### The normal distribution 3F

1. **a.** Yes, since $n = 120$ is large (> 50) and $p = 0.6$ is close to 0.5.  
   
   $$\mu = np = 120 \times 0.6 = 72 \quad \text{and} \quad \sigma = \sqrt{np(1-p)} = \sqrt{72 \times 0.4} = \sqrt{28.8} = 5.37 \ (3 \ \text{s.f.})$$

   $$X \sim B(72, 5.37^2)$$

   **b.** No, $n = 6$ is not large enough (< 50).  

   **c.** Yes, since $n = 250$ is large (> 50) and $p = 0.52$ is close to 0.5.  

   $$\mu = np = 250 \times 0.52 = 130 \quad \text{and} \quad \sigma = \sqrt{np(1-p)} = \sqrt{130 \times 0.48} = \sqrt{62.4} = 7.90 \ (3 \ \text{s.f.})$$

   $$X \sim B(130, 7.90^2)$$

   **d.** No, $p = 0.98$ is too far from 0.5.  

   **e.** Yes, since $n = 400$ is large (> 50) and $p = 0.48$ is close to 0.5.  

   $$\mu = np = 400 \times 0.48 = 192 \quad \text{and} \quad \sigma = \sqrt{np(1-p)} = \sqrt{192 \times 0.52} = \sqrt{99.84} = 9.99 \ (3 \ \text{s.f.})$$

   $$X \sim B(192, 9.99^2)$$

   **f.** Yes, since $n = 1000$ is large (> 50) and $p = 0.58$ is close to 0.5.  

   $$\mu = np = 1000 \times 0.58 = 580 \quad \text{and} \quad \sigma = \sqrt{np(1-p)} = \sqrt{580 \times 0.42} = \sqrt{243.6} = 15.6 \ (3 \ \text{s.f.})$$

   $$X \sim B(580, 15.6^2)$$

2. A normal approximation is valid since $n = 150$ is large (> 50) and $p = 0.45$ is close to 0.5.  

   $$\mu = np = 150 \times 0.45 = 67.5 \quad \text{and} \quad \sigma = \sqrt{np(1-p)} = \sqrt{67.5 \times 0.55} = \sqrt{37.125} = 6.093 \ (4 \ \text{s.f.})$$

   **a.** $P(X \leq 60) \approx P(Y < 60.5) = 0.1253 \ (4 \ \text{s.f.})$

   **b.** $P(X > 75) \approx P(Y > 75.5) = 0.0946 \ (4 \ \text{s.f.})$

   **c.** $P(65 \leq X \leq 80) \approx P(64.5 < Y < 80.5) = 0.6723 \ (4 \ \text{s.f.})$

3. A normal approximation is valid since $n = 200$ is large (> 50) and $p = 0.53$ is close to 0.5.  

   $$\mu = np = 200 \times 0.53 = 106 \quad \text{and} \quad \sigma = \sqrt{np(1-p)} = \sqrt{106 \times 0.47} = \sqrt{49.82} = 7.058 \ (4 \ \text{s.f.})$$

   **a.** $P(X < 90) \approx P(Y < 89.5) = 0.0097 \ (4 \ \text{s.f.})$

   **b.** $P(100 \leq X < 110) \approx P(99.5 < Y < 109.5) = 0.5115 \ (4 \ \text{s.f.})$

   **c.** $P(X = 105) \approx P(104.5 < Y < 105.5) = 0.0559 \ (4 \ \text{s.f.})$
4 A normal approximation is valid since \( n = 100 \) is large (> 50) and \( p = 0.6 \) is close to 0.5.
\[ \mu = np = 100 \times 0.6 = 60 \quad \text{and} \quad \sigma = \sqrt{np(1 - p)} = \sqrt{60 \times 0.4} = \sqrt{24} = 4.899 \quad (4 \text{ s.f.}) \]

a \( P(X > 58) \approx P(Y > 58.5) = 0.6203 \quad (4 \text{ s.f.}) \)

b \( P(60 < X \leq 72) \approx P(60.5 < Y < 72.5) = 0.4540 \quad (4 \text{ s.f.}) \)

c \( P(X = 70) \approx P(69.5 < Y < 70.5) = 0.0102 \quad (4 \text{ s.f.}) \)

5 Let \( X \) = number of heads in 70 tosses of a fair coin, so \( X \sim B(70, 0.5) \).
Since \( p = 0.5 \) and 70 is large, \( X \) can be approximated by the normal distribution \( Y \sim N(\mu, \sigma^2) \),
where \( \mu = 70 \times 0.5 = 35 \) and \( \sigma = \sqrt{70 \times 0.5 \times 0.5} = \sqrt{17.5} \)
So \( Y \sim N(35, 17.5) \)
\[ P(X > 45) \approx P(Y \geq 45.5) = 0.0060 \]

6 A normal approximation is valid since \( n = 1200 \) is large and \( p \) is close to 0.5.
\[ \mu = np = 1200 \times 0.50 = 594.059 \]
and \( \sigma = \sqrt{np(1 - p)} = \sqrt{594.059 \times 0.51} = \sqrt{299.97059} = 17.32 \quad (4 \text{ s.f.}) \)
So \( Y \sim N(594.059, 299.97\ldots) \)
\[ P(X \geq 600) \approx P(Y > 599.5) = 0.3767 \quad (4 \text{ s.f.}) \]

7 a The number of trials, \( n \), must be large (> 50), and the success probability, \( p \), must be close to 0.5.

b Using the binomial distribution, \( P(X = 10) = \binom{20}{10} \times 0.45^{10} \times 0.55^{10} = 0.1593 \quad (4 \text{ s.f.}) \)

c A normal approximation is valid since \( n = 240 \) is large and \( p = 0.45 \) is close to 0.5.
\[ \mu = np = 240 \times 0.45 = 108 \quad \text{and} \quad \sigma = \sqrt{np(1 - p)} = \sqrt{108 \times 0.55} = \sqrt{59.4} = 7.707 \quad (4 \text{ s.f.}) \]
So \( Y \sim N(108, 59.4) \)
\[ P(X < 110) \approx P(Y < 109.5) = 0.5772 \quad (4 \text{ s.f.}) \]

d \( P(X \geq q) = 0.2 \Rightarrow P(Y > (q - 0.5)) = 0.2 \)
Using the inverse normal function,
\[ P(Y > (q - 0.5)) = 0.2 \Rightarrow q - 0.5 = 114.485 \Rightarrow q = 114.985 \]
So \( q = 115 \)

8 a Using the cumulative binomial function with \( N = 30 \) and \( p = 0.52 \),
\[ P(X < 17) = P(X \leq 16) = 0.6277 \quad (4 \text{ s.f.}) \]

b A normal approximation is valid since \( n = 600 \) is large and \( p = 0.52 \) is close to 0.5.
\[ \mu = np = 600 \times 0.52 = 312 \quad \text{and} \quad \sigma = \sqrt{np(1 - p)} = \sqrt{312 \times 0.48} = \sqrt{149.76} = 12.24 \quad (4 \text{ s.f.}) \]
So \( Y \sim N(312, 149.76) \)
\[ P(300 \leq X \leq 350) \approx P(299.5 < Y < 350.5) = 0.8456 \quad (4 \text{ s.f.}) \]
9 a Using the binomial distribution, \( P(X = 55) = \binom{100}{55} \times 0.56^{55} \times 0.44^{45} = 0.07838 \) (4 s.f.)

b A normal approximation is valid since \( n = 100 \) is large and \( p = 0.56 \) is close to 0.5.
\[
\mu = np = 100 \times 0.56 = 56 \quad \text{and} \quad \sigma = \sqrt{np(1 - p)} = \sqrt{56 \times 0.44} = \sqrt{24.64} = 4.964 \quad (4 \text{ s.f.})
\]
So \( Y \sim \text{B}(56, 24.64) \)
\[ P(X = 55) \approx P(54.5 < Y < 55.5) = 0.07863 \quad (4 \text{ s.f.}) \]

Percentage error = \( \frac{0.07838... - 0.07863...}{0.07838...} \times 100 = -0.31\% \) (2 d.p.)