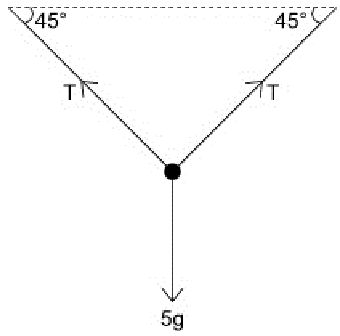
Applications of forces 7B

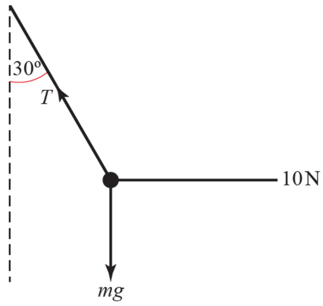
1 From symmetry the tension in both strings is the same.



$R(\uparrow)$

$$T\sin 45° + T\sin 45° - 5g = 0$$
$$\therefore 2T\sin 45° = 5g$$
$$T = \frac{5g}{2\sin 45°}$$
$$= \frac{49\sqrt{2}}{2}$$
$$T = 34.6\text{ N (3 s.f.)}$$

2 **a** Let the tension in the string be T N



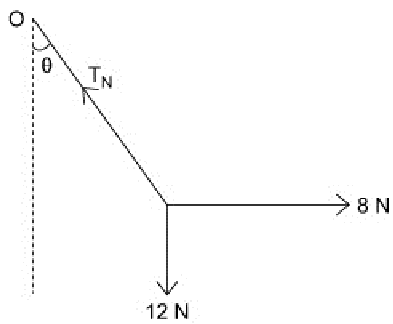
$R(\leftarrow)$

$$T\sin 30° - 10 = 0$$
$$\therefore T = \frac{10}{\sin 30°}$$
$$T = 20\text{ N}$$

b $R(\uparrow)$

$$T\cos 30° - mg = 0$$
$$mg = 20\cos 30° \quad (\text{since } T = 20\text{ N})$$
$$\therefore m = \frac{20\cos 30°}{g}$$
$$= \frac{10\sqrt{3}}{g}$$
$$= 1.8\text{ kg (2 s.f.)}$$

3 Let the tension in the string be T N.



$R(\rightarrow)$

$$8 - T\sin\theta = 0$$
$$\therefore T\sin\theta = 8 \qquad \textbf{(1)}$$

$R(\uparrow)$

$$T\cos\theta - 12 = 0$$
$$\therefore T\cos\theta = 12 \qquad \textbf{(2)}$$

3 **a** Divide equation (**1**) by equation (**2**) to eliminate the tension *T*.

$$\frac{T\sin\theta}{T\cos\theta}=\frac{8}{12}$$

$$\therefore \tan\theta=\frac{2}{3}$$

$$\therefore \theta=33.7° \text{ (3 s.f.)}$$

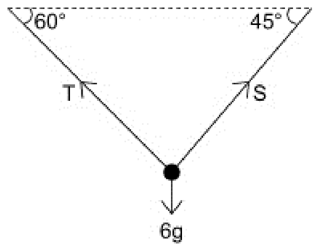
b Substitute into equation (**1**)

$$T\sin 33.7° = 8$$

$$T=\frac{8}{\sin 33.7°}$$

$$=14.4 \text{ (3 s.f.)}$$

4 Let the tension in the strings be *T* N and *S* N as shown in the figure.



$R(\leftarrow)$

$$T\cos 60° - S\cos 45° = 0$$

$$\therefore \frac{T}{2}-\frac{S}{\sqrt{2}}=0$$

$$\therefore T=S\sqrt{2} \qquad \textbf{(1)}$$

$R(\uparrow)$

$$T\sin 60° + S\sin 45° - 6g = 0$$

$$T\frac{\sqrt{3}}{2}+S\frac{1}{\sqrt{2}}=6g \qquad \textbf{(2)}$$

Substitute $T=S\sqrt{2}$ from (**1**) into equation (**2**)

$$S\left(\sqrt{2}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\right)=6g$$

$$S\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)=6g$$

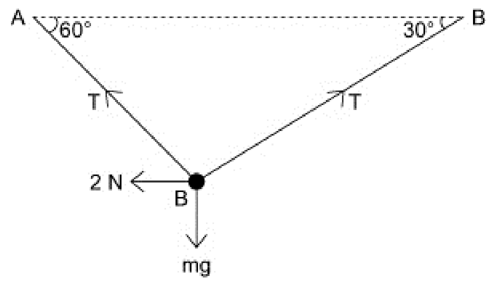
$$S=\frac{6g\sqrt{2}}{\left(\sqrt{3}+1\right)}$$

$$=3g\sqrt{2}\left(\sqrt{3}-1\right)$$

$$=30 \text{ (2 s.f.)}$$

and $T=6g\left(\sqrt{3}-1\right)=43$ (2 s.f.)

5 a Let the tension in the string be T and the mass of the bead be m.



Resolve horizontally first to find T:

$R(\rightarrow)$

$$T\cos 30° - T\cos 60° - 2 = 0$$

$$T(\cos 30° - \cos 60°) = 2$$

$$\therefore T = \frac{2}{\cos 30° - \cos 60°} = \frac{4}{\sqrt{3}-1} = \frac{4\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)} = \frac{4\left(\sqrt{3}+1\right)}{2} = 2\left(\sqrt{3}+1\right) = 5.46\text{ N (3 s.f.)}$$

b $R(\uparrow)$

$$T\sin 60° + T\sin 30° - mg = 0$$

$$mg = T(\sin 60° + \sin 30°)$$

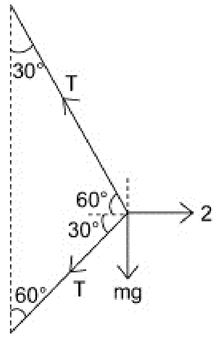
$$m = \frac{2}{g}\left(\sqrt{3}+1\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right) \quad \text{(using } T = 2\left(\sqrt{3}+1\right) \text{ from part } \textbf{a}\text{)}$$

$$= \frac{4+2\sqrt{3}}{g}$$

$$= 0.76\text{kg (2 s.f.)}$$

c Modelling the bead as smooth assumes there is no friction between it and the string.

6 Let the tension in the string be T and the mass of the bead be m.



a Resolve horizontally first to find T.

$R(\rightarrow)$

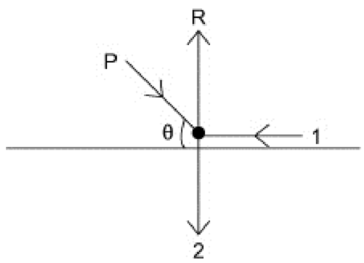
$$2 - T\cos 60° - T\cos 30° = 0$$
$$T(\cos 60° + \cos 30°) = 2$$
$$\therefore T = \frac{2}{\cos 60° + \cos 30°}$$
$$= \frac{4}{1+\sqrt{3}}$$
$$= \frac{4}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \quad \text{(to rationalise the denomiator)}$$
$$= 2\left(\sqrt{3}-1\right)$$
$$= 1.46 \text{ (3 s.f.)}$$

b $R(\uparrow)$

$$T\sin 60° - T\sin 30° - mg = 0$$
$$mg = T(\sin 60° - \sin 30°)$$
$$= 2\left(\sqrt{3}-1\right)\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) \quad \left(\text{using } T = 2\left(\sqrt{3}-1\right) \text{ from } \mathbf{a}\right)$$
$$= \left(\sqrt{3}-1\right)^2$$
$$= 4 - 2\sqrt{3}$$
$$m = \frac{\left(4-2\sqrt{3}\right)}{g}$$
$$= 0.055\,\text{kg} = 55\,\text{g}$$

7 $\tan\theta = \dfrac{12}{5} \Rightarrow \sin\theta = \dfrac{12}{13}$ and $\cos\theta = \dfrac{5}{13}$

Let the normal reaction be R N.



a $R(\rightarrow)$

$$P\cos\theta - 1 = 0$$
$$\therefore P = \frac{1}{\cos\theta}$$
$$= \frac{13}{5}$$
$$P = 2.6$$

7 b $R(\uparrow)$

$R - P\sin\theta - 2 = 0$

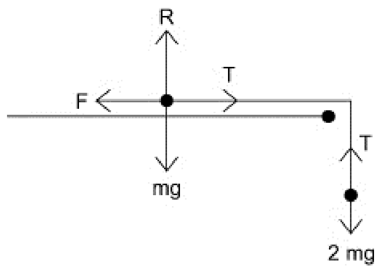
$\therefore R = P\sin\theta + 2$

$= 2.6 \times \frac{12}{13} + 2$

$= 2.4 + 2$

$= 4.4$

8 a Consider the particle of mass $2m$ kg first, as it has only two forces acting on it. This enables you to find the tension.



$R(\uparrow)$

$T - 2mg = 0$

$\therefore T = 2mg$

Consider the particle of mass m kg:

$R(\rightarrow)$

$T - F = 0$

$\therefore F = T = 2mg$

$= 19.6m$

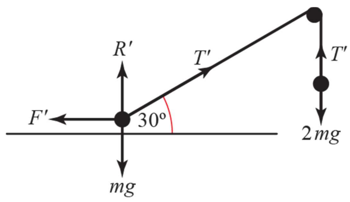
$R(\uparrow)$

$R - mg = 0$

$\therefore R = mg$

$= 9.8m$

b Let T' be the new tension in the string.



Consider the particle of mas $2m$ kg:

$R(\uparrow)$: $T' = 2mg$

Consider the particle of mass m kg:

$R(\rightarrow)$

$T'\cos 30° - F' = 0$

$\therefore F' = 2mg \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}mg$

$= 17m$ (2 s.f.)

$R(\uparrow)$

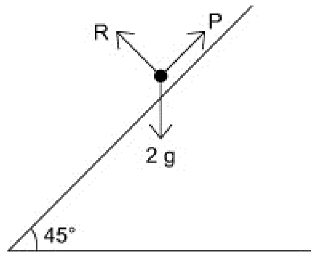
$R' + T'\sin 30 - mg = 0$

$\therefore R' = mg - T'\sin 30$

$= mg - 2mg \times \frac{1}{2}$ (using $T' = 2mg$)

$= 0$

9 Let the normal reaction be R N.



$R(\nearrow)$:

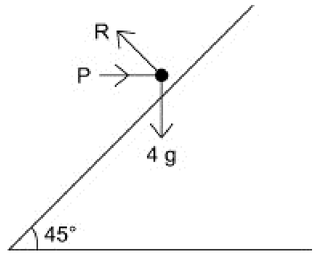
$P - 2g\sin 45° = 0$

$\therefore P = 2g\sin 45°$

$= g\sqrt{2}$

$= 14\text{ N}$ (2 s.f.)

10 Let the normal reaction be R N.



$R(\nearrow)$:

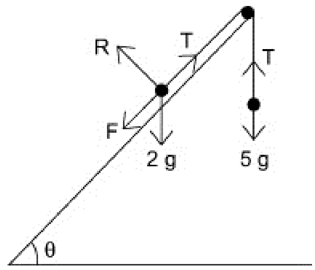
$P\cos 45° - 4g\sin 45° = 0$

$$\therefore P = \frac{4g\sin 45°}{\cos 45°}$$

$= 4g$

$= 39$ (2 s.f.)

11 a Let the normal reaction between the particle P and the plane be R N.
Let the tension in the string be T N.



Consider first the 5 kg mass.

$R(\uparrow)$

$T - 5g = 0$

$\therefore T = 5g$

Consider the 2 kg mass.

$R(\nwarrow)$

$R - 2g\cos\theta = 0$

$$R = 2g \times \frac{4}{5}$$

$$= \frac{8g}{5}$$

$= 16\text{ N}$ (2 s.f.)

b $R(\nearrow)$

$T - F - 2g\sin\theta = 0$

$F = T - 2g\sin\theta$

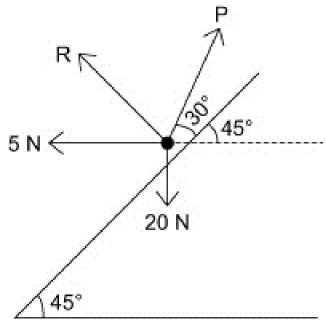
$= 5g - 2g \times \frac{3}{5}$ (using $T = 5g$ from above)

$$= \frac{19g}{5}$$

$= 37\text{ N}$ (2 s.f.)

c Assuming the pulley is smooth means there is no friction between it and the string.

12 Let the normal reaction be R N.



First, resolve along the plane to find P as it is the only unknown when resolving in that direction.

$R(\nearrow)$

$P\cos 30° - 5\cos 45° - 20\sin 45° = 0$

$$\begin{aligned}\therefore P &= \frac{5\cos 45° + 20\sin 45°}{\cos 30°}\\ &= \left(5\times\frac{\sqrt{2}}{2} + 20\times\frac{\sqrt{2}}{2}\right)\times\frac{2}{\sqrt{3}}\\ &= \frac{25\sqrt{2}}{\sqrt{3}}\\ &= \frac{25\sqrt{6}}{3}\\ &= 20.4 \text{ (3 s.f.)}\end{aligned}$$

$R(\nwarrow)$

$R + P\sin 30° + 5\sin 45° - 20\cos 45° = 0$

$$R = 20\cos 45° - 5\sin 45° - P\sin 30° \quad (\text{as } P = \frac{25\sqrt{6}}{3})$$

$$\begin{aligned}R &= \frac{15}{\sqrt{2}} - \frac{25\sqrt{6}}{6}\\ &= \frac{45\sqrt{2} - 25\sqrt{6}}{6}\\ &= 0.400 \text{ (3 s.f.)}\end{aligned}$$