Applications of forces 7A

1 a i $Q - 5\cos 30° = 0$

ii $P - 5\sin 30° = 0$

iii $Q = 5\cos 30° = \frac{5\sqrt{3}}{2} = 4.33\text{N}$ (3 s.f.)

$P = 5\sin 30° = 2.5\text{N}$

Give exact answers using $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$ or give decimal answers using your calculator.

b i $P\cos\theta + 8\sin 40° - 7\cos 35° = 0$

ii $P\sin\theta + 7\sin 35° - 8\cos 40° = 0$

iii $P\cos\theta = 7\cos 35° - 8\sin 40°$

$= 0.5918 \quad (\mathbf{1})$

$P\sin\theta = 8\cos 40° - 7\sin 35°$

$= 2.113 \quad (\mathbf{2})$

Divide equation (**2**) by equation (**1**)

Use $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$ to eliminate P from the equations obtained in **i** and **ii.**

$$\frac{P\sin\theta}{P\cos\theta} = \frac{8\cos 40° - 7\sin 35°}{7\cos 35° - 8\sin 40°}$$

$$\therefore \tan\theta = \frac{2.113}{0.5918}$$

$$= 3.57$$

$$\therefore \theta = 74.4° \text{ (3 s.f.)}$$

Substitute θ into equation (**1**)

$$P\cos 74.3569° = 0.5918$$

$$\therefore P = \frac{0.5918}{\cos 74.3569°}$$

$$= 2.19 \text{ (3 s.f.)}$$

c i $9 - P\cos 30° = 0$

ii $Q + P\sin 30° - 8 = 0$

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1 c iii Using result from part **i**,

Use part **i** to find P, then substitute into **ii** to find **i**.

$$P = \frac{9}{\cos 30°}$$
$$= 9 \times \frac{2}{\sqrt{3}}$$
$$= \frac{9 \times 2}{\sqrt{3}}$$
$$= \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{18\sqrt{3}}{3}$$
$$= 6\sqrt{3}$$
$$= 10.4\,\text{N} \quad (3 \text{ s.f.})$$

$\cos 30° = \frac{\sqrt{3}}{2}$

Substitute into result from part **ii**

$$Q + 6\sqrt{3}\sin 30° - 8 = 0$$
$$\therefore Q = 8 - 6\sqrt{3} \times \frac{1}{2}$$
$$= 8 - 3\sqrt{3}$$
$$= 2.80\,\text{N} \quad (3 \text{ s.f.})$$

$\sin 30° = \frac{1}{2}$

d i $Q\cos 60° + 6\cos 45° - P = 0$

Use angles on a straight line to find Q makes an angle of 60° with the x-axis.

ii $Q\sin 60° - 6\sin 45° = 0$

iii Using result from part **ii**,

$\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 60° = \frac{1}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$

$$Q = \frac{6\sin 45°}{\sin 60°}$$
$$= 6 \times \frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}}$$
$$= \frac{12}{\sqrt{6}}$$
$$= \frac{12}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$
$$= 2\sqrt{6}$$
$$= 4.90\,\text{N} \quad (3 \text{ s.f.})$$

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1 d iii Substitute into result from part **i**:

$$2\sqrt{6}\times\frac{1}{2}+6\times\frac{1}{\sqrt{2}}-P=0$$

$$\therefore P=\sqrt{6}+\frac{6}{\sqrt{2}}$$

$$=\sqrt{6}+\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\sqrt{6}+3\sqrt{2}$$

$$=6.69\,\text{N}\ \ (3\text{ s.f.})$$

e i $6\cos 45°-2\cos 60°-P\sin\theta=0$

ii $6\sin 45°+2\sin 60°-P\cos\theta-4=0$

iii Using result from **i**:

$P\sin\theta=6\cos 45°-2\cos 60°$ **(1)**

Using result from **ii**:

$P\cos\theta=6\sin 45°+2\sin 60°-4$ **(2)**

(1) ÷ **(2)**:

$$\frac{P\sin\theta}{P\cos\theta}=\frac{6\cos 45°-2\cos 60°}{6\sin 45°+2\sin 60°-4}$$

$$\therefore \tan\theta=\frac{3.24264}{1.97469...}$$

$$=1.642$$

$$\therefore \theta=58.7°\ (3\text{ s.f.})$$

Use $\frac{P\sin\theta}{P\cos\theta}=\tan\theta$ to eliminate P from the equations after resolving

Substitute into (**1**):

$P\sin 58.65°=6\cos 45°-2\cos 60°$

$$\therefore P=\frac{3.24264}{\sin 58.65°}$$

$$P=3.80\,\text{N}\ \ (3\text{ s.f.})$$

f i $9\cos 40°+3-P\cos\theta-8\sin 20°=0$

ii $P\sin\theta+9\sin 40°-8\cos 20°=0$

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1 f iii Using result from **i**:

$P\cos\theta = 9\cos 40° + 3 - 8\sin 20°$ **(1)**

Using result from **ii**:

$P\sin\theta = 8\cos 20° - 9\sin 40°$ **(2)**

$(\mathbf{2}) \div (\mathbf{1})$:

$$\frac{P\sin\theta}{P\cos\theta} = \frac{8\cos 20° - 9\sin 40°}{9\cos 40° + 3 - 8\sin 20°}$$

$$\therefore \tan\theta = \frac{1.732}{7.158}$$

$$= 0.242$$

$$\therefore \theta = 13.6° \text{ (3 s.f.)}$$

Substitute into (**2**):

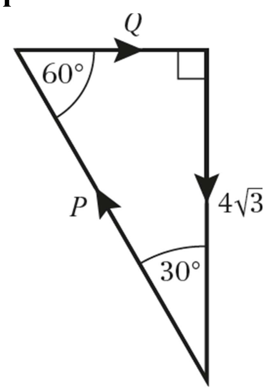
$$P\cos 13.6° = 9\cos 40° + 3 - 8\sin 20°$$

$$= 7.158$$

$$\therefore P = \frac{7.158}{\cos 13.6°}$$

$$= 7.36 \text{ (3 s.f.)}$$

2 a i



ii $R(\rightarrow)$, $Q - P\cos 60° = 0$ **(1)**

$R(\uparrow)$, $P\sin 60° - 4\sqrt{3} = 0$ **(2)**

From (**2**):

$$P = \frac{4\sqrt{3}}{\sin 60°}$$

$$= 4\sqrt{3} \times \frac{2}{\sqrt{3}}$$

$$= 8\,\text{N}$$

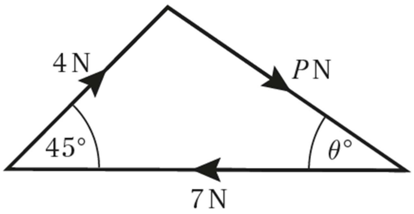
$\sin 60° = \frac{\sqrt{3}}{2}$

2 a ii Substitute $P = 8$ N into (**1**):

$$Q = 8\cos 60°$$
$$= 8 \times \frac{1}{2}$$
$$= 4\text{ N}$$

$\cos 60° = \frac{1}{2}$

b i



ii $R(\rightarrow)$, $4\cos 45° + P\cos\theta - 7 = 0$

$\therefore P\cos\theta = 7 - 4\cos 45°$ (**1**)

$R(\uparrow)$, $4\sin 45° - P\sin\theta = 0$

$\therefore P\sin\theta = 4\sin 45°$ (**2**)

(**2**) ÷ (**1**):

$$\frac{P\sin\theta}{P\cos\theta} = \frac{4\sin 45°}{7 - 4\cos 45°}$$
$$\therefore \tan\theta = \frac{2.828}{4.172}$$
$$= 0.678$$
$$\therefore \theta = 34.1° \text{ (3 s.f.)}$$

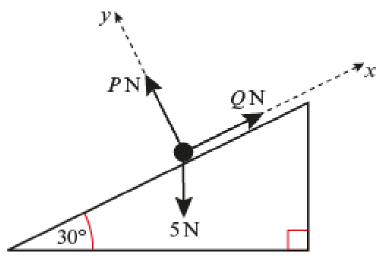
Use $\frac{P\sin\theta}{P\cos\theta} = \tan\theta$ to eliminate P from the equations after resolving

Substitute $\theta = 34.1°$ into equation (**2**):

$$P = \frac{2.828}{\sin 34.1°}$$
$$P = 5.04\text{ N (3 s.f.)}$$

3 a $R(\rightarrow)$, $P = 5\cos 30° = 4.33\text{ N}$

$R(\uparrow)$, $Q = 5\sin 30° = 2.5\text{ N}$



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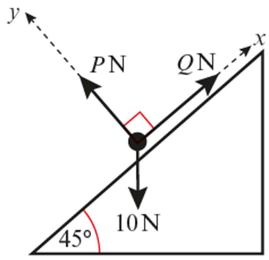
3 b $R(\nearrow)$, $Q - 10\sin 45° = 0$ **(1)**

$R(\nwarrow)$, $P - 10\cos 45° = 0$ **(2)**



From (**2**), $P = 10\cos 45°$

$= 5\sqrt{2}$

$= 7.07\text{ N}$ (3 s.f.)

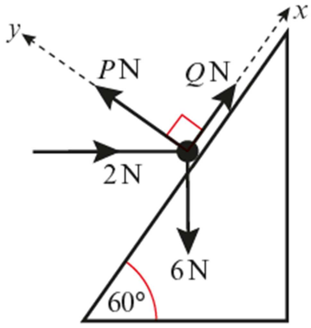
From (**1**), $Q = 10\sin 45°$

$= 5\sqrt{2}$

$= 7.07\text{ N}$ (3 s.f.)

c $R(\nearrow)$, $Q + 2\cos 60° - 6\sin 60° = 0$ **(1)**

$R(\nwarrow)$, $P - 2\sin 60° - 6\cos 60° = 0$ **(2)**



From (**2**), $P = 2\sin 60° + 6\cos 60°$

$P = 4.73$ (3 s.f.)

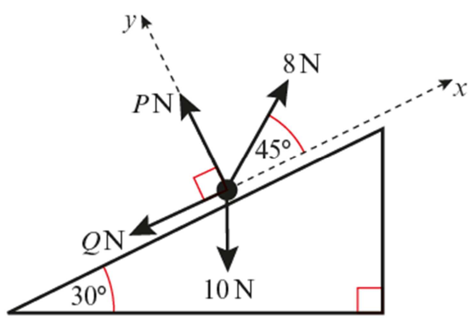
From (**1**), $Q = 6\sin 60° - 2\cos 60°$

$Q = 4.20$ (3 s.f.)

You may give your answers as exact answers using surds as $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$, and $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ or you may give answers to 3 significant figures, using a calculator.

d $R(\nearrow)$, $8\cos 45° - 10\sin 30° - Q = 0$ **(1)**

$R(\nwarrow)$, $P + 8\sin 45° - 10\cos 30° = 0$ **(2)**



From (**2**), $P = 10\cos 30° - 8\sin 45°$

$= 5\sqrt{3} - 4\sqrt{2}$

$= 3.00\text{ N}$ (3 s.f.)

From (**1**), $Q = 8\cos 45° - 10\sin 30°$

$= 4\sqrt{2} - 5$

$= 0.657\text{ N}$ (3 s.f.)

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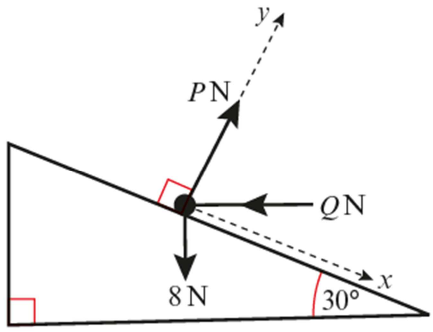
3 e $R(\searrow)$, $8\sin 30° - Q\cos 30° = 0$ **(1)**

$R(\nearrow)$, $P - Q\sin 30° - 8\cos 30° = 0$ **(2)**



From (**1**), $Q = \dfrac{8\sin 30°}{\cos 30°}$

$= 8\tan 30°$

$= \dfrac{8\sqrt{3}}{3}$

$= 4.62\,\text{N}$ (3 s.f.)

$\tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

Substitute into (**2**):

$P = Q\sin 30° + 8\cos 30°$

$= \dfrac{8\sqrt{3}}{3} \times \dfrac{1}{2} + 8 \times \dfrac{\sqrt{3}}{2}$

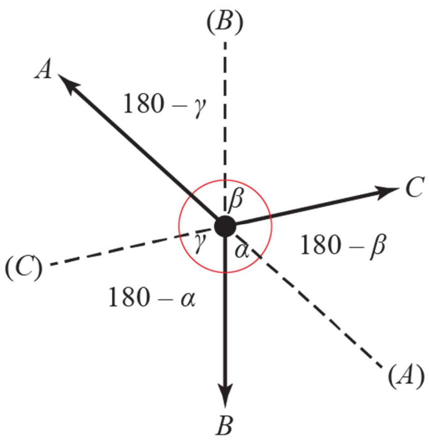
$= \dfrac{4\sqrt{3}}{3} + 4\sqrt{3}$

$= \dfrac{16\sqrt{3}}{3}$

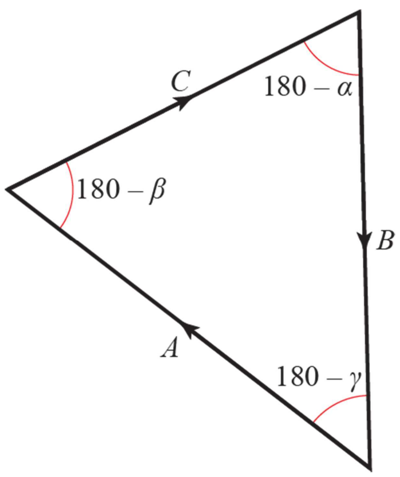
$= 9.24\,\text{N}$ (3 s.f.)

$\sin 30° = \dfrac{1}{2}$, $\cos 30° = \dfrac{\sqrt{3}}{2}$

Challenge
By extending the line of action of each force backwards through the centre, we can find the acute angles between the lines of action of each of the forces.



Since the body is in equilibrium, the forces A, B and C form a closed triangle as shown below:



Using the sine rule:

$$\frac{A}{\sin(180-\alpha)} = \frac{B}{\sin(180-\beta)} = \frac{C}{\sin(180-\gamma)}$$

But, for any angle θ, $\sin(180-\theta) = \sin\theta$

Hence,

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$