The normal distribution 3C

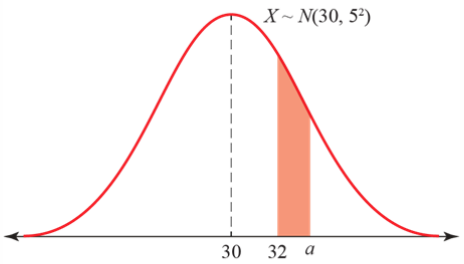
1 Use the inverse normal distribution function on your calculator, with $\mu = 30$ and $\sigma = 5$.

a $P(X < a) = 0.3 \Rightarrow a = 27.377... = 27.38$ (2 d.p.).

b $P(X < a) = 0.75 \Rightarrow a = 33.372... = 33.37$ (2 d.p.).

c $P(X > a) = 0.4 \Rightarrow P(X < a) = 0.6 \Rightarrow a = 31.266... = 31.27$ (2 d.p.).

d Since $P(32 < X < a) = 0.2$, it must be that $a > 32$.



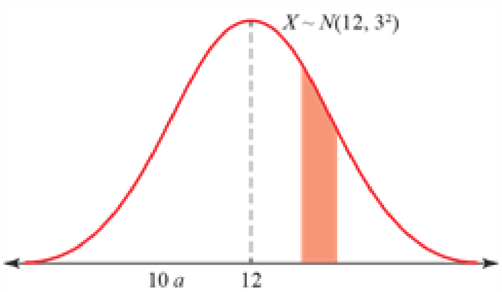
$P(32 < X < a) = 0.2$
$\Rightarrow P(X < a) - P(X < 32) = 0.2$
$\Rightarrow P(X < a) = 0.2 + P(X > 32) = 0.2 + 0.6554 = 0.8554$
$\Rightarrow a = 35.299... = 35.30$ (2 d.p.)

2 Use the inverse normal distribution function on your calculator, with $\mu = 12$ and $\sigma = 3$.

a $P(X < a) = 0.1 \Rightarrow a = 8.155... = 8.16$ (2 d.p.).

b $P(X > a) = 0.65 \Rightarrow P(X < a) = 0.35 \Rightarrow a = 10.844... = 10.84$ (2 d.p.).

c



$P(10 \leqslant X \leqslant a) = P(X \leqslant a) - P(X \leqslant 10) = 0.25$
$\Rightarrow P(X < a) = 0.25 + P(X < 10) = 0.25 + 0.2525 = 0.5025$
$\Rightarrow a = 12.018... = 12.02$ (2 d.p.)

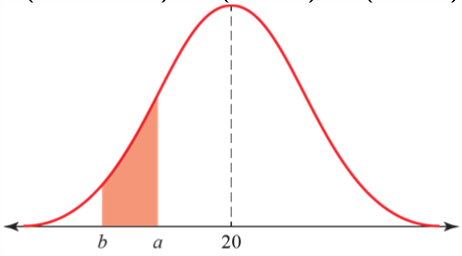
d $P(a < X < 14) = P(X < 14) - P(X < a) = 0.32$
$\Rightarrow P(X < a) = P(X < 14) - 0.32 = 0.7475 - 0.32 = 0.4275$
$\Rightarrow a = 11.451... = 11.45$ (2 d.p.)

3 Use the inverse normal distribution function on your calculator, with $\mu = 20$ and $\sigma = \sqrt{12}$.

a i $P(X < a) = 0.40 \Rightarrow a = 19.12... = 19.1$ (1 d.p.)

ii $P(X > b) = 0.6915 \Rightarrow P(X < b) = 0.3085 \Rightarrow b = 18.26... = 18.3$ (1 d.p.)

3 b $P(b < X < a) = P(X < a) - P(X < b) = 0.40 - (1 - 0.6915) = 0.0915$

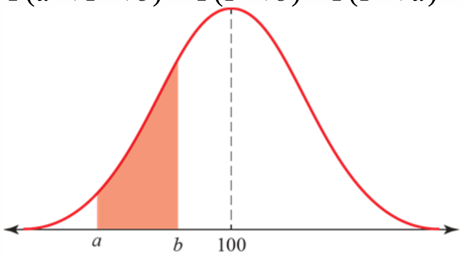


4 Use the inverse normal distribution function on your calculator, with $\mu = 100$ and $\sigma = 15$.

a i $P(Y > a) = 0.975 \Rightarrow P(Y < a) = 0.025 \Rightarrow a = 70.60... = 70.6$ (1 d.p.)

ii $P(Y < b) = 0.10 \Rightarrow b = 80.77... = 80.8$ (1 d.p.)

b $P(a < Y < b) = P(Y < b) - P(Y < a) = 0.10 - (1 - 0.975) = 0.10 - 0.025 = 0.075$

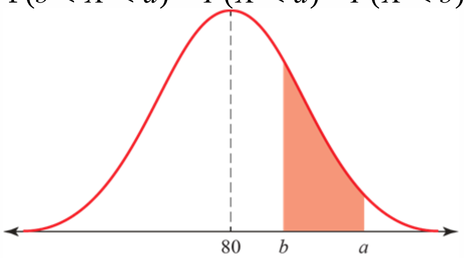


5 Use the inverse normal distribution function on your calculator, with $\mu = 80$ and $\sigma = \sqrt{16} = 4$.

a i $P(X > a) = 0.40 \Rightarrow P(X < a) = 0.60 \Rightarrow a = 81.01... = 81.0$ (1 d.p.)

ii $P(X < b) = 0.5636 \Rightarrow b = 80.64... = 80.6$ (1 d.p.)

b $P(b < X < a) = P(X < a) - P(X < b) = (1 - 0.40) - 0.5636 = 0.60 - 0.5636 = 0.0364$ (4 d.p.)



6 Use the inverse normal distribution function on your calculator, with $\mu = 4.5$ and $\sigma = 0.6$.

a By definition, the lower quartile is the point Q_1 such that $P(M < Q_1) = 0.25$.
$P(M < Q_1) = 0.25 \Rightarrow Q_1 = 4.0953... = 4.095$ kg (3 d.p.).

b Let a be the 80th percentile, so that $P(M < a) = 0.8$.
$P(M < a) = 0.8 \Rightarrow a = 5.0049... = 5.005$ kg (3 d.p.)

c The mean is 4.5, and since the data is normally distributed, this means that 50% of the badgers will have a mass less than 4.5 kg, i.e. $Q_2 = 4.5$ kg.

7 Use the inverse normal distribution function on your calculator, with $\mu = 72$ and $\sigma = 6$.

a $P(X < a) = 0.6 \Rightarrow a = 73.520... = 73.52$ (2 d.p.).

7 b $P(X < Q_1) = 0.25 \Rightarrow Q_1 = 67.953...$ and $P(X < Q_3) = 0.75 \Rightarrow Q_3 = 76.046...$
So the interquartile range = $Q_3 - Q_1 = 76.046 - 67.953 = 8.093 = 8.09$ (2 d.p.)

8 Use the inverse normal distribution function on your calculator, with $\mu = 60$ and $\sigma = 2$.

a $\mathrm{P}(Y > y) = 0.2 \Rightarrow \mathrm{P}(Y < y) = 0.8 \Rightarrow y = 61.683... = 61.68$ (2 d.p.)

b $\mathrm{P}(X < a) = 0.1 \Rightarrow a = 57.436...$ and $\mathrm{P}(X < b) = 0.9 \Rightarrow b = 62.563...$
So the 10% to 90% interpercentile range of masses is $b - a = 5.127 = 5.13$ grams (2 d.p.).

c Tom is correct: the data is assumed to be normally distributed, so the median is equal to the mean.

9 a The short coat should be suitable for the shortest 30% of the men.
Since $\mathrm{P}(H < a) = 0.3 \Rightarrow a = 165$, this means that the short coat should be suitable for the men who are up to 165 cm tall.
The long coat should be suitable for the tallest 20% of the men.
Since $\mathrm{P}(H > b) = 0.2 \Rightarrow \mathrm{P}(H < b) = 0.8 \Rightarrow b = 178$, this means that the long coat should be suitable for the men who are more than 178 cm tall.
The regular coat is then suitable for those in between, i.e. those men who are between 165 cm and 178 cm tall.

b There are many assumptions made by the model. It is assumed, for example, that the men likely to shop at the stores selling these frock coats have the same distribution of heights as those in the 'large group'; that the men's measurements from their necks to near the floor are normally distributed; that arm lengths are also normally distributed; that people are 'in proportion' (that, generally, taller men have longer arm lengths and higher shoulders than shorter men); and that the population follows the normal distribution over the whole range of values, i.e. that there are no extreme outliers.