Conditional probability Mixed exercise 2

1 a \( P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.35 - 0.2 = 0.55 \)

b \( P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.55 = 0.45 \)

c \( \frac{P(B|A)}{P(A)} = \frac{0.2}{0.4} = 0.5 \)

d \( \frac{P(A'|B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{0.15}{0.35} = 0.429 \) (3 s.f.)

2 a Work out each region of the Venn diagram from the information provided in the question.

First, as \( J \) and \( L \) are mutually exclusive, \( P(J \cap L) = \emptyset \)
So \( P(J' \cap K' \cap L') = P(J) - P(J \cap K) = 0.25 - 0.1 = 0.15 \)

As \( K \) and \( L \) are independent \( P(K \cap L) = P(K) \times P(L) = 0.45 \times 0.15 = 0.0675 \)
So \( P(L' \cap K') = P(L) - P(L \cap K) = 0.15 - 0.0625 = 0.0825 \)
And \( P(K' \cap J' \cap L') = P(K) - P(J \cap K) - P(K \cap L) = 0.45 - 0.1 - 0.0675 = 0.2825 \)

Find the outer region by subtracting the sum of all the other regions from 1
\( P(J' \cap K' \cap L') = 1 - 0.15 - 0.1 - 0.2825 - 0.0675 - 0.0825 = 0.3175 \)

b i \( P(J \cup K) = 0.15 + 0.1 + 0.2825 + 0.0675 = 0.6 \)

ii \( P(J' \cap L') = 0.2825 + 0.3175 = 0.6 \)

iii \( P(J|K) = \frac{P(J \cap K)}{P(K)} = \frac{0.1}{0.45} = 0.222 \) (3 s.f.)

iv \( P(K|J' \cap L') = \frac{P(K \cap (J' \cap L'))}{P(J' \cap L')} = \frac{0.2825}{0.6} = 0.471 \) (3 s.f.)
3  

a \( P(F \cap S^\prime) + P(S \cap F^\prime) = P(F) - P(F \cap S) + P(F) - P(F \cap S) \)
\[
= \frac{35 - 27 + 45 - 27}{60} = \frac{26}{60} = 0.433 \text{ (3 s.f.)}
\]

b \( P(F|S) = \frac{P(F \cap S)}{P(S)} = \frac{27}{45} = 0.6 \)

c \( P(S|F^\prime) = \frac{P(S \cap F^\prime)}{P(F^\prime)} = \frac{45 - 27}{60 - 35} = \frac{18}{25} = 0.72 \)

d There are 6 students that study just French and wear glasses \( (8 \times 0.75 = 6) \) and 9 students that study just Spanish and wear glasses \( (18 \times 0.5 = 9) \), so the required probability is
\[
P(\text{studies one language and wears glasses}) = \frac{6 + 9}{60} = \frac{15}{60} = 0.25
\]

e There are 26 students studying one language (from part a). Of these, 15 wear glasses (from part d).
\[
P(\text{wears glasses|studies one language}) = \frac{15}{26} = 0.577 \text{ (3 s.f.)}
\]

4  

a

\[
\begin{array}{c}
\text{G} \\
\text{R} \\
\end{array}
\begin{array}{c}
\frac{5}{14} \\
\frac{9}{14} \\
\end{array}
\begin{array}{c}
\text{R} \\
\text{G} \\
\end{array}
\begin{array}{c}
\frac{6}{14} \\
\frac{8}{14} \\
\end{array}
\begin{array}{c}
\text{G} \\
\text{R} \\
\end{array}
\begin{array}{c}
\frac{5}{14} \\
\frac{9}{14} \\
\end{array}
\]

b i \( P(GG) = \frac{9}{15} \times \frac{8}{14} = \frac{3 \times 4}{5 \times 7} = \frac{12}{35} = 0.343 \text{ (3 s.f.)} \)

ii There are two different ways to obtain balls that are different colours:
\[
P(RG) + P(GR) = \left( \frac{6}{15} \times \frac{9}{14} \right) + \left( \frac{9}{15} \times \frac{6}{14} \right) = \frac{2 \times 9}{5 \times 7} = \frac{18}{35} = 0.514 \text{ (3 s.f.)}
\]

c There are 4 different outcomes:
\[
P(\text{RRR}) + P(\text{RGR}) + P(\text{GRR}) + P(\text{GGR})
\]
\[
= \left( \frac{6}{15} \times \frac{5}{14} \times \frac{4}{13} \right) + \left( \frac{6}{15} \times \frac{9}{14} \times \frac{5}{13} \right) + \left( \frac{9}{15} \times \frac{6}{14} \times \frac{5}{13} \right) + \left( \frac{9}{15} \times \frac{8}{14} \times \frac{6}{13} \right)
\]
\[
= \frac{120 + 270 + 270 + 432}{2730} = \frac{1092}{2730} = 0.4
\]
4 d The only way for this to occur is to draw a green ball each time. The corresponding probability is:

\[ P(GGGG) = \frac{9}{15} \times \frac{8}{14} \times \frac{7}{13} \times \frac{6}{12} = \frac{3 \times 2}{5 \times 3} = \frac{6}{65} = 0.0923 \text{ (3 s.f.)} \]

5 a Either Colin or Anne must win both sets. Therefore the required probability is:

\[ P(\text{match over in two sets}) = (0.7 \times 0.8) + (0.3 \times 0.6) = 0.56 + 0.18 = 0.74 \]

b \[ P(\text{Anne wins}\mid \text{match over in two sets}) = \frac{0.7 \times 0.8}{0.74} = \frac{0.56}{0.74} = 0.757 \text{ (3 s.f.)} \]

c The three ways that Anne can win the match are: wins first set, wins second set; wins first set, loses second set, wins tiebreaker; loses first set, wins second set, wins tiebreaker.

\[ P(\text{Anne wins match}) = (0.7 \times 0.8) + (0.7 \times 0.2 \times 0.55) + (0.3 \times 0.4 \times 0.55) = 0.56 + 0.077 + 0.066 = 0.703 \]

6 a There are 20 kittens with neither black nor white paws (75 – 26 – 14 – 15 = 20).

\[ P(\text{neither white or black paws}) = \frac{20}{75} = \frac{4}{15} = 0.267 \text{ (3 s.f.)} \]

b There are 41 kittens with some black paws (26 + 15 = 41).

\[ P(\text{black and white paws}\mid \text{some black paws}) = \frac{15}{41} = 0.366 \text{ (3 s.f.)} \]

c This is selection without replacement (since the first kitten chosen is not put back).

\[ P(\text{both kittens have all black paws}) = \frac{26}{75} \times \frac{25}{74} = \frac{13}{3 \times 37} = \frac{13}{111} = 0.117 \text{ (3 s.f.)} \]

d There are 29 kittens with some white paws (14 + 15 = 29).

\[ P(\text{both kittens have some white paws}) = \frac{29}{75} \times \frac{28}{74} = \frac{812}{5550} = 0.146 \text{ (3 s.f.)} \]

7 a Using the fact that \( A \) and \( B \) are independent: \( P(A) \times P(B) = P(A \cap B) \Rightarrow P(B) = P(A \cap B) = \frac{0.12}{0.4} = 0.3 \)

b Use the addition formula to find \( P(A \cup B) \)

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.3 - 0.12 = 0.58 \]

\[ P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.58 = 0.42 \]
7 e As \( A \) and \( C \) are mutually exclusive
\[
P(A \cap B' \cap C') = P(A) - P(A \cap B) = 0.4 - 0.12 = 0.28
\]
\[
P(C \cap A' \cap B') = P(C) - P(B \cap C) = 0.4 - 0.1 = 0.3
\]
\[
P(B \cap A' \cap C') = P(B) - P(A \cap B) - P(B \cap C) = 0.3 - 0.12 - 0.1 = 0.08
\]
Find the outer region by subtracting the sum of all the other regions from 1
\[
P(A' \cap B' \cap C') = 1 - 0.28 - 0.12 - 0.08 - 0.1 - 0.3 = 0.12
\]
\[
\begin{align*}
\text{A} & \quad 0.28 \\
\text{B} & \quad 0.08 \\
\text{C} & \quad 0.1 \\
\text{0.12} & \quad 0.3 \\
\text{0.12} & \quad \text{ } \\
\end{align*}
\]

7 f i \[
P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{0.1}{0.4} = 0.25
\]

7 f ii The required region must be contained within \( A \), and not include \( B \) (the condition on \( C \) is irrelevant since \( A \) and \( C \) are mutually exclusive). Therefore, \( P(A \cap (B' \cup C)) = 0.28 \)

8 a It may be that neither team scores in the match, and it is a 0–0 draw.

8 b \[
P(\text{team A scores first}) = P(\text{team A scores first and wins}) + P(\text{team A scores first and does not win})
\]
So \[
P(\text{team A scores first and does not win}) = 0.6 - 0.48 = 0.12
\]

8 c From the question \( P(\text{A wins|B scores first}) = 0.3 \). Using the multiplication formula gives
\[
P(\text{A wins|B scores first}) = \frac{P(\text{A wins \cap B scores first})}{P(B \text{ scores first})} = 0.3
\]
\[
\Rightarrow P(\text{A wins \cap B scores first}) = 0.3 \times 0.35 = 0.105
\]
Now find the required probability
\[
P(\text{B scores first|A wins}) = \frac{P(\text{A wins \cap B scores first})}{P(\text{A wins})} = \frac{0.105}{0.48 + 0.105} = \frac{0.105}{0.585} = 0.179 \text{ (3 s.f.)}
\]

Challenge

8 a Let \( P(A \cap B) = k \)
As \( P(A \cap B) \leq P(B) \Rightarrow k \leq 0.2 \)
\( A \) and \( B \) could be mutually exclusive, meaning \( P(A \cap B) = 0 \), so \( 0 \leq k \leq 2 \)
Now, \( P(A \cap B') = P(A) - P(A \cap B) \), so \( p = 0.6 - k \Rightarrow 0.4 \leq p \leq 0.6 \)

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.
Challenge

b Use the fact that \( P(A \cap C) = P(A \cap B \cap C) + P(A \cap B' \cap C) \)
So \( P(A \cap B' \cap C) = P(A \cap C) - P(A \cap B \cap C) = P(A \cap C) - 0.1 \)

Consider the range of \( P(A \cap C) \)
\( P(A \cap C) \leq P(A) \Rightarrow P(A \cap C) \leq 0.6 \)

By the multiplication formula \( P(A \cup C) = P(A) + P(C) - P(A \cap C) \)
So \( P(A \cap C) = P(A) + P(C) - P(A \cup C) = 1.3 - P(A \cup C) \)
As \( P(A \cup C) \leq 1 \Rightarrow P(A \cap C) \geq 0.3 \)

So \( 0.3 \leq P(A \cap C) \leq 0.6 \) and as \( P(A \cap B' \cap C) = P(A \cap C) - 0.1 \) this gives the result that
\( 0.3 - 0.1 \leq P(A \cap B' \cap C) \leq 0.6 - 0.1 \), so \( 0.2 \leq q \leq 0.5 \)