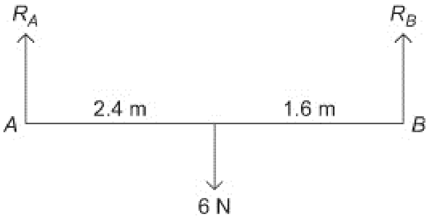
Moments 4D

1



Resolving vertically:

$6 = R_A + R_B$

Taking moments about A:

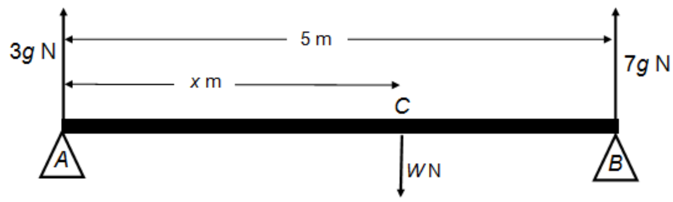
$6 \times 2.4 = 4 \times R_B$

$\Rightarrow R_B = 3.6\text{ N}$

So $R_A = 2.4\text{ N}$

The reactions at A and B are 2.4 N and 3.6 N respectively.

2 Centre of mass is at C, a distance x m from A.
The bar is in equilibrium.



a Resolving vertically:

$W = 3g + 7g$

$= 10g$

The weight of the bar is $10g$ N

b Taking moments about C:

$3gx = 7g(5 - x)$

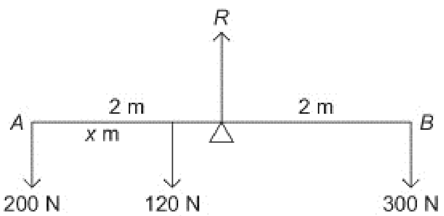
$3gx = 35g - 7gx$

$10x = 35$

$x = 3.5$

The centre of mass is 3.5 m from A.

3



Let the centre of mass be x m from A.
Taking moments about the mid-point:

$120 \times (2 - x) + 200 \times 2 = 300 \times 2$

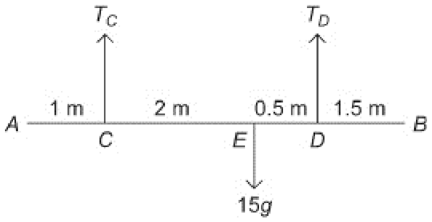
$240 - 120x + 400 = 600$

$120x = 40$

$\Rightarrow x = \frac{40}{120} = \frac{1}{3}$

The centre of mass is $\frac{1}{3}$ m from A.

4 a



Taking moments about C:

$T_D \times 2.5 = 15g \times 2$

$2.5T_D = 30g$

$T_D = 12g$

$= 118\,\text{N}$ (3 s.f.)

Resolving vertically:

$T_C + T_D = 15g$

$T_C = 3g$

$= 29.4\,\text{N}$

b Let distance $AF = x$ m.

The bar is in equilibrium.

Resolve vertically:

$T + 2T = 9g + 15g$

$3T = 24g$

$T = 8g$ and $2T = 16g$

Taking moments about A:

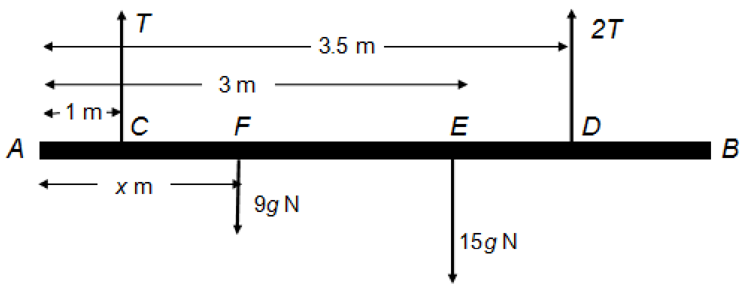
$9gx + (15g \times 3) = (8g \times 1) + (16g \times 3.5)$

$9x = 8 + 56 - 45$

$x = \frac{19}{9}$

$= 2.11$

The distance AF is 2.11 m (3s.f.).



5 a Let the tensions in the ropes be T_A and T_B respectively.

The plank is in equilibrium.

Taking moments about A:

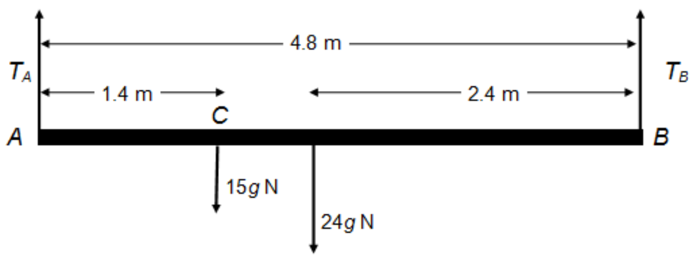
$T_B \times 4.8 = (1.4 \times 15g) + (2.4 \times 24g)$

$4.8T_B = 21g + 57.6g$

$T_B = \frac{78.6 \times 9.8}{4.8}$

$= 160$

The tension in the rope at B is 160 N.



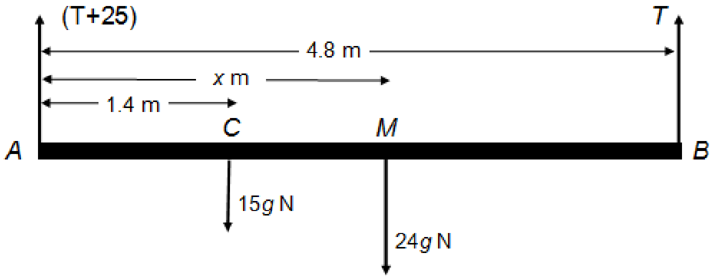
5 b Centre of mass is at M, x m from A.
The plank is in equilibrium.
Resolving vertically:

$$T + T + 25 = 15g + 24g$$
$$2T = 39g - 25$$
$$T = \frac{(39 \times 9.8) - 25}{2}$$
$$= 178.6$$

Taking moments about A:

$$(15g \times 1.4) + 24gx = 178.6 \times 4.8$$
$$(147 \times 1.4) + 235.2x = 857.28$$
$$x = \frac{857.28 - 205.8}{235.2}$$
$$= 2.77$$

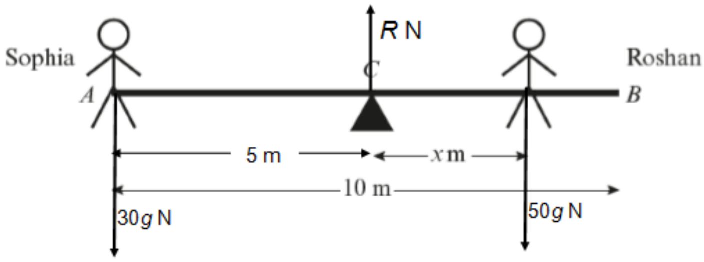
The centre of mass is 2.77 m from A (3s.f.).



6 a Taking moments about C:

$$30g \times 5 = 50gx$$
$$150g = 50gx$$
$$x = 3$$

The seesaw is in equilibrium when Roshan sits 3 m from C.

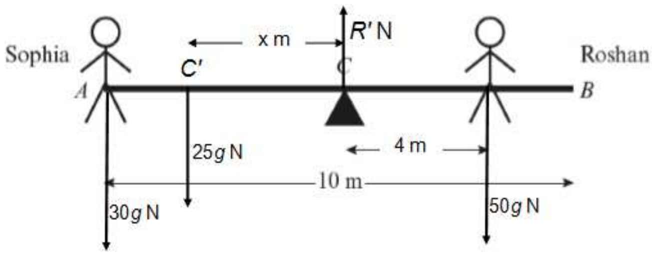


b Modelling the beam as uniform means that the centre of mass of the seesaw is at C, and so weight of the seesaw can be ignored when taking moments about C.

c Centre of mass is at C', y m from C.
Taking moments about C:

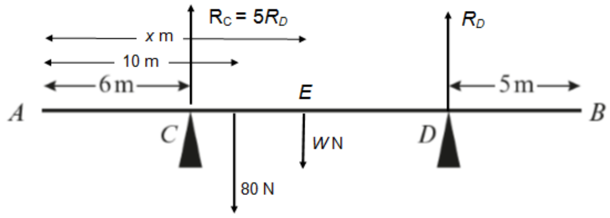
$$(30g \times 5) + 25gx = 50g \times 4 \quad \text{(divide by 5g)}$$
$$30 + 5x = 40$$
$$x = \frac{40 - 30}{5} = 2$$

The centre of mass is 2 m to the left of C (towards Sophia).



7 The rod is in equilibrium.
Letting $R_D = R$ gives $R_c = 5R$
Resolving vertically:

$$R_c + R_D = 80 + W$$
$$5R + R = 80 + W$$
$$R = \frac{80 + W}{6}$$



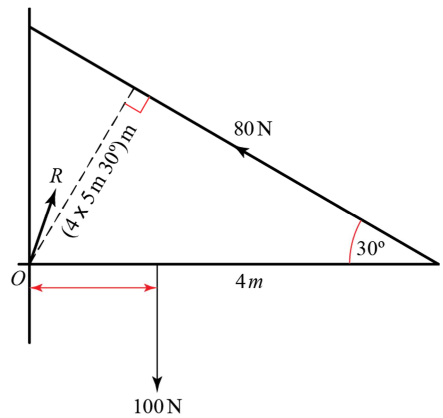
Taking moments about A:

$$(6 \times 5R) + 20R = (80 \times 10) + Wx$$
$$50R = 800 + Wx$$
$$50\left(\frac{80 + W}{6}\right) = 800 + Wx \qquad \text{(multiply by 6 and expand)}$$
$$4000 + 50W = 4800 + 6Wx$$
$$(50 - 6x)W = 4800 - 4000 \qquad \text{(divide by 2 and rearrange)}$$
$$W = \frac{400}{25 - 3x} \qquad \text{as required.}$$

8 The rod is attached to the wall at O.
Let the distance from the wall to the centre of mass be x m.
The rod is in equilibrium.
Taking moments about O:

$$100x = 80 \times 4 \sin 30^\circ$$
$$100x = 160$$
$$x = 1.6$$

The centre of mass of the rod is 1.6 m from the wall.



Challenge
Let the distance from M to the beam's centre of mass be x m.
The beam is in equilibrium.
Taking moments about M:

$$120 \times \left(x \cos 40^\circ\right) = 30 \times \left(5 \sin 80^\circ\right)$$
$$4x \cos 40^\circ = 5 \sin 80^\circ$$
$$x = \frac{5 \sin 80^\circ}{4 \cos 40^\circ}$$
$$= 1.61$$

The centre of mass of the beam is 1.61 m from M (3s.f.).