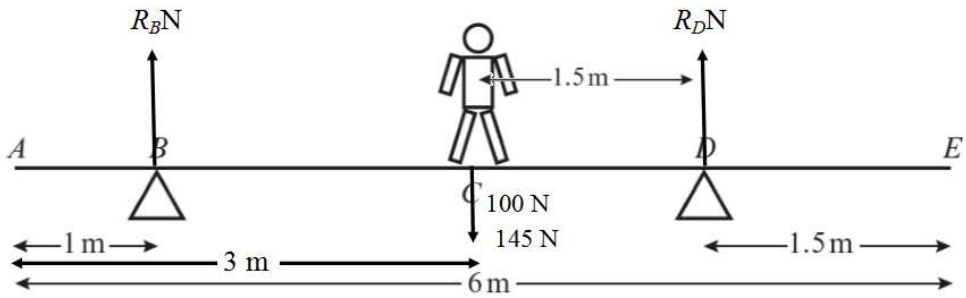
Moments Mixed exercise 4

1 a



The plank is in equilibrium.
Let the reaction forces at the supports be R_B and R_D.
Considering moments about point D:

$$R_B \times (6-1.5-1) = (100+145)\times(3-1.5)$$
$$3.5R_B = 245\times 1.5$$
$$3.5R_B = 367.5$$
$$R_B = 105$$

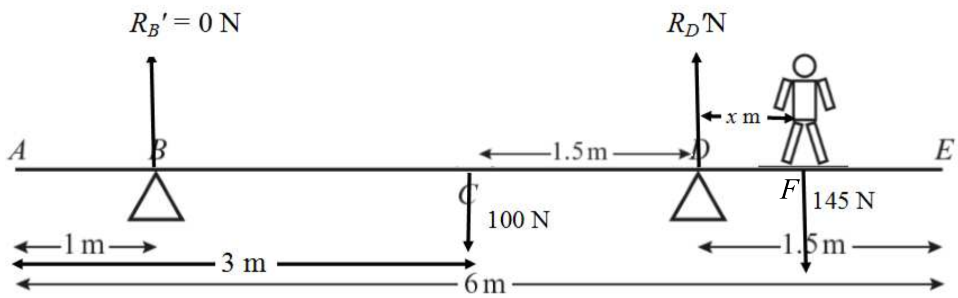
The support at B exerts a force of 105 N on the plank.

b The plank is in equilibrium.
Resolving vertically:

$$R_B + R_D = 100+145$$
$$R_D = 245-105$$
$$R_D = 140$$

The support at D exerts a force of 140 N on the plank.

c

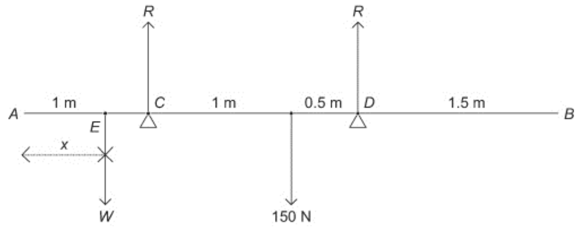


When the plank is on the point of tilting, the new reaction force at support B, $R'_B = 0$ N and plank is again in equilibrium. The child stands a distance x m from support D.
Considering moments about point D:

$$145x = 100\times(3-1.5)$$
$$145x = 150$$
$$x = 1.03$$

The distance DF is 1.03 m.

2



a Since the rod is uniform, the centre of mass is at the mid-point.
Taking moments about A:

$$Wx + 150 \times 2 = R \times 1 + R \times 2.5,$$

$$Wx + 300 = 3.5R \qquad \textbf{(1)}$$

$$R(\uparrow):\ W + 150 = R + R,$$

$$2R = W + 150$$

$$R = \frac{W + 150}{2} \qquad \textbf{(2)}$$

Sub (**2**) into (**1**) gives:

$$Wx + 300 = \frac{7}{2} \times \frac{W + 150}{2}$$

$$4(Wx + 300) = 7W + 7 \times 150$$

$$4Wx + 1200 = 7W + 1050$$

$$1200 - 1050 = 7W - 4Wx$$

$$W(7 - 4x) = 150$$

$$W = \frac{150}{7 - 4x}$$

b The range of values of x are:

$$x \geq 0 \text{ and } \frac{150}{7 - 4x} > 0$$

$$\Rightarrow 7 - 4x > 0$$

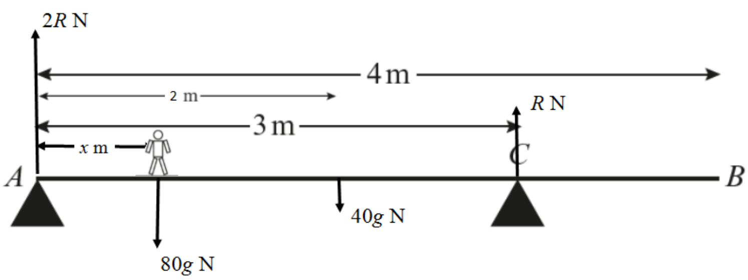
$$4x < 7$$

$$x < \frac{7}{4}$$

$$x < 1.75$$

So $0 \leq x < 1.75$

3 a



The plank is in equilibrium.

3 a Resolving vertically:

$$2R + R = 40g + 80g$$
$$3R = 120 \times 9.8$$
$$3R = 1176$$
$$R = 392$$

The value of R is 392 N.

b Taking moments about A:

$$80gx + (40g \times 2) = 3R$$
$$80g(x+1) = 3 \times 392$$
$$x + 1 = \frac{1176}{80 \times 9.8}$$
$$x + 1 = 1.5$$

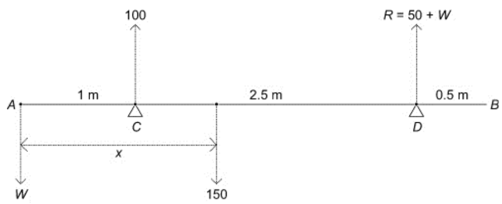
The man stands 0.5 m from A.

c i Assuming the plank is uniform means the weight of the plank acts at its centre of mass: i.e. halfway along the plank.

ii Assuming the plank is a rod means its width can be ignored.

iii Assuming the man is a particle means all his weight acts at the point at which he stands.

4 a



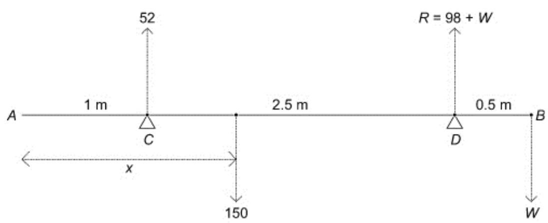
$R(\uparrow)$:

$$100 + R = W + 150$$
$$R = W + 50$$

Taking moments about A:

$$100 \times 1 + (W + 50) \times 3.5 = 150 \times x$$
$$100 + 175 + 3.5W = 150x$$
$$275 + 3.5W = 150x$$
$$550 + 7W = 300x$$

b



$R(\uparrow)$

4 b

$$52 + R = 150 + W$$
$$R = 150 + W - 52$$
$$= 98 + W$$

Taking moments about B:

$$52 \times 3 + (98 + W) \times 0.5 = 150 \times (4 - x)$$
$$156 + 49 + 0.5W = 600 - 150x$$

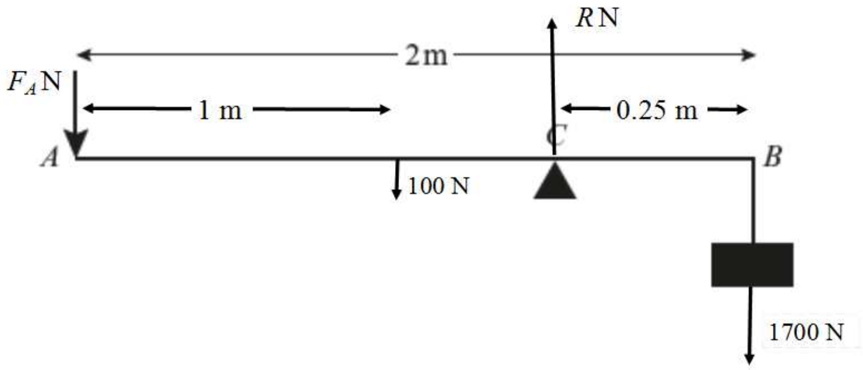
Doubling,

$$410 + W = 1200 - 300x$$
$$W = 790 - 300x$$

c Solving the simultaneous equations obtained in **a** and **b**:

$$\Rightarrow W = 790 - (550 + 7W)$$
$$8W = 790 - 550$$
$$= 240$$
$$\Rightarrow W = 30$$
$$\Rightarrow 410 + 30 = 1200 - 300x$$
$$300x = 760$$
$$x = 2.53 \text{ (3 s.f.)}$$

5 a

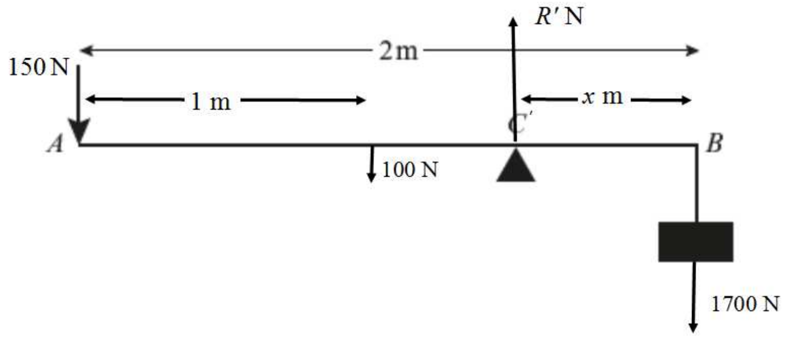


The lever is in equilibrium.
Considering moments about point C:

$$F_A(2 - 0.25) + 100(1 - 0.25) = 1700 \times 0.25$$
$$1.75F_A + 75 = 425$$
$$F_A = \frac{425 - 75}{1.75}$$
$$F_A = 200$$

The force at A is 200 N.

5 b

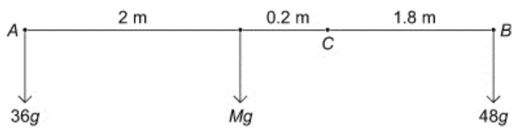


The lever is again in equilibrium. Let x be the distance of the pivot from B.
Considering moments about the new support position C':

$$150(2-x)+100(1-x)=1700x$$
$$300-150x+100-100x=1700x$$
$$400=1700x+250x$$
$$400=1950x$$
$$x=0.205$$

The pivot is now 0.21 m from B (to the nearest cm).

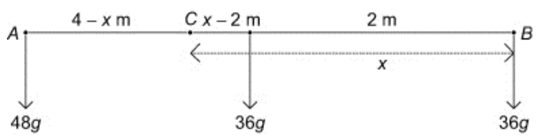
6 a Let the mass of the plank be M. Since the plank is uniform, its centre of mass is at its mid-point.



Taking moments about C:

$$48g\times 1.8 = Mg\times 0.2+36g\times 2.2$$
$$86.4g=0.2Mg+79.2g$$
$$86.4=0.2M+79.2$$
$$0.2M=86.4-79.2$$
$$=7.2$$
$$\Rightarrow M=36\text{ kg}$$

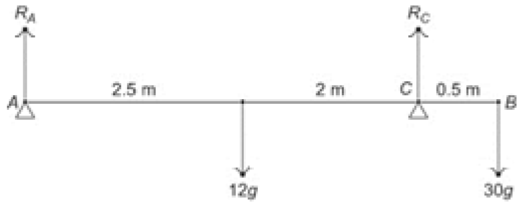
b Let the distance BC be x



Taking moments about C:

$$36gx+36g(x-2)=48g(4-x)$$
$$3x+3(x-2)=4(4-x)$$
$$6x-6=16-4x$$
$$10x=22$$
$$x=2.2\text{ m}$$

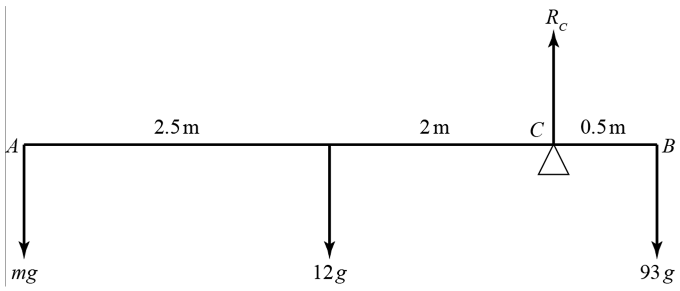
7 a



Taking moments about C:

$$R_A \times 4.5 + 30g \times 0.5 = 12g \times 2$$
$$R_A \times 4.5 = 24g - 15g$$
$$= 9g$$
$$\Rightarrow R_A = 2g$$
$$= 19.6\text{ N}$$

b



The plank is about to tilt about C

$\Rightarrow$reaction at $A = 0$

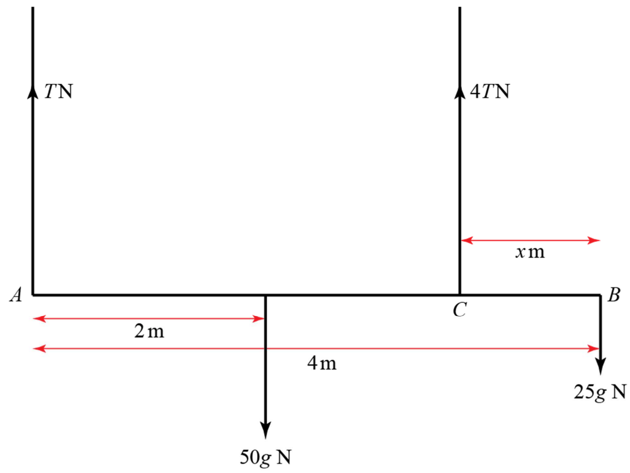
Taking moments about C:

$$mg \times 4.5 + 12g \times 2 = 93g \times 0.5$$
$$4.5m = 93 \times 0.5 - 24$$
$$= 22.5$$
$$m = 5$$

8 The plank is in equilibrium.

Resolving vertically:

$$T + 4T = 50g + 25g$$
$$5T = 75g$$
$$T = 15g$$
$$4T = 60g$$

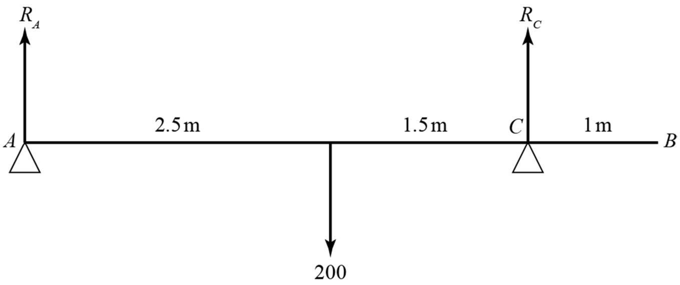


8 Considering moments about B:

$$(50g \times 2) = 60gx + (15g \times 4)$$
$$100g = 60gx + 60g$$
$$100g - 60g = 60gx$$
$$x = \frac{40g}{60g}$$
$$x = 0.666...$$

The distance from B to C is 0.67 m (to the nearest cm).

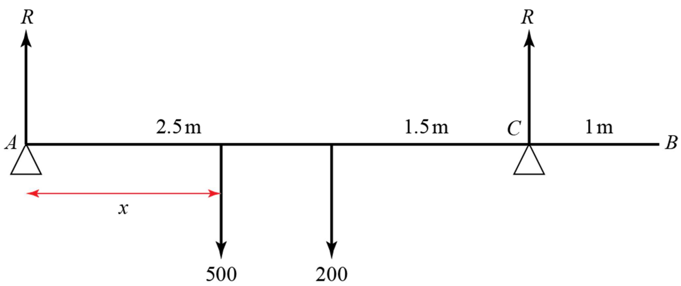
9 a



Taking moments about A:

$$200 \times 2.5 = R_C \times 4$$
$$R_C = 125\,\text{N}$$

b



Let the distance AD be x

$R(\uparrow)$

$$2R = 500 + 200$$
$$= 700$$
$$R = 350\,\text{N}$$

Taking moments about A:

$$R \times 4 = 200 \times 2.5 + 500 \times x$$
$$1400 = 4R$$
$$= 500 + 500x$$
$$900 = 500x$$
$$x = 1.8\,\text{m}$$

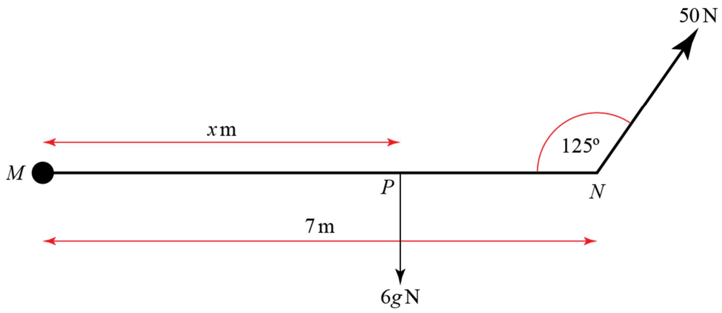
10 Distance $MP = x$ m
The plank is in equilibrium.
Taking moments about M:

$$6gx = 7 \times 50 \sin 55^\circ$$

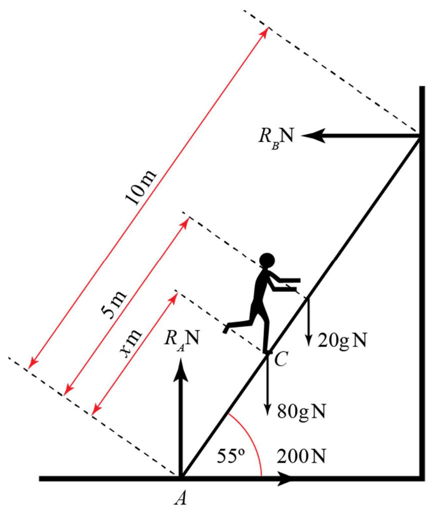
$$x = \frac{350 \sin 55^\circ}{6 \times 9.8}$$

$$= 4.8759...$$

The distance MP is 4.88 m (3s.f.).



11



The ladder is in equilibrium. Let x be the distance AC.
Resolving horizontally:

$R_B = 200$ N

Considering moments about A:

$$(80g \times x \cos 55^\circ) + (20g \times 5 \cos 55^\circ) = 200 \times 10 \sin 55^\circ$$

$$(784 \cos 55^\circ)x = 2000 \sin 55^\circ - 980 \cos 55^\circ$$

$$x = \frac{2000 \sin 55^\circ - 980 \cos 55^\circ}{784 \cos 55^\circ}$$

$$x = 2.3932...$$

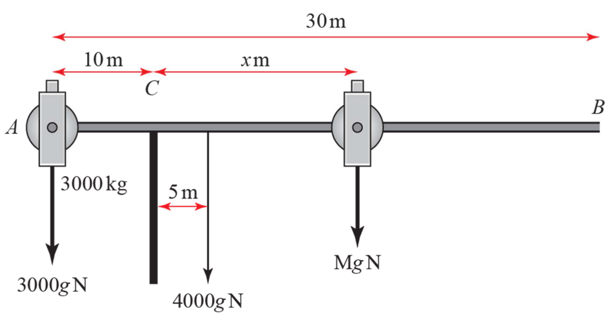
The distance AC is 2.39 m (to the nearest cm).

12 a Centre of mass of beam is 5 m from C.
Taking moments about C:

$$3000g \times 10 = (4000g \times 5) + Mgx$$

$$30\,000 = 20\,000 + Mx$$

$$M = \frac{10\,000}{x}$$



12 b Maximum load is when $x = 5$ m:

$$M = \frac{10\,000}{5} = 2000 \text{ kg}$$

Minimum load is when $x = 20$ m:

$$M = \frac{10\,000}{20} = 500 \text{ kg}$$

c It is not very accurate to model the beam as a uniform rod. Since the beam may taper at one end, the centre of mass of the beam may not lie in the middle of the beam.

Challenge

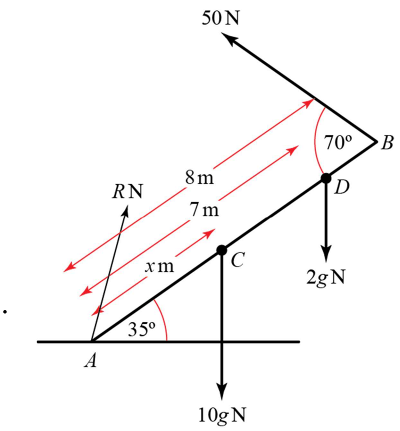
1 Let x be the distance from A to the centre of mass.
The beam is in equilibrium.
Taking moments about A:

$$10g \times x\cos 35^\circ + (2g \times 7\cos 35^\circ) = 8 \times 50\sin 70^\circ$$

$$10gx\cos 35^\circ = 400\sin 70^\circ - 14g\cos 35^\circ$$

$$x = \frac{400\sin 70^\circ - 137.2\cos 35^\circ}{98\cos 35^\circ} = 3.2822...$$

The centre of mass of the beam is 3.28 m from A (3s.f.).



2 a When force is a minimum, system is in limiting equilibrium.
Taking moments about P:

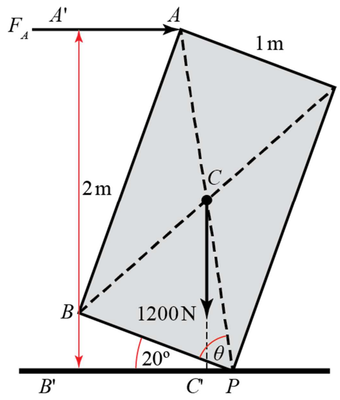
$$F_A \times (A'B') = 1200 \times PC' \quad \textbf{(1)}$$

Finding $A'B'$:

$$A'B = 2\cos 20^\circ$$

$$BB' = 1\sin 20^\circ$$

$$\therefore A'B' = 2\cos 20^\circ + \sin 20^\circ$$



Finding PC':

$$PC' = PC\cos(\theta + 20)$$

$$(PC)^2 = 1^2 + 0.5^2$$

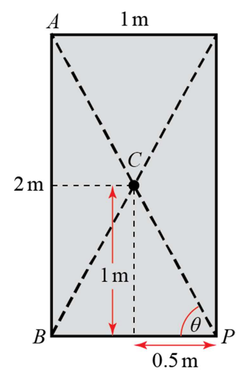
$$PC = \sqrt{1.25}$$

$$\tan\theta = \frac{1}{0.5}$$

$$\theta = 63.434...^\circ$$

$$PC' = \sqrt{1.25} \times \cos(63.4 + 20)^\circ$$

$$PC' = \sqrt{1.25} \times \cos 83.434...^\circ$$



Substituting values for $A'B'$ and PC' into equation **(1)**

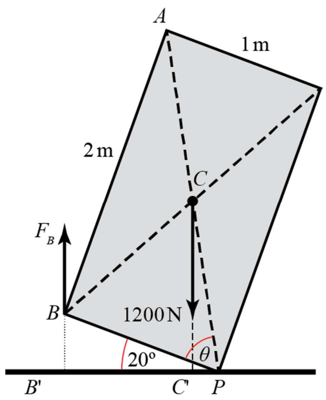
$$F_A \times \left(2\cos 20^\circ + \sin 20^\circ\right) = 1200 \times \sqrt{1.25} \times \cos 83.434...^\circ$$

$$F_A = \frac{1200 \times \sqrt{1.25} \times \cos 83.434...^\circ}{2\cos 20^\circ + \sin 20^\circ}$$

$$F_A = 69.051...$$

A horizontal force of 69.0 N at A will tip the refrigerator (3s.f.).

Challenge
2 b



When force is a minimum, system is in limiting equilibrium.
Taking moments about P:

$$F_B \times (PB') = 1200 \times \sqrt{1.25} \times \cos 83.434...^\circ$$

$$F_B \times 1\cos 20^\circ = 1200 \times \sqrt{1.25} \times \cos 83.434...^\circ$$

$$F_B = \frac{1200 \times \sqrt{1.25} \times \cos 83.434...^\circ}{\cos 20^\circ}$$

$$F_B = 163.25...$$

A vertical force of 163 N at *B* will tip the refrigerator (3s.f.).