The normal distribution 3B

1. Use the Normal CD function on your calculator, with $\mu = 30$ and $\sigma = 2$.

   a. Set a small value for the lower limit, e.g. 0.
   
   $P(X < 33) = 0.93319... = 0.9332$ (4 d.p.)

   ![Diagram of normal distribution with mean 30 and standard deviation 2, showing the area under the curve for $X < 33$.]

   b. Set a large value for the upper limit, e.g. 1000.
   
   $P(X > 26) = 0.97724... = 0.9772$ (4 d.p.)

   ![Diagram of normal distribution with mean 30 and standard deviation 2, showing the area under the curve for $X > 26$.]

   c. Set a large value for the upper limit, e.g. 1000.
   
   $P(X \geq 31.6) = P(X > 31.6) = 0.21185... = 0.2119$ (4 d.p.)

   ![Diagram of normal distribution with mean 30 and standard deviation 2, showing the area under the curve for $X \geq 31.6$.]

2. Use the Normal CD function on your calculator, with $\mu = 40$ and $\sigma = \sqrt{3} = 3$.

   a. Set a large value for the upper limit, e.g. 1000.
   
   $P(X > 45) = 0.04779... = 0.0478$ (4 d.p.)

   ![Diagram of normal distribution with mean 40 and standard deviation $\sqrt{3}$, showing the area under the curve for $X > 45$.]
2  
\( b \) Set a small value for the lower limit, e.g. 0.
\[ P(X \leq 38) = P(X < 38) = 0.25249... = 0.2525 \text{ (4 d.p.)} \]

\[ \text{Graph showing the normal distribution with } X \text{ between 38 and 40.} \]

\( c \) \( P(41 \leq X \leq 44) = P(41 < X < 44) = 0.27823... = 0.2782 \text{ (4 d.p.)} \)

\[ \text{Graph showing the normal distribution with } X \text{ between 40 and 44.} \]

3  
Use the Normal CD function on your calculator, with \( \mu = 25 \) and \( \sigma = \sqrt{25} = 5. \)

\( a \) Set a small value for the lower limit, e.g. 0.
\[ P(Y < 20) = 0.15865... = 0.1587 \text{ (4 d.p.)} \]

\[ \text{Graph showing the normal distribution with } Y \text{ between 20 and 25.} \]

\( b \) \( P(18 < Y < 26) = 0.49850... = 0.4985 \text{ (4 d.p.)} \)

\[ \text{Graph showing the normal distribution with } Y \text{ between 18 and 26.} \]

\( c \) Set a large value for the upper limit, e.g. 1000.
\[ P(Y > 23.8) = 0.59483... = 0.5948 \text{ (4 d.p.)} \]

4  
Use the Normal CD function on your calculator, with \( \mu = 18 \) and \( \sigma = \sqrt{10}. \)

\( a \) Set a large value for the upper limit, e.g. 1000.
\[ P(X \geq 20) = P(X > 20) = 0.26354... = 0.2635 \text{ (4 d.p.)} \]

\( b \) Set a small value for the lower limit, e.g. 0.
\[ P(X < 15) = 0.17139... = 0.1714 \text{ (4 d.p.)} \]
4\ c\ \ P(18.4 < X < 18.7) = 0.03726... = 0.0373 (4\ d.p.)

5\ Use\ the\ Normal\ CD\ function\ on\ your\ calculator,\ with\ \(\mu = 15\) and \(\sigma = 1.5\).

\ a\ \ i\ \ \text{Set a large value for the upper limit, e.g. 1000.}\ 
\hspace{1cm} P(M > 14) = 0.74750... = 0.7474 (4\ d.p.)

\ ii\ \ \text{Set a small value for the lower limit, e.g. 0.}\ 
\hspace{1cm} P(M < 14) = 0.25249... = 0.2525 (4\ d.p.)

\ b\ \ P(M > 14) + P(M < 14) = 0.7475 + 0.2525 = 1

The sum is 1, as the combined probabilities include every possible value.

6\ a\ \ Use\ the\ Normal\ CD\ function\ on\ your\ calculator,\ with\ \(\mu = 4.5, \sigma = \sqrt{0.4}\) and a small value for the lower limit.
\hspace{1cm} P(T < 4.2) = 0.31762... = 0.3176 (4\ d.p.)

\ b\ \ P(T > 4.2) = 1 - P(T < 4.2) = 1 - 0.3176 = 0.6824 (4\ d.p.)

7\ Use\ the\ Normal\ CD\ function\ on\ your\ calculator,\ with\ \(\mu = 45\) and \(\sigma = 2\).

\ a\ \ P(Y < 41 \text{ or } Y > 47) = 1 - P(41 < Y < 47)\ 
\hspace{1cm} \text{Using your calculator, } P(41 < Y < 47) = 0.81859...\ 
\hspace{1cm} \text{So } P(Y < 41 \text{ or } Y > 47) = 1 - 0.81859... = 0.1814 (4\ d.p.)

\ b\ \ P(Y < 44 \text{ or } 46.5 < Y < 47.5) = P(Y < 44) + P(46.5 < Y < 47.5)\ 
\hspace{1cm} \text{Using your calculator, } P(Y < 44) = 0.30853...\ \text{and } P(46.5 < Y < 47.5) = 0.12097...\ 
\hspace{1cm} \text{so } P(Y < 44 \text{ or } 46.5 < Y < 47.5) = 0.30853... + 0.12097... = 0.4295 (4\ d.p.)

8\ Use\ the\ Normal\ CD\ function\ on\ your\ calculator,\ with\ \(\mu = 6\) and \(\sigma = 0.8\).

\ a\ \ i\ \ A\ suitable\ upper\ limit\ is\ 10,\ giving\ P(X < 7) = 0.10564... = 0.1056 (4\ d.p.)

\ ii\ \ A\ suitable\ lower\ limit\ is\ 2,\ giving\ P(X < 5) = 0.10564... = 0.1056 (4\ d.p.)

\ b\ \ Since\ these\ are\ independent\ events,\ the\ probability\ is\ P(X < 5)^3,\ i.e.\ 
\hspace{1cm} (0.10564...)^3 = 0.00117... = 0.0012 (4\ d.p.)
9 Use the Normal CD function on your calculator, with \( \mu = 500 \) and \( \sigma = 14 \).

a i A suitable upper limit is 570, giving \( P(W > 505) = 0.36049... = 0.3605 \) (4 d.p.)

ii A suitable lower limit is 430, giving \( P(W < 490) = 0.23752... = 0.2375 \) (4 d.p.)

b Since these are independent events, the probability is \( P(W > 490)^4 \).
\[
P(W > 490) = 1 - P(W < 490) = 1 - 0.23752... = 0.76248...
\]
So the probability is \((0.76248...)^4 = 0.33799... = 0.3380\) (4 d.p.).

10 Use the Normal CD function on your calculator, with \( \mu = 165 \) and \( \sigma = 3.5 \).

a A suitable lower limit is 10, giving \( P(h < 160) = 0.07656... = 0.0766 \) (4 d.p.)

b \( P(168 < h < 174) = 0.19061... = 0.1906 \) (4 d.p.)

c Use the binomial distribution \( X \sim B(20, 0.1906) \).
Using the binomial CD function on your calculator:
\[
P(X \geq 5) = 1 - P(X < 4) = 1 - 0.67035... = 0.3296 \) (4 d.p.)

11 Use the Normal CD function on your calculator, with \( \mu = 13 \) and \( \sigma = 0.1 \).

a A suitable lower limit is 12.5, giving \( P(D < 12.8) = 0.02274... = 0.0227 \) (4 d.p.)

b \( P('perfect') = P(12.9 < D < 13.1) = 0.68268... = 0.6827 \) (4 d.p.)
Use the binomial distribution \( X \sim B(40, 0.6827) \).
Using the binomial CD function on your calculator:
\[
P(X > 25) = 1 - P(X \leq 25) = 1 - 0.26549... = 0.7345 \) (4 d.p.)

12 Use the Normal CD function on your calculator, with \( \mu = 480 \) and \( \sigma = 40 \).

a A suitable upper limit is 680, giving \( P(X > 490) = 0.40129... = 0.4013 \) (4 d.p.)

b \( P(470 < X < 490) = 0.19741... = 0.1974 \) (4 d.p.)
Use the binomial distribution \( Y \sim B(30, 0.1974) \).
Using the binomial CD function on your calculator:
\[
P(X \geq 15) = 1 - P(X \leq 14) = 1 - 0.99980141... = 0.0001986 \) (4 s.f.)