting friction $F = \mu R$ acts down the plane:

$R(\nearrow)$

$$T_{MAX}\cos 20^\circ - 0.5g\sin 40^\circ - F = 0$$

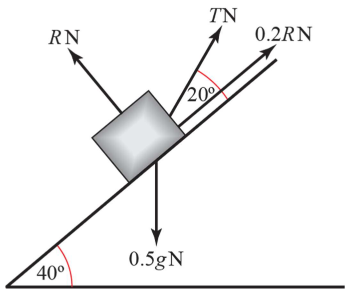
$$T_{MAX}\cos 20^\circ - 0.5g\sin 40^\circ - \frac{1}{5}\left(0.5g\cos 40^\circ - T\sin 20^\circ\right) = 0$$

$$T_{MAX}\cos 20^\circ + 0.2T_{MAX}\sin 20^\circ = 0.5g\sin 40^\circ + 0.1g\cos 40^\circ$$

$$T_{MAX} = \frac{0.5g\sin 40^\circ + 0.1g\cos 40^\circ}{\cos 20^\circ + 0.2\sin 20^\circ}$$

$$= 3.8690...$$

11 T_{MIN} occurs when the particle is on the point of moving down the plane.
At this point, limiting friction $F = \mu R$ acts up the plane:



$R(\nearrow)$

$$T_{\text{MIN}} \cos 20^\circ - 0.5g \sin 40^\circ - F = 0$$

$$T_{\text{MIN}} \cos 20^\circ - 0.5g \sin 40^\circ + \frac{1}{5}\left(0.5g \cos 40^\circ - T \sin 20^\circ\right) = 0$$

$$T_{\text{MIN}} \cos 20^\circ - 0.2T_{\text{MIN}} \sin 20^\circ = 0.5g \sin 40^\circ - 0.1g \cos 40^\circ$$

$$T_{\text{MAX}} = \frac{0.5g \sin 40^\circ - 0.1g \cos 40^\circ}{\cos 20^\circ - 0.2 \sin 20^\circ}$$

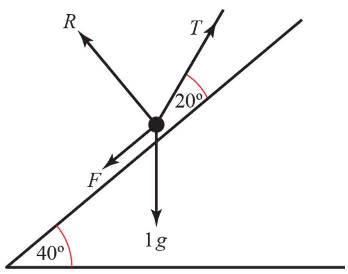
$$= 2.7533...$$

T lies between 2.75 N and 3.87 N (both values to 3s.f.).

12 $R(\nwarrow)$

$R + 10 \sin 20° - g \cos 40° = 0$

$$R = g \cos 40° - 10 \sin 20°$$
$$= 4.087$$



$R(\nearrow)$

$10 \cos 20° - F - g \sin 40° = 0$

$$F = 10 \cos 20° - g \sin 40°$$
$$= 3.0976...$$

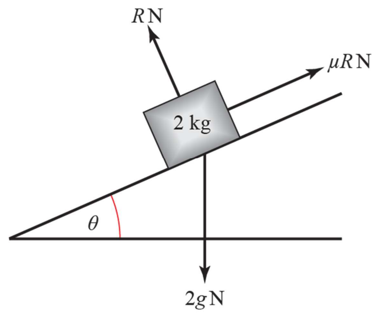
As the friction is limiting, $F = \mu R$

$\therefore \mu = \dfrac{F}{R}$

$= \dfrac{3.0976}{4.087}$

$= 0.758 \quad$ (3 s.f.)

13 $\tan\theta = \frac{3}{4}$ so $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$



Before P is applied, the particle will be on the point of moving down the slope and the limiting frictional force therefore acts up the slope:

$R(\nwarrow)$

$R = 2g\cos\theta$

$R(\nearrow)$

$2g\sin\theta = F_{\text{Max}}$

$2g\sin\theta = \mu R$

$2g\sin\theta = \mu \times 2g\cos\theta$

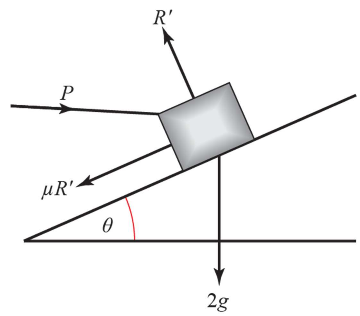
$\mu = \tan\theta$

$\mu = \frac{3}{4}$

After applying the max value of P that will allow the particle to remain in equilibrium, particle is on the point of moving up the slope. Therefore, the frictional force acts down the slope.

$R(\nwarrow)$

$R' = 2g\cos\theta + P\sin\theta$



$R(\nearrow)$

$P\cos\theta = F_{\text{Max}} + 2g\sin\theta$

$P\cos\theta = \mu R' + 2g\sin\theta$

$P\cos\theta = \mu(2g\cos\theta + P\sin\theta) + 2g\sin\theta$

$P(\cos\theta - \mu\sin\theta) = 2g\left(\mu\cos\theta + \sin\theta\right)$

$P\left(\frac{4}{5} - \frac{3}{4}\times\frac{3}{5}\right) = 2\times 9.8\times\left(\frac{3}{4}\times\frac{4}{5} + \frac{3}{5}\right)$

$0.35P = 23.52$

$P = 67.2 \text{ N}$

So max. P is 67.2 N