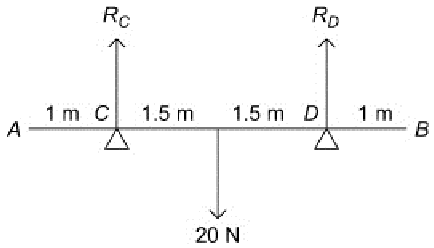
## Moments 4C

**1 a**




Resolving vertically:

$R_C + R_D = 20$

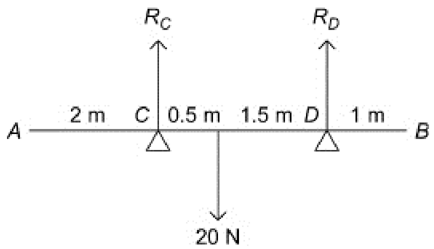
Taking moments about $C$:

$3 \times R_D = 1.5 \times 20$

$= 30$

$\Rightarrow R_D = 10\,\text{N}$ and $R_C = 10\,\text{N}$

**b**




Resolving vertically:

$R_C + R_D = 20$

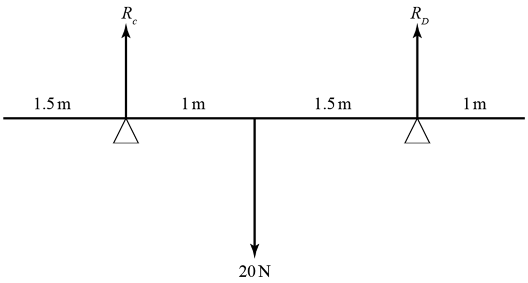
Taking moments about $C$:

$R_D \times 2 = 20 \times 0.5$

$= 10$

$\Rightarrow R_D = 5\,\text{N}$ and $R_C = 15\,\text{N}$

**1 c**




Resolving vertically:

$R_C + R_D = 20$

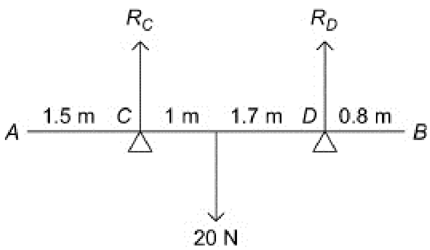
Taking moments about $C$:

$R_D \times 2.5 = 20 \times 1$

$= 20$

$\Rightarrow R_D = \frac{20}{2.5} = 8\text{ N}$ and $R_C = 12\text{ N}$

**d**




Resolving vertically:

$R_C + R_D = 20$

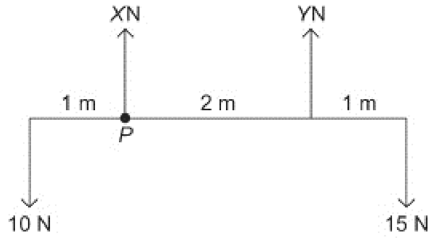
Taking moments about $C$:

$2.7 \times R_D = 20 \times 1$

$= 20$

$\Rightarrow R_D = \frac{20}{2.7} = 7.4\text{ N}$ and $R_C = 12.6\text{ N}$

**2 a**




Resolving vertically:

$X + Y = 10 + 15$

$\quad\quad = 25$

Taking moments about $P$:

$15 \times (2 + 1) = 10 \times 1 + Y \times 2$

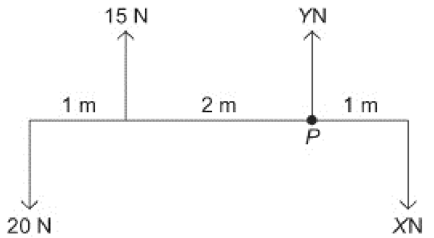
$45 = 10 + 2Y$

$2Y = 35$

$Y = 17.5$

$\Rightarrow X = 7.5$ and $Y = 17.5$

**b**




Resolving vertically:

$15 + Y = 20 + X$

$Y - X = 5$

Taking moments about $P$:

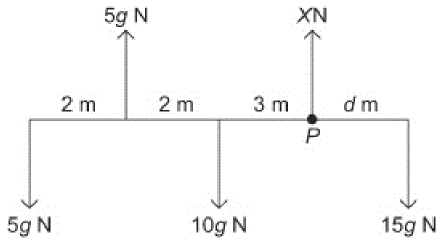
$20 \times (2 + 1) = 15 \times 2 + X \times 1$

$60 = 30 + X$

$X = 30$

$\Rightarrow X = 30$ and $Y = 35$

**2 c**



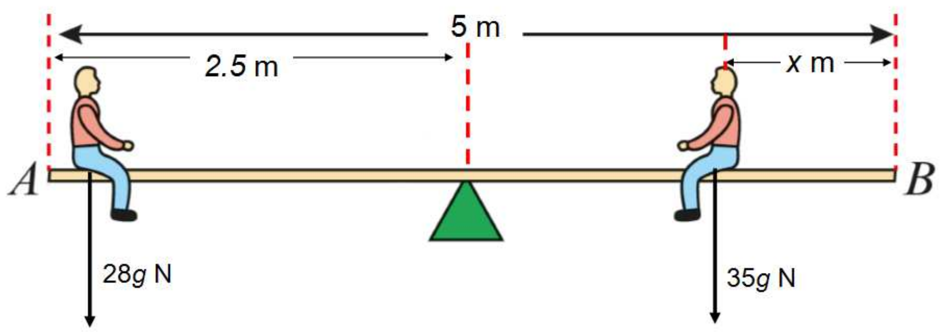

Resolving vertically:

$$5g + X = 5g + 10g + 15g$$
$$= 30g$$
$$\Rightarrow X = 25g = 245$$

Taking moments about $P$:

$$15g \times d + 5g \times (2+3) = 10g \times 3 + 5g \times (2+2+3)$$
$$15gd + 25g = 30g + 35g$$
$$15d = 40$$
$$d = 2\tfrac{2}{3}$$

**3**

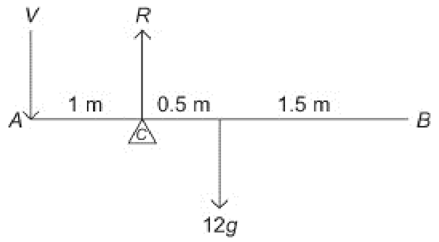



Seesaw is in equilibrium so
clockwise moment about pivot = anticlockwise moment about pivot

$$35g(2.5 - x) = 28g \times 2.5 \qquad \text{(divide both sides by } 7g\text{)}$$
$$5(2.5 - x) = 4 \times 2.5$$
$$5x = 2.5(5 - 4)$$
$$x = \frac{2.5}{5} = 0.5$$

Jack sits 0.5 m from B.

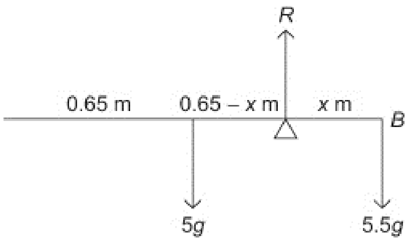
**4**




Suppose that the force required is $V$ N acting vertically downwards at $A$.
Taking moments about the pivot ($C$):
$V \times 1 = 0.5 \times 12g$

$\Rightarrow V = 6g = 59\,\text{N}$ (2 s.f.)

**5**

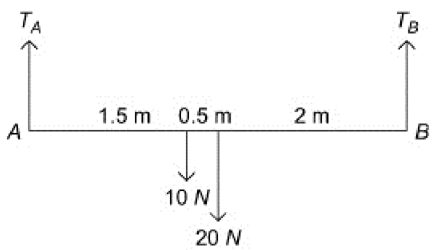



Let the support be $x$ m from the broomhead.
Taking moments about the support:

$$5.5g \times x = 5g \times (0.65 - x)$$
$$5.5x = 5 \times 0.65 - 5x$$
$$10.5x = 3.25$$
$$x = 0.31$$

The support should be 31 cm from the broomhead.

**6 a**

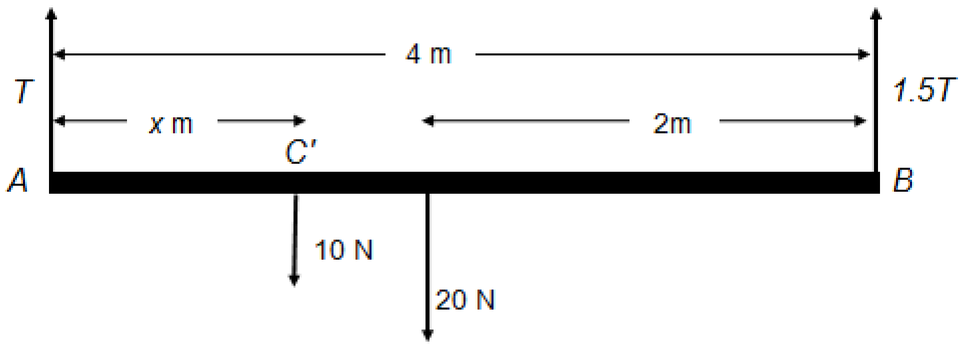



Let the tensions in the two strings be $T_A$ and $T_B$ respectively.
Resolving vertically:
$T_A + T_B = 10 + 20 = 30$
Taking moments about point $A$:

$$10 \times 1.5 + 20 \times (1.5 + 0.5) = 4 \times T_B$$
$$\Rightarrow 4T_B = 15 + 40$$
$$= 55$$

$T_B = 13.75\,\text{N}$ and $T_A = 16.25\,\text{N}$

**6 b** Particle is now at $C'$ where $AC' = x$ m.




Beam is in equilibrium.
Resolving vertically:

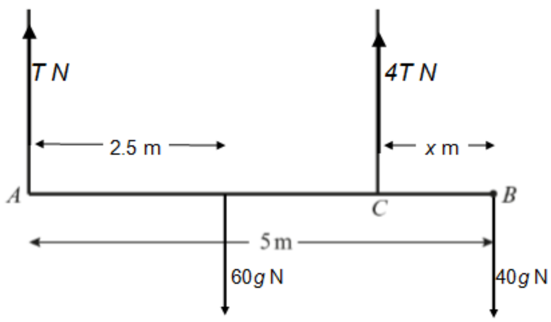
$$T + 1.5T = 10 + 20$$
$$2.5T = 30$$
$$T = 12$$

Taking moments about $A$:

$$10x + (20 \times 2) = (1.5 \times 12) \times 4$$
$$10x + 40 = 18 \times 4$$
$$10x = 72 - 40$$
$$x = \frac{32}{10} = 3.2$$

The particle is now 3.2 m from $A$.

**7** $BC = x$ m.
Beam is in equilibrium.




**a** Resolving vertically:

$$4T + T = 40g + 60g$$
$$5T = 100g$$
$$T = 20g$$
$$\text{So } 4T = 80g$$
$$4T = 80 \times 9.8 = 784$$

The tension in the wire at $C$ is 784 N.

**b** Taking moments about $B$:

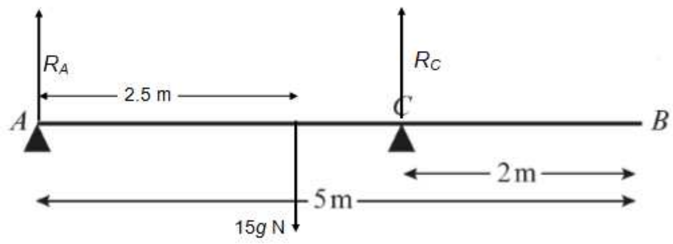
$$(20g \times 5) + 80gx = 60g \times 2.5 \qquad \text{(divide by } 20g\text{)}$$
$$5 + 4x = 7.5$$
$$4x = 2.5$$
$$x = \frac{2.5}{4}$$
$$= 0.625$$

The distance $CB$ is 0.625 m.

**8 a** Plank is in equilibrium.
Let the reactions at $A$ and $C$ be $R_A$ and $R_C$ respectively.
Taking moments about $A$:

$$15g \times 2.5 = R_c \times 3$$
$$R_c = 2.5 \times 5g$$
$$R_c = 12.5 \times 9.8$$
$$= 122.5$$

The reaction at $C$ is 122.5 N.



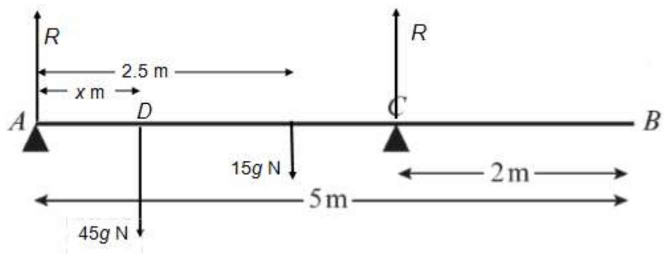

**b** Let $AD = x$ m
Let $R_A = R_C = R$
Plank remains in equilibrium.
Resolving vertically:

$$2R = 45g + 15g = 60g$$
$$R = 30g$$

Taking moments about A:

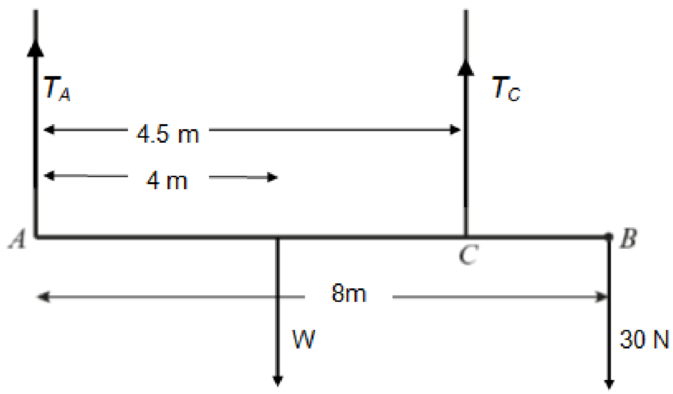
$$45gx + (15g \times 2.5) = 30g \times 3 \qquad \text{(divide by } 15g\text{)}$$
$$3x + 2.5 = 6$$
$$3x = 3.5$$
$$x = \frac{3.5}{3}$$
$$= 1.17$$

The distance $AD$ is 1.17 m (3s.f.).




**9 a** Beam is in equilibrium.
Let tension in wire at $C$ be $T_C$
Taking moments about $A$:

$$4.5T_C = 4W + (8 \times 30)$$
$$\frac{9}{2}T_C = 4W + 240$$
$$9T_C = 8W + 480$$
$$T_C = \frac{8}{9}W + \frac{160}{3} \quad \text{as required.}$$




**b** Let tension in wire at $A$ be $T_A$
Resolving vertically:

$$W + 30 = T_A + T_C$$
$$W + 30 = T_A + \frac{8}{9}W + \frac{160}{3}$$
$$9W + 270 = 9T_A + 8W + 480$$
$$W + 270 - 480 = 9T_A$$
$$T_A = \frac{W - 210}{9}$$
$$T_A = \frac{W}{9} - \frac{70}{3}$$

**9 c**

$$T_C = 12T_A$$
$$\frac{8W}{9} + \frac{160}{3} = \frac{12W}{9} - \frac{12\times 70}{3}$$
$$8W + (160\times 3) = 12W - (12\times 70\times 3)$$
$$480 + 2520 = 12W - 8W$$
$$4W = 3000$$
$$W = 750$$

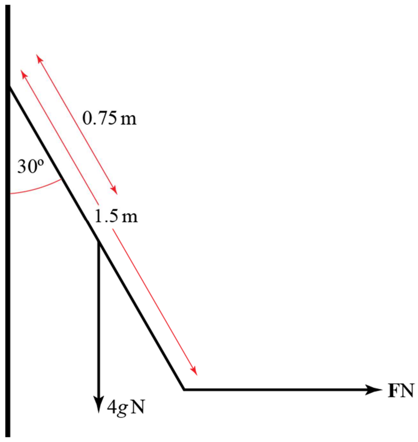
The weight of the beam is 750 N.

**10** The lever is in equilibrium.

Taking moments about point where lever is attached to the wall:

$$5g\times 0.75\sin 30^\circ = F\times 1.5\cos 30^\circ$$
$$F = \frac{5g\times 0.75\sin 30^\circ}{1.5\cos 30^\circ}$$
$$F = \frac{5}{2}g\tan 30^\circ$$
$$F = \frac{5}{2}\times 9.8\tan 30^\circ = 14.1$$

The force F is 14.1 N (3s.f.).




**11 a** The ladder is in equilibrium.

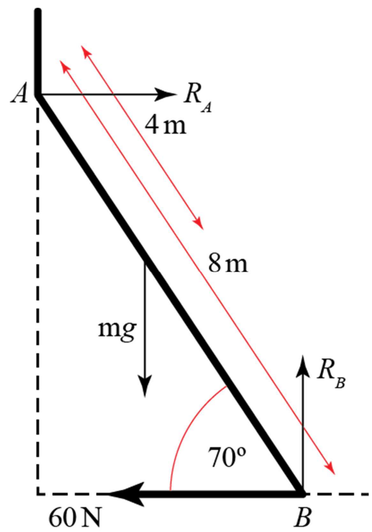
Resolving horizontally:

The reaction of the ladder on the wall at $A$ = 60 N.

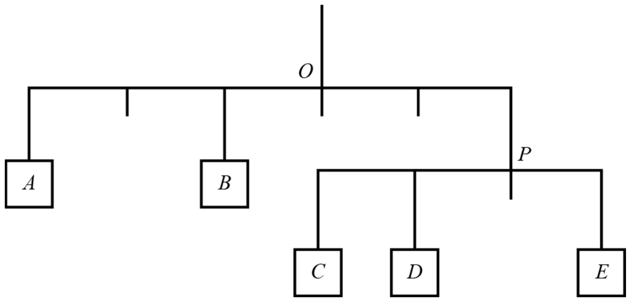
**b** Taking moments about $B$:

$$60\times 8\sin 70^\circ = 4mg\cos 70^\circ$$
$$m = \frac{60\times 8\sin 70^\circ}{4g\cos 70^\circ}$$
$$m = \frac{120}{g}\tan 70^\circ$$
$$m = \frac{120}{9.8}\tan 70^\circ$$
$$= 33.6$$

The mass of the ladder is 33.6 kg (3s.f.).



**Challenge**
Let the masses of the hanging components be $A$, $B$, $C$, $D$ and $E$ kg as shown.




Treating $CDE$ as a single component and taking moments about $O$:
$(3A + B)g = 2(C + D + E)g$
Since all the numbers are whole, $2(C + D + E)$ is even, so $3A + B$ must be even.
This means that **A & B are either both even or both odd**.
The minimum possible value of $C + D + E = 1 + 2 + 3 = 6$
So $3A + B \geqslant 12$
Maximum value of $B$ is 5
So $3A \geqslant 7$
i.e. $A \geqslant \frac{7}{3} \Rightarrow$ **A cannot be 1 or 2.**

Taking moments about P:
$(2C + D)g = Eg$
Smallest possible value of $2C + D$ is $(2 \times 1) + 2 = 4$
So ***E* must be 4 or 5**

If $E = 4$ then $C = 1$ and $D = 2$
This leaves $A$ & $B$ as 3 and 5.
Either option allowed by rules above.
$2(C + D + E) = 2(1 + 2 + 4) = 14$
since $3 \times 5 > 14$, this means $A$ must be 3 and $B$ must be 5.
To check: $3A + B = (3 \times 3) + 5 = 14$
**Therefore this combination works.**

However, best to check other possibilities:
If $E = 5$ then either $C = 2$ & $D = 1$ or $C = 2$ & $D = 1$.
First case means $A$ & $B$ are 3 & 4, which **is not allowed** as one odd and one even.
In second case, since $A$ cannot be 2, $A = 4$ and $B = 2$.
Then:
$2(C + D + E) = 2(2 + 1 + 5) = 16$
$3A + B = (3 \times 4) + 2 = 14$
Since these are **not equal**, this combination does not work either.

The masses, from left to right, are: 3 kg, 5 kg, 1 kg, 2 kg and 4 kg.