

Vectors 12A

$$\begin{aligned}
 1 \quad OP &= \sqrt{2^2 + 8^2 + (-4)^2} \\
 &= \sqrt{4 + 64 + 16} = \sqrt{84} \\
 &= 2\sqrt{21} \approx 9.17 \text{ (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad OP &= \sqrt{7^2 + 7^2 + 7^2} \\
 &= \sqrt{49 + 49 + 49} = \sqrt{147} \\
 &= 7\sqrt{3} \approx 12.1 \text{ (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad a \quad AB &= \sqrt{(3-1)^2 + (0-(-1))^2 + (5-8)^2} \\
 &= \sqrt{2^2 + 1^2 + (-3)^2} \\
 &= \sqrt{14} \approx 3.74 \text{ (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 b \quad AB &= \sqrt{(8-(-3))^2 + (11-1)^2 + (8-6)^2} \\
 &= \sqrt{11^2 + 10^2 + 2^2} \\
 &= \sqrt{225} = 15
 \end{aligned}$$

$$\begin{aligned}
 c \quad AB &= \sqrt{(3-3)^2 + (5-10)^2 + (-2-3)^2} \\
 &= \sqrt{0^2 + (-5)^2 + (-5)^2} \\
 &= \sqrt{50} = 5\sqrt{2} \approx 7.07 \text{ (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 d \quad AB &= \sqrt{(-1-4)^2 + (-2-(-1))^2 + (5-3)^2} \\
 &= \sqrt{(-5)^2 + (-1)^2 + 2^2} \\
 &= \sqrt{30} \approx 5.48 \text{ (3 s.f.)}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad AB &= \sqrt{(7-k)^2 + (-1-0)^2 + (2-4)^2} = 3 \\
 &= \sqrt{(49-14k+k^2)+1+4} = 3 \\
 49-14k+k^2+1+4 &= 9 \\
 k^2-14k+45 &= 0 \\
 (k-5)(k-9) &= 0 \\
 k &= 5 \text{ or } k = 9
 \end{aligned}$$

$$\begin{aligned}
 5 \quad AB &= \sqrt{(5-1)^2 + (3-k)^2 + (-8-(-3))^2} \\
 &= 3\sqrt{10} \\
 \sqrt{16+(9-6k+k^2)+25} &= 3\sqrt{10} \\
 16+9-6k+k^2+25 &= 9 \times 10 \\
 k^2-6k-40 &= 0 \\
 (k+4)(k-10) &= 0 \\
 k &= -4 \text{ or } k = 10
 \end{aligned}$$

Challenge

- a Coordinates of other points in the plane $x = 1$ will be $(1, -3, 4)$ and $(1, -3, -2)$.

Coordinates of other points in the plane $x = 7$ will be $(7, 3, 4)$, $(7, 3, -2)$ and $(7, -3, -2)$.

- b Shortest route for the ant will be from A to half way along one of the opposite edges and then across the next face to C .

$$\begin{aligned}
 \text{Distance} &= 2 \times \sqrt{6^2 + 3^2} = 2 \times \sqrt{45} \\
 &= 2 \times 3\sqrt{5} = 6\sqrt{5}
 \end{aligned}$$