Forces and friction 5C

1 a i $R(\uparrow)$

$R - 5g = 0$

$R = 5g$

$= 49 \text{ N}$

$\therefore F_{MAX} = \frac{1}{7} \times 49$

$= 7 \text{ N}$

Since the driving force is only 3 N, the friction will only need to be 3 N to prevent the block from slipping, so $F = 3 \text{ N}$.

ii Since driving force is equal to frictional force, body remains at rest in equilibrium.

b i $F_{MAX} = 7 \text{ N}$ (from part **a**), and driving force is 7 N, so friction will need to be at its maximum value to prevent the block from slipping, i.e. $F = 7 \text{ N}$.

ii F is equal to the driving force of 7 N, so the body remains at rest in limiting equilibrium.

c i $F_{MAX} = 7 \text{ N}$ (from part **a**), and driving force is 12 N, so friction will be at its maximum value of 7 N.

ii Since the driving force is greater than the frictional force, there is a resultant force and the body accelerates.

iii $R(\rightarrow)$

$F = ma$

$12 - 7 = 5a$

$a = 1 \text{ms}^{-2}$

Body accelerates at 1ms^{-2}

d i $R(\uparrow)$

$R - 14 - 5g = 0$

$R = 63 \text{ N}$

$\therefore F_{MAX} = \mu R$

$= \frac{1}{7} \times 63$

$= 9 \text{ N}$

Since the driving force is only 6 N, the friction will only need to be 6 N to prevent the block from slipping, so $F = 6 \text{ N}$.

ii Since driving force is equal to frictional force, body remains at rest in equilibrium.

e i $F_{MAX} = 9 \text{ N}$ (from part **d**), and driving force is 9 N, so friction will need to be at its maximum value to prevent the block from slipping, i.e. $F = 9 \text{ N}$.

ii F is equal to the driving force of 9 N, so the body remains at rest in limiting equilibrium.

1 f i $F_{MAX} = 9\text{ N}$ (from part **d**), and driving force is 12 N, so friction will be at its maximum value of 9 N.

ii Since the driving force is greater than the frictional force, there is a resultant force and the body accelerates.

iii $R(\rightarrow)$

$$F = ma$$
$$12 - 9 = 5a$$
$$a = 0.6\text{ ms}^{-2}$$

Body accelerates at 0.6 ms^{-2}

g i $R(-)$

$$R + 14 - 5g = 0$$
$$R = 35\text{ N}$$
$$\therefore F_{MAX} = \mu R$$
$$= \tfrac{1}{7} \times 35$$
$$= 5\text{ N}$$

Since the driving force is only 3 N, the friction will only need to be 3 N to prevent the block from slipping, so $F = 3\text{ N}$.

ii Since driving force is equal to frictional force, body remains at rest in equilibrium.

h i $F_{MAX} = 5\text{ N}$ (from part **g**), and driving force is 5 N, so friction will need to be at its maximum value to prevent the block from slipping, i.e. $F = 5\text{ N}$.

ii F is equal to the driving force of 5 N, so the body remains at rest in limiting equilibrium.

i i $F_{MAX} = 5\text{ N}$ (from part **g**), and driving force is 6 N, so friction will be at its maximum value of 5 N

ii Since the driving force is greater than the frictional force, there is a resultant force and the body accelerates.

iii $R(\rightarrow)$

$$F = ma$$
$$6 - 5 = 5a$$
$$a = 0.2\text{ m s}^{-2}$$

Body accelerates at 0.2 m s^{-2}

1 j i $R(-)$

$$R + 14\sin 30° - 5g = 0$$
$$R = 42 \text{ N}$$
$$\therefore F_{MAX} = \mu R$$
$$= \frac{1}{7} \times 42$$
$$= 6 \text{ N}$$

Considering horizontal forces:

Driving force $- F_{MAX} = 14\cos 30° - 6 > 0$, so $F = F_{MAX} = 6$ N

ii Since the driving force is greater than the frictional force, there is a resultant force and the body accelerates.

iii $R(\rightarrow)$

$$F = ma$$
$$14\cos 30° - 6 = 5a$$
$$a = 1.22 \text{ m s}^{-2} \text{ (3 s.f.)}$$

Body accelerates at 1.22 m s^{-2} (3 s.f.)

k i $R(-)$

$$R + 28\sin 30° - 5g = 0$$
$$R = 35 \text{ N}$$
$$\therefore F_{MAX} = \mu R$$
$$= \frac{1}{7} \times 35$$
$$= 5 \text{ N}$$

Considering horizontal forces:

Driving force $- F_{MAX} = 28\cos 30° - 5 > 0$, so $F = F_{MAX} = 5$ N

ii Since the driving force is greater than the frictional force, there is a resultant force and the body accelerates.

iii $R(\rightarrow)$

$$F = ma$$
$$28\cos 30° - 5 = 5a$$
$$a = 3.85 \text{ m s}^{-2} \text{ (3 s.f.)}$$

Body accelerates at 3.85 m s^{-2} (3 s.f.)

1 l i $R(-)$

$R - 56\cos 45° - 5g = 0$

$\therefore R = 88.6 \text{ N} \ \ (3 \text{ s.f.})$

$\therefore F_{MAX} = \mu R$

$= \frac{1}{7} \times 88.6$

$= 12.657 \text{ N}$

Considering horizontal forces:

Driving force $- F_{MAX} = 56\sin 45° - 12.657 > 0$, so $F = F_{MAX} = 12.7 \text{ N} \ \ (3 \text{ s.f.})$

ii Since the driving force is greater than the frictional force, there is a resultant force and the body accelerates.

iii $R(\rightarrow)$

$F = ma$

$56\sin 45° - 12.657 = 5a$

$5a = 26.941$

$a = 5.388 \text{ m s}^{-2}$

So the acceleration is 5.39 m s^{-2} (3 s.f.)

2 a $R(-)$

$R + 20\sin 30° - 10g = 0$

$R = 88 \text{ N}$

$R(\rightarrow)$

$F = ma$

$20\cos 30° - \mu \times 88 = 10 \times 1$

$\mu = 0.083$ (2 s.f.)

b $R(-)$

$R + 20\cos 30° - 10g = 0$

$R = 80.679... \text{N}$

$R(\rightarrow)$

$F = ma$

$20\cos 60° - \mu \times 80.679 = 10 \times 0.5$

$\mu = 0.062$ (2 s.f.)

c $R(-)$

$R - 20\sqrt{2}\sin 45° - 10g = 0$

$R = 118 \text{N}$

$R(\rightarrow)$

$20\sqrt{2}\cos 45° - m´ \ 118 = 10´ \ 0.5$

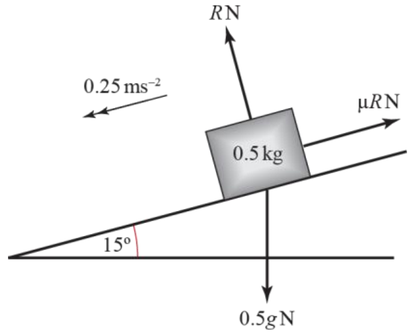
$m = 0.13$ (2 s.f.)

3 R(↖):

$R = 0.5g\cos 15^\circ$

$= 0.5 \times 9.8\cos 15^\circ$

$= 4.7330...$



Using Newton's second law of motion and R(↙):

$$F = ma$$

$$0.5g\sin 15^\circ - \mu R = 0.5 \times 0.25$$

$$\mu R = (0.5 \times 9.8\sin 15^\circ) - 0.125$$

$$\mu = \frac{1.2682... - 0.125}{4.7330...}$$

$$= 0.24153...$$

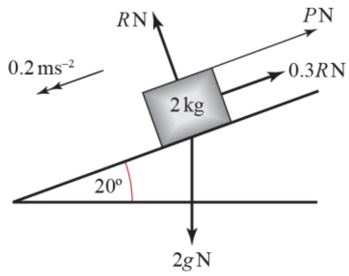
The coefficient of friction is 0.242 (3s.f.).

4 R(↖):

$R = 2g\cos 20^\circ$

$= 2 \times 9.8\cos 20^\circ$

$= 18.418...$



Using Newton's second law of motion ($F = ma$) and R(↙):

$$2g\sin 20^\circ - 0.3R - P = 2 \times 0.2$$

$$(2 \times 9.8\sin 20^\circ) - (0.3 \times 18.418...) - 0.4 = P$$

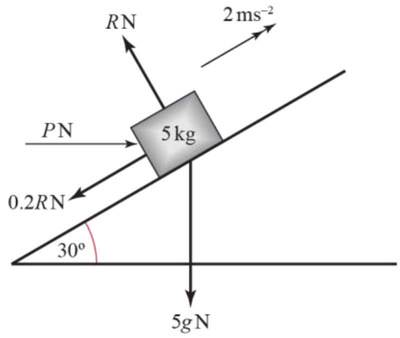
$$P = 0.7782...$$

The force P is 0.778 N (3s.f.).

5 R(↖):

$R = 5g\cos 30^\circ + P\sin 30^\circ$

$= \frac{49\sqrt{3}}{2} + \frac{P}{2}$



Using Newton's second law of motion and R(↗):

$P\cos 30^\circ - 5g\sin 30^\circ - 0.2R = 5 \times 2$

$$P\cos 30^\circ = 10 + 5g\sin 30^\circ + \frac{1}{5}\left(\frac{P}{2} + \frac{49\sqrt{3}}{2}\right)$$

$$\left(\frac{\sqrt{3}}{2} - \frac{1}{10}\right)P = 10 + \frac{5 \times 9.8}{2} + \frac{49\sqrt{3}}{10}$$

$$\left(5\sqrt{3} - 1\right)P = 100 + 245 + 49\sqrt{3}$$

$$P = \frac{429.8704896}{7.6602...} = 56.117...$$

The force P is 56.1 N (3s.f.).

6 Resolving vertically:

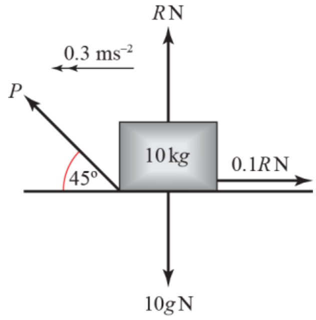
$R + P\sin 45^\circ = 10g$

$P\sin 45^\circ = 10g - R$ **(1)**

Resolving horizontally and using $F = ma$:

$P\cos 45^\circ - 0.1R = 10 \times 0.3$

$P\cos 45^\circ = 3 + 0.1R$ **(2)**



Since $\sin 45^\circ = \cos 45^\circ$, we can equate (**1**) and (**2**):

$10g - R = 3 + 0.1R$

$1.1R = 10g - 3$

$R = \dfrac{(10 \times 9.8) - 3}{1.1}$

$= 86.3636...$

Sub $R = 86.36$ into (**1**):

$P\sin 45^\circ = 10g - 86.36$

$P = \dfrac{(10 \times 9.8) - 86.36}{\sin 45^\circ} = 16.45...$

The force P is 16.5 N (3s.f.).

7 **a** $v = 0 \text{ ms}^{-1}$, $u = 30 \text{ ms}^{-1}$, $t = 20$ s, a = ?

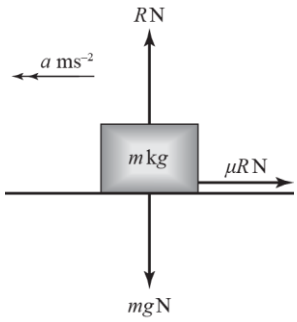
$v = u + at$

$0 = 30 + 20a$

$a = -\dfrac{20}{30} = -\dfrac{2}{3}$

Resolving vertically:

$R = mg$



Since the wheels lock up, the force which causes the deceleration is the maximum frictional force between the wheels and the track.

Resolving horizontally and using Newton's second law:

$-\mu R = -\dfrac{2}{3}m$

$-\mu mg = -\dfrac{2}{3}m$

$\mu g = \dfrac{2}{3}$

$\mu = \dfrac{2}{3g}$

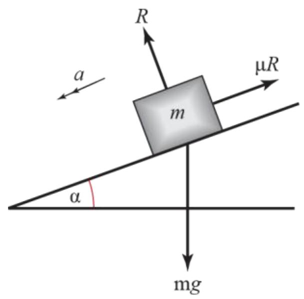
7 b Suppose there is an added constant resistive force of air resistance, A, where $A > 0$
Resolving horizontally and using Newton's second law:

$$\mu mg + A = \frac{2}{3}m$$

$$\mu = \frac{2}{3g} - \frac{A}{mg} < \frac{2}{3g}$$

So the coefficient of friction found by the second model is less than the coefficient of friction found by the first model.

Challenge



R(↖):
$R = mg\cos\alpha$

Using Newton's second law of motion and R(↙):

$$mg\sin\alpha - \mu R = ma$$
$$mg\sin\alpha - \mu mg\cos\alpha = ma$$
$$g(\sin\alpha - \mu\cos\alpha) = a$$

Since *m* does not appear in this expression, *a* is independent of *m*.