Conditional probability 2B

1 a There are 29 male students out of a total of 60 students.

$$\text{P(Male)} = \frac{29}{60}$$

b Restrict the sample space to the 29 male students; 18 of these prefer curry.

$$\text{P(Curry|Male)} = \frac{18}{29}$$

c Restrict the sample space to the 35 students that prefer curry; 18 of these are male.

$$\text{P(Male|Curry)} = \frac{18}{35}$$

d Restrict the sample space to the 31 female students; 14 of these prefer pizza.

$$\text{P(Pizza|Female)} = \frac{14}{31}$$

2 a By simple subtraction, there are 43 male members of the club (75 – 32 = 43). Of these 21 play badminton (43 – 22 = 21).

	Badminton	**Squash**	**Total**
Male	21	22	43
Female	15	17	32
Total	36	39	75

b i Restrict the sample space to the 39 members that play squash; 22 of these are male.

$$\text{P(Male|Squash)} = \frac{22}{39}$$

ii Restrict the sample space to the 36 members that play badminton; 15 of these are female.

$$\text{P(Female|Badminton)} = \frac{15}{36} = \frac{5}{12}$$

iii Restrict the sample space to the 32 members that are female; 17 of these play squash.

$$\text{P(Squash|Female)} = \frac{17}{32}$$

3 a There are 35 boys (80 – 45 = 35), of which 10 like chocolate (35 – 2 – 23 = 10).
Of the girls, 20 like strawberry (45 – 13 – 12 = 20).

	Girls	**Boys**	**Total**
Vanilla	13	2	15
Chocolate	12	10	22
Strawberry	20	23	43
Total	45	35	80

b i Restrict the sample space to the 43 children that like strawberry; 23 of these are boys.

$$\text{P(Boy|Strawberry)} = \frac{23}{43}$$

ii Restrict the sample space to the 15 children that like vanilla; 13 of these are girls.

$$\text{P(Girl|Vanilla)} = \frac{13}{15}$$

iii Restrict the sample space to the 35 boys; 10 of these like chocolate.

$$\text{P(Chocolate|Boy)} = \frac{10}{35} = \frac{2}{7}$$

4 a

Blue spinner

Red spinner	**1**	**2**	**3**	**4**
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

b i There 4 outcomes where $X = 5$, and 16 possible outcomes in total.

$$\text{P}(X = 5) = \frac{4}{16} = \frac{1}{4}$$

ii There are 4 equally likely outcomes where the red spinner is 2; and for one of these $X = 3$.

$$\text{P}(X = 3|\text{Red spinner is 2}) = \frac{1}{4}$$

iii There are 4 equally likely outcomes where $X = 5$, and for one of these the blue spinner is 3.

$$\text{P(Blue spinner is 3}|X = 5) = \frac{1}{4}$$

5 a

Dice 1

Dice 2

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

b There are 6 outcomes where Dice 1 shows 5, and for one of these the product is 20.

$$\text{P(Product is 20|Dice 1 shows a 5)} = \frac{1}{6}$$

c There are 4 outcomes where the product is 2, and for one of these Dice 2 shows a 6.

$$\text{P(Dice 2 shows a 6|Product is 12)} = \frac{1}{4}$$

d All outcomes are equally likely.

6 $$\text{P(Ace|Diamond)} = \frac{\text{P(Ace of diamonds)}}{\text{P(Diamond)}} = \frac{\frac{1}{52}}{\frac{13}{52}} = \frac{1}{13}$$

7 Drawing a sample space diagram can be helpful in answering this question.

Coin 1

Coin 2

	H	**T**
H	HH	TH
T	HT	TT

a Note there are three outcomes where at least one coin lands on a head.

$$\text{P(HH|H)} = \frac{\text{P(Head and Head)}}{\text{P(Head)}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

7 b $$\text{P(Head and Tail|Head)} = \frac{\text{P(Head and Tail)}}{\text{P(Head)}}$$

$$= \frac{\frac{2}{4}}{\frac{3}{4}} = \frac{2}{3}$$

c Assume that the coins are not biased.

8 a 64 students do not watch sport (120 – 56 = 64).
43 students do not watch drama (120 – 77 = 43).

Use the fact that of those who watch drama, 18 also watch sport to complete the table.
For example, this means that 38 students who watch sport do not watch drama (56 – 18 = 38), and 59 students who watch drama do not watch sport (77 – 18 = 59).

Given that 43 students do not watch drama, but 38 students who do not watch drama watch sport, this means 5 students do not watch drama or sport (43 – 38 = 5).

	Watches drama (D)	**Does not watch drama (D′)**	**Total**
Watches sport (S)	18	38	56
Does not watch sport (S′)	59	5	64
Total	77	43	120

b i The probability that the student does not watch drama.

$$\text{P}(D') = \frac{43}{120}$$

ii The probability that the student does not watch sport ot drama.

$$\text{P}(S' \cap D') = \frac{5}{120} = \frac{1}{24}$$

iii The probability that the student also watches sport if they watch drama.

$$\text{P}(S|D) = \frac{18}{77}$$

iv The probability that the student does not watch drama if they watch sport.

$$\text{P}(D'|S) = \frac{38}{56} = \frac{19}{28}$$

9 a

	Women	**Men**	**Total**
Stick	26	18	44
No stick	37	29	66
Total	63	47	110

b i $\text{P(Uses a stick)} = \frac{44}{110} = \frac{2}{5}$

ii Restrict the sample space to the 63 women; 26 of these use a stick.

$\text{P(Uses a stick|Female)} = \frac{26}{63}$

iii Restrict the sample space to those who use a stick; 18 of these are men.

$\text{P(Male|Uses a stick)} = \frac{18}{44} = \frac{9}{22}$

10 Build up a table to show the options as follows. First note that as there are 450 female owners, so there are 300 male owners (750 – 450 = 300). Consider those who own cats. 320 owners in total own a cat. Since no one owns more than one type of pet, this means that 430 owners do not own a cat (750 – 320 = 430).

175 female owners have a cat. Since there are 450 female owners, this means that 275 female owners do not own a cat (450 – 175 = 275). 145 male owners own a cat (320 – 175 = 145) and so 155 male owners do not own a cat (300 – 145 = 155). This gives this table:

	Owns a cat	**Does not own a cat**	**Total**
Female	175	275	450
Male	145	155	300
Total	320	430	750

Of the 430 owners who do not own a cat, 250 of them own a dog. Therefore 180 of the owners own another type of pet (430 – 250 = 180). Since 25 males own another type of pet, this means that 155 women own another type of pet (180 – 25 = 155).

10 Finally, of the 450 women, 175 own a cat and 155 own something other than a cat or a dog. Therefore 120 women own a dog (450 – 175 –155 = 120) and 130 men own a dog (300 – 145 – 25 = 130). This information is summarised in this table:

	Owns a cat	Owns a dog	Owns another type of pet	Total
Female	175	120	155	450
Male	145	130	25	300
Total	320	250	180	750

a The probability that the owner does not own a dog or a cat.

$$\mathrm{P}(D' \cap C') = \frac{180}{750} = \frac{6}{25}$$

b The probability that a male owner (i.e. not female) owns a dog.

$$\mathrm{P}(D|F') = \frac{130}{300} = \frac{13}{30}$$

c The probability that a cat owner is male (i.e. not female).

$$\mathrm{P}(F'|C) = \frac{145}{320} = \frac{29}{64}$$

d The probability that a female owner does not own a dog or a cat.

$$\mathrm{P}((D' \cap C')|F) = \frac{155}{450} = \frac{31}{90}$$