Projectiles 6C

Unless otherwise stated, the positive direction is upwards.

1 Resolving the initial velocity vertically:

$R(\uparrow),\ u_y = 35\sin 60°$

$u = 35\sin 60°,\quad v = 0,\quad a = -9.8,\quad t = ?$

$$v = u + at$$
$$0 = 35\sin 60° - 9.8t$$
$$t = \frac{35\sin 60°}{9.8}$$
$$= 3.092...$$

The time the particle takes to reach its greatest height is 3.1 s (2 s.f.).

2 Resolving the initial velocity vertically:

$R(\uparrow),\ u_y = 18\sin 40°$

$u = 18\sin 40°,\quad a = -9.8,\quad t = 2,\quad s = ?$

$$s = ut + \frac{1}{2}at^2$$
$$= 18\sin 40° \times 2 - 4.9 \times 2^2$$
$$= 3.540...$$

The height of the ball above the ground 2 s after projection is $(5 + 3.5)\,\text{m} = 8.5\,\text{m}$ (2 s.f.).

3 Taking the downwards direction as positive.
Resolving the initial velocity horizontally and vertically:

$R(\rightarrow)\ u_x = 32\cos 10°$

$R(\uparrow)\ u_y = 32\sin 10°$

a $R(\uparrow)$

$u = 32\sin 10°,\quad a = -9.8,\quad t = 2.5,\quad s = ?$

$$s = ut + \frac{1}{2}at^2$$
$$= 32\sin 10° \times 2.5 + 4.9 \times 2.5^2$$
$$= 44.517...$$

The stone is projected from 44.5 m above the ground.

b $R(\rightarrow)$

$u = 32\cos 10°,\quad t = 2.5,\quad s = ?$

$$s = vt$$
$$= 2.5 \times 32\cos 10°$$
$$= 78.785...$$

The stone lands 78.8 m away from the point on the ground vertically below where it was projected from.

4 Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\ u_x = 150\cos 10°$

$R(\uparrow)\ u_y = 150\sin 10°$

a $R(\uparrow)$

$u = 150\sin 10°,\quad v = 0,\quad a = -9.8,\quad t = ?$

$$v = u + at$$
$$0 = 150\sin 10° - 9.8t$$
$$t = \frac{150\sin 10°}{9.8}$$
$$= 2.657...$$

The time taken to reach the projectile's highest point is 2.7 s (2 s.f.).

b First, resolve vertically to find the time of flight:

$R(\uparrow)\ u = 150\sin 10°,\quad s = 0,\quad a = -9.8,\quad t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$0 = 150t\sin 10° - 4.9t^2$$
$$0 = t(150\sin 10° - 4.9t)$$
$$t = 0 \text{ s} \quad \text{or} \quad t = \frac{150\sin 10°}{4.9}$$
$$= 5.316... \text{ s}$$

[Note that, alternatively, you can consider the symmetry of the projectile's path:
The time of flight is twice as long as the time it takes to reach the highest point, that is

$$t = 2.657... \times 2$$
$$= 5.315 \text{ s}]$$

$R(\rightarrow)$

$u = 150\cos 10°,\ t = 5.315,\ s = ?$

$$s = ut$$
$$= 150\cos 10° \times 5.315$$
$$= 785.250...$$

The range of the projectile is 790 m (2 s.f.).

5 Resolving the initial velocity horizontally and vertically:

$R(\rightarrow)\ u_x = 20\cos 45° = 10\sqrt{2}$

$R(\uparrow)\ u_y = 20\sin 45° = 10\sqrt{2}$

a $R(\uparrow)$

$u = 10\sqrt{2},\ v = 0,\ a = -9.8,\ s = ?$

$$v^2 = u^2 + 2as$$
$$0 = 200 - 19.6s$$
$$s = \frac{200}{19.6}$$
$$= 10.204...$$

The greatest height above the plane reached by the particle is 10 m (2 s.f.).

b To find the time taken to move from O to X, first find the time of flight:

$R(\uparrow)$

$u = 10\sqrt{2},\ s = 0,\ a = -9.8,\ t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$0 = 10\sqrt{2}t - 4.9t^2$$
$$0 = t\left(10\sqrt{2} - 4.9t\right)$$
$$t = \frac{10\sqrt{2}}{4.9} \qquad \text{(ignore } t = 0\text{)}$$
$$= 2.886...\text{ s}$$

$R(\rightarrow)$

$u = 10\sqrt{2},\ t = 2.886..,\ s = ?$

$$s = ut$$
$$= 10\sqrt{2} \times 2.886...$$
$$= 40.86...$$

$\Rightarrow OX = 41\text{ m}\ \ (2\text{ s.f.})$

6 $\sin\theta = \frac{4}{5} \Rightarrow \cos\theta = \frac{3}{5}$

Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\ u_x = 24\cos\theta = 14.4$

$R(\uparrow)\ u_y = 24\sin\theta = 19.2$

a $R(\uparrow)$

$u = 19.2,\ s = 0,\ a = -9.8,\ t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$0 = 19.2t - 4.9t^2$$
$$= t(19.2 - 4.9t)$$
$$t = \frac{19.2}{4.9} \qquad \text{(ignore } t = 0\text{)}$$
$$= 3.918...$$

The time of flight of the ball is 3.9 s (2 s.f.).

b $R(\rightarrow)$

$u = 14.4,\ t = 3.918,\ s = ?$

$$s = ut$$
$$= 14.4 \times 3.918...$$
$$= 56.424...$$

$AB = 56\,\text{m}$ (2 s.f.)

7 Resolving the initial velocity vertically,

$u_y = 21\sin\alpha$

$R(\uparrow)$: $u = 21\sin\alpha,\ v = 0,\ a = -9.8,\ s = 15$

$$v^2 = u^2 + 2as$$
$$0 = (21\sin\alpha)^2 - 2\times 9.8 \times 15$$
$$441\sin^2\alpha = 294$$
$$\sin^2\alpha = \frac{294}{441} = \frac{2}{3}$$
$$\sin\alpha = \sqrt{\frac{2}{3}} = 0.816$$
$$\alpha = 54.736°$$
$$= 55° \quad \text{(nearest degree)}$$

8 a $R(\rightarrow)$

$u = 12,\ t = 3,\ s = ?$

$$s = ut = 12\times 3 = 36$$

$R(\uparrow)$

$u = 24,\ a = -g,\ t = 3,\ s = ?$

$$s = ut + \frac{1}{2}at^2 = 24\times 3 - 4.9\times 9 = 27.9$$

The position vector of P after 3 s is $(36\mathbf{i} + 27.9\mathbf{j})$ m

b $R(\rightarrow)$ $u_x = 12$, throughout the motion

$R(\uparrow)$ $v = u + at$

$$v_y = 24 - 9.8\times 3 = -5.4$$

Let the speed of P after 3 s be V m s^{-1}

$$V^2 = u_x^2 + v_y^2 = 12^2 + (-5.4)^2 = 173.16$$

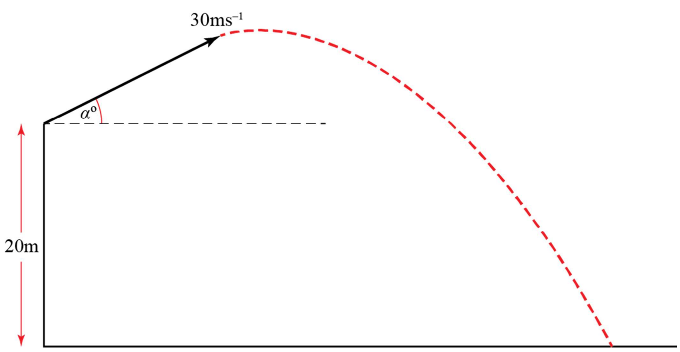
$$V = \sqrt{173.16} = 13.159...$$

The speed of P after 3 s is 13 m s^{-1} (2 s.f.).

9 Let α be the angle of projection above the horizontal. Resolving the initial velocity horizontally and vertically.

$R(\rightarrow)$ $u_x = 30\cos\alpha$

$R(\uparrow)$ $u_y = 30\sin\alpha$



a $R(\uparrow)$

$u = 30\sin\alpha,\ s = -20,\ a = -9.8,\ t = 3.5$

$$s = ut + \frac{1}{2}at^2$$

$$-20 = 30\sin\alpha\times 3.5 - 4.9\times 3.5^2$$

$$\sin\alpha = \frac{4.9\times 3.5^2 - 20}{30\times 3.5} = 0.381190...$$

$$\alpha = 22.407...^\circ$$

The angle of projection of the stone is 22° (2 s.f.) above the horizontal.

9 **b** $R(\rightarrow)$

$u = 30\sin 22.407...°, \quad t = 3.5, \quad s = ?$

$$s = ut$$
$$= 30\sin 22.407...° \times 3.5$$
$$= 97.072...$$

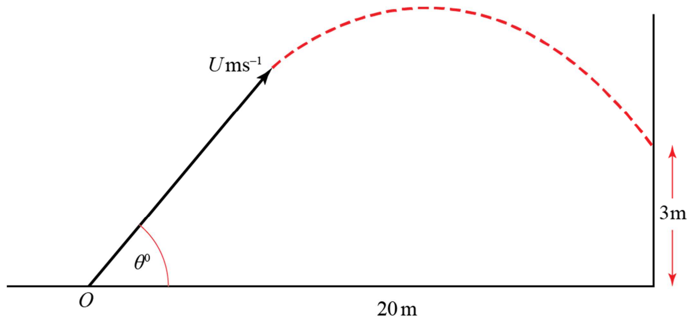
The horizontal distance from the window to the point where the stone hits the ground is 97 m (2 s.f.).

10 $\tan\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}$

Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\ u_x = U\cos\theta = \frac{4U}{5}$

$R(\uparrow)\ u_y = U\sin\theta = \frac{3U}{5}$



a $R(\rightarrow)$

$u = \frac{4U}{5}, \quad s = 20, \quad t = ?$

$$s = ut$$
$$20 = \frac{4tU}{5}$$
$$t = \frac{25}{U} \qquad \textbf{(1)}$$

$R(\uparrow)$

$u = \frac{3U}{5}, \quad s = 3, \quad a = -g, \quad t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$3 = \frac{3U}{5} \times t - 4.9t^2 \qquad \textbf{(2)}$$

Substituting $t = \frac{25}{U}$ from (**1**) into (**2**):

$$3 = \frac{3U}{5} \times \frac{25}{U} - 4.9 \times \frac{25^2}{U^2}$$
$$3 = 15 - \frac{3062.5}{U^2}$$
$$\Rightarrow U^2 = \frac{3062.5}{12}$$
$$= 255.208...$$
$$U = 15.975...$$
$$= 16 \text{ (2 s.f.)}$$

10 b $R(\rightarrow)$

$t = \frac{25}{U}$

$= \frac{25}{15.975...}$

$= 1.5649...$

The time from the instant the ball is thrown to the instant that it strikes the wall is 1.6 s (2 s.f.).

11 a Resolve vertically for motion between A and B:

$R(\uparrow)$

$u = 4u,\quad s = 20 - 12 = -8,\quad a = -g,\quad t = 4$

$$s = ut + \frac{1}{2}at^2$$

$$-8 = 4u \times 4 - 4.9 \times 4^2$$

$$u = \frac{4.9 \times 4^2 - 8}{16}$$

$$= 4.4$$

b Resolve horizontally for motion between A and B:

$R(\rightarrow)$

$u = 5u = 5 \times 4.4 = 22,\quad t = 4,\quad s = k$

$$s = ut$$

$$k = 22 \times 4$$

$$= 88$$

c $u_x = 22\ \text{ms}^{-1}$ throughout the motion.

Resolve vertically to find v_y at C:

$R(\uparrow)$

$u = 4 \times 4.4,\quad a = -g,\quad s = -20,\quad v = ?$

$$v^2 = u^2 + 2as$$

$$v_y^2 = (4 \times 4.4)^2 + 2 \times (-9.8) \times (-20)$$

$$= 16 \times 4.4^2 + 392$$

$$= 701.76$$

Let θ be angle that the path of P makes with the x-axis as it reaches C.

$$\tan\theta = \frac{v_y}{u_x}$$

$$= \frac{\sqrt{701.76}}{22}$$

$$= 1.204...$$

$$\theta = 50.291...$$

The angle the path of P makes with the x-axis as it reaches C is $50°$ (2 s.f.).

12 Take downwards as the positive direction.
Resolving the initial velocity horizontally and vertically:

$R(\rightarrow)\ u_x = 30\cos 15°$

$R(\uparrow)\ u_y = 30\sin 15°$

a $R(\downarrow)$

$u = 30\sin 15°,\quad s = 14,\quad a = 9.8,\quad t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$14 = 30t\sin 15° + 4.9t^2$$

$$4.9t^2 + 30t\sin 15° - 14 = 0$$

Using the formula for solving the quadratic,

$$t = \frac{-30\sin 15° \sqrt{(900\sin^2 15 + 4\times 14\times 4.9)}}{9.8}$$

$= 1.074...$ (the negative solution can be ignored)

The time the particle takes to travel from A to B is 1.1 s (2 s.f.).

b $R(\rightarrow)$

$u = 30\cos 15°,\quad t = 1.074..,\quad s = ?$

$$s = ut$$

$$= (30\cos 15°)\times 1.074$$

$$= 31.136...$$

$$AB^2 = 14^2 + (31.136...)^2$$

$$= 1165.456...$$

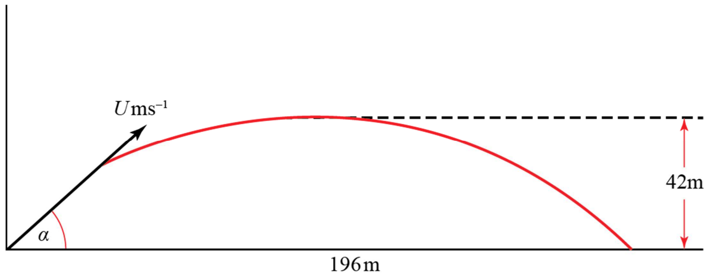
$$AB = 34.138...$$

The distance AB is 34 m (2 s.f.).

13 Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\ u_x = U\cos\alpha$

$R(\uparrow)\ u_y = U\sin\alpha$



To get one equation in U and α, resolve vertically when particle reaches its maximum height of 42 m:

$R(\uparrow)$

$u = U\sin\alpha,\quad a = -g,\quad s = 42,\quad v = 0$

$$v^2 = u^2 + 2as$$

$$0 = U^2\sin^2\alpha - 2g\times 42$$

$$U^2\sin^2\alpha = 84g \qquad \textbf{(1)}$$

To get a second equation in U and α, we must resolve both horizontally and vertically to find expressions for t when the particle hits the ground. We can then equate these expressions and eliminate t:

$R(\rightarrow)$

$u = U\cos\alpha,\quad s = 196,\quad t = ?$

$$s = ut$$

$$196 = U\cos\alpha\times t$$

$$t = \frac{196}{U\cos\alpha} \qquad (*)$$

$R(\uparrow)$

$u = U\sin\alpha,\quad a = -g,\quad s = 0,\quad t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = Ut\sin\alpha - \frac{1}{2}gt^2$$

$$= t\left(U\sin\alpha - \frac{1}{2}gt\right)$$

$$\frac{1}{2}gt = U\sin\alpha \qquad (\text{ignore } t = 0)$$

$$t = \frac{2U\sin\alpha}{g} \qquad (**)$$

(*) = (**):

$$\frac{196}{U\cos\alpha} = \frac{2U\sin\alpha}{g}$$

$$U^2\sin\alpha\cos\alpha = 98g \qquad \textbf{(2)}$$

Now we have two equations in U and α, (**1**) and (**2**), that we can solve simultaneously.

(**1**) ÷ (**2**):

13 (cont.)

$$\frac{U^2\sin^2\alpha}{U^2\sin\alpha\cos\alpha}=\frac{84g}{98g}$$

$$\tan\alpha=\frac{6}{7}$$

$$\alpha=40.6° \text{ (3 s.f.)}$$

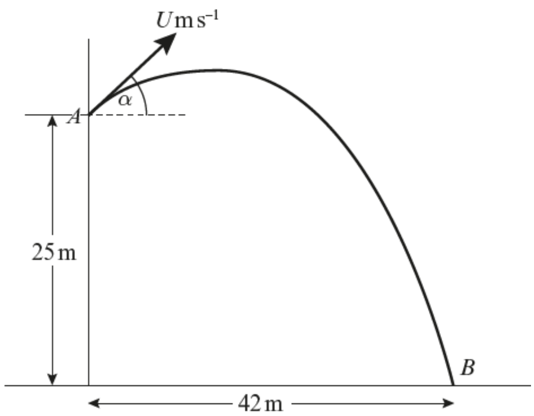
Sub $\alpha=40.6°$ in (1):

$U\sin 40.6°=\sqrt{84g}$ (discard the negative square root as U is a scalar, so must be positive)

$$U=\frac{\sqrt{84\times 9.8}}{\sin 40.6°}$$

$$=44 \quad \text{(2 s.f.)}$$

14 $\tan\alpha=\frac{5}{12}$ so $\sin\alpha=\frac{5}{13}$ and $\cos\alpha=\frac{12}{13}$



R(→): $u_x=U\cos\alpha=\frac{12}{13}U$

R(↑): $u_y=U\sin\alpha=\frac{5}{13}U$

a Resolve horizontally to find time at which particle hits the ground:

R(→): $v=u_x=\frac{12}{13}U$ ms^{-1}, $s=42$ m, $t=?$

$$s=vt$$

$$42=\frac{12}{13}Ut$$

$$t=\frac{13\times 42}{12U}$$

$$=\frac{91}{2U}$$

Resolve vertically with $t=\frac{91}{2U}$:

R(↑): $u_y=\frac{5}{13}U$, $t=\frac{91}{2U}$, $a=g=-10$, $s=-25$

$$s=ut+\frac{1}{2}at^2$$

$$-25=\left(\frac{5}{13}U\times\frac{91}{2U}\right)+\frac{1}{2}\left(-10\times\left(\frac{91}{2U}\right)^2\right)$$

$$-25=\frac{35}{2}-5\left(\frac{91}{2U}\right)^2$$

14 a (cont.)

$$\frac{85}{2} = 5\left(\frac{91}{2U}\right)^2$$

$$\frac{85}{10} = \left(\frac{91}{2U}\right)^2$$

$$85 \times 4U^2 = 10 \times 91^2$$

$$U = \sqrt{\frac{82810}{340}}$$

$$= 15.606...$$

The speed of projection is 15.6 ms^{-1} (3s.f.).

b From **a**:

$$t = \frac{91}{2U}$$

$$= \frac{91}{2 \times 15.606...}$$

$$= 2.9154...$$

The object takes 2.92 s (3s.f.) to travel from A to B.

c At 12.4 m above the ground:

$v_x = u_x = \frac{12}{13}U$ ms^{-1} and

v_y is found by resolving vertically with $s = -25 + 12.4 = -12.6$ m

R(↑): $u_y = \frac{5}{13}U$, $a = g = -10$, $s = -12.6$ m, $v = v_y$

$$v^2 = u^2 + 2as$$

$$v_y^{\ 2} = \left(\frac{5}{13}U\right)^2 + 2(-10)(-12.6)$$

$$v_y^{\ 2} = \left(\frac{5}{13}U\right)^2 + 252$$

The speed at 12.4 m above the ground is given by:

$$v^2 = v_x^2 + v_y^2$$

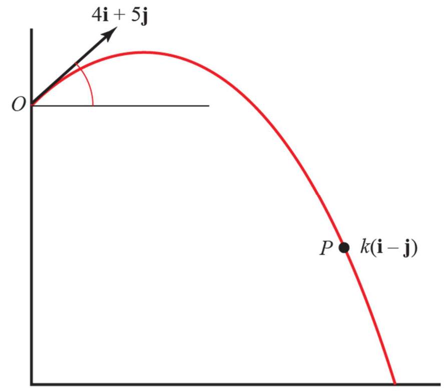
$$v^2 = \left(\frac{12}{13}U\right)^2 + \left(\frac{5}{13}U\right)^2 + 252$$

$$v^2 = U^2 + 252$$

$$v = \sqrt{15.606...^2 + 252}$$

$$v = 22.261...$$

The speed of the object when it is 12.4 m above the ground is 22.3 ms^{-1} (3s.f).



15 a First, resolve horizontally to find the time at which object reaches P:

R(→): $v = u_x = 4$, $s = k$, $t = ?$

$$s = vt$$
$$k = 4t$$
$$t = \frac{k}{4}$$

Now resolve vertically at the instant when object reaches P:

R(↑): $u = u_y = 5$, $t = \frac{k}{4}$, $a = g = -9.8$, $s = -k$

$$s = ut + \frac{1}{2}at^2$$
$$-k = \frac{5k}{4} + \frac{1}{2}\left(-9.8 \times \frac{k^2}{16}\right)$$
$$\frac{9}{4} = 4.9\frac{k}{16} \qquad \text{(We have divided through by } k, \text{ since } k > 0)$$
$$k = \frac{4 \times 9}{4.9}$$
$$k = 7.3469...$$

The value of k is 7.35 (3s.f.).

15 b i At P:

$$v_x = u_x = 4\text{ms}^{-1}$$

v_y is found by resolving vertically with $s = -k = -7.3469...$

R(↑): $u_y = 5$, $a = g = -9.8$, $s = -k$, $v = v_y$

$$v^2 = u^2 + 2as$$
$$v_y^{\ 2} = 5^2 + 2(-9.8)(-k)$$
$$v_y^{\ 2} = 25 + 19.6k$$

The speed at P is given by:

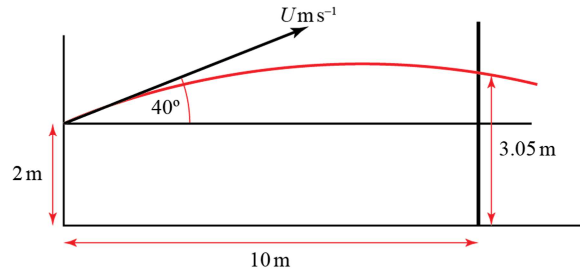
$$v^2 = v_x^2 + v_y^2$$
$$v^2 = 4^2 + 25 + 19.6k$$
$$v^2 = 41 + (19.6 \times 7.3469...)$$
$$v = \sqrt{185}$$
$$v = 13.601...$$

The speed of the object at P is 13.6 ms^{-1} (3s.f.).

15 b ii The object passes through P at an angle α where:

$$\cos\alpha = \frac{v_x}{v} \qquad \left(\text{alternatively, } \tan\alpha = \frac{v_y}{v_x} \text{ or } \sin\alpha = \frac{v_y}{v}\right)$$
$$\cos\alpha = \frac{4}{\sqrt{185}}$$
$$\alpha = 72.897...$$

The object passes through P travelling at an angle of 72.9° below the horizontal (to 3s.f.).



16 a Let U be the speed at which the basketball is thrown. Resolve horizontally to find, in terms of U, the time at which the ball reaches the basket:

R(→): $v = u_x = U\cos 40^\circ$, $s = 10$, $t = ?$

$$s = vt$$

$$10 = Ut\cos 40^\circ$$

$$t = \frac{10}{U\cos 40^\circ}$$

Now resolve vertically at the instant when the ball passes through the basket:

R(↑): $u = u_y = U\sin 40^\circ$, $t = \dfrac{10}{U\cos 40^\circ}$ s, $a = g = -9.8$, $s = 3.05 - 2 = 1.05$

$$s = ut + \frac{1}{2}at^2$$

$$1.05 = \frac{10U\sin 40^\circ}{U\cos 40^\circ} + \frac{1}{2}\left(-9.8 \times \left(\frac{10}{U\cos 40^\circ}\right)^2\right)$$

$$1.05 = 10\tan 40^\circ - \frac{490}{(U\cos 40^\circ)^2}$$

$$(U\cos 40^\circ)^2 = \frac{490}{10\tan 40^\circ - 1.05}$$

$$U^2 = \frac{490}{(10\tan 40^\circ - 1.05)(\cos 40^\circ)^2}$$

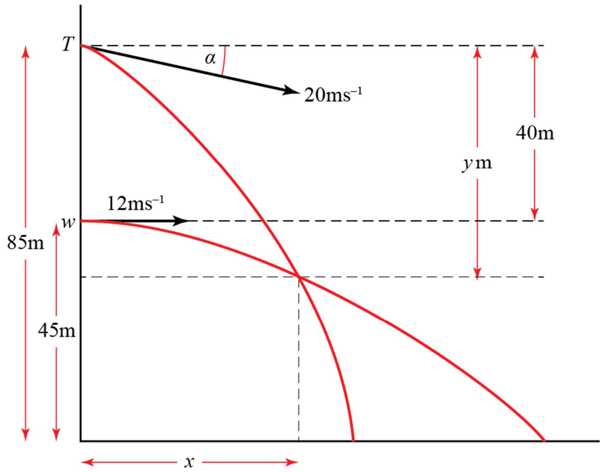
$$U = 10.665...$$

The player throws the ball at 10.7 ms^{-1} (3s.f.).

b By modelling the ball as a particle, we can ignore the effects of air resistance, the weight of the ball and any energy or path changes caused by the spin of the ball.

Challenge
Let the positive direction be downwards.

The stone thrown from the top of the tower is T, and that from the window is W.
Let u_{T_x} denote the horizontal component of the initial velocity of T, and u_{W_y} denote the vertical component of the initial velocity of W, etc.



The stones collide at time t at a horizontal distance x m from the tower.

For T, R(→): $v = u_{T_x} = 20\cos\alpha$ ms^{-1}, $s = x$, $t = t$

For W, R(→): $v = u_{W_x} = 12$ ms^{-1}, $s = x$, $t = t$

$s = vt$

$x = u_{T_x}t = u_{W_x}t$

$20\cos\alpha = 12$

$\cos\alpha = \frac{3}{5} \Rightarrow \sin\alpha = \frac{4}{5}$

For T, R(↓): $u = u_{T_y} = 20\sin\alpha = 16$ ms^{-1}, $a = g$, $s = s_{T_y}$, $t = t$

For W, R(↓): $u = u_{W_y} = 0$, $a = g$, $s = s_{T_W} = s_{T_y} - 40$, $t = t$

$s_{W_y} = s_{T_y} - 40$

$u_{W_y}t + \frac{1}{2}gt^2 = u_{T_y}t + \frac{1}{2}gt^2 - 40$ (since $s = ut + \frac{1}{2}at^2$)

$0 = 16t - 40$ (subtracting $\frac{1}{2}gt^2$ from each side in line above, and sub values for u)

$t = \frac{40}{16}$

$= 2.5$

The stones collide after 2.5 s of flight.