

## Integration 11A

$$\begin{aligned}
 \mathbf{1 \ a} \quad & \int \left( 3 \sec^2 x + \frac{5}{x} + \frac{2}{x^2} \right) dx \\
 &= \int \left( 3 \sec^2 x + \frac{5}{x} + 2x^{-2} \right) dx \\
 &= 3 \tan x + 5 \ln |x| - \frac{2}{x} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & \int (5e^x - 4 \sin x + 2x^3) dx \\
 &= 5e^x + 4 \cos x + \frac{2x^4}{4} + c \\
 &= 5e^x + 4 \cos x + \frac{x^4}{2} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad & \int 2(\sin x - \cos x + x) dx \\
 &= \int (2 \sin x - 2 \cos x + 2x) dx \\
 &= -2 \cos x - 2 \sin x + x^2 + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad & \int \left( 3 \sec x \tan x - \frac{2}{x} \right) dx \\
 &= 3 \sec x - 2 \ln |x| + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} \quad & \int \left( 5e^x + 4 \cos x - \frac{2}{x^2} \right) dx \\
 &= \int (5e^x + 4 \cos x - 2x^{-2}) dx \\
 &= 5e^x + 4 \sin x + \frac{2}{x} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{f} \quad & \int \left( \frac{1}{2x} + 2 \operatorname{cosec}^2 x \right) dx \\
 &= \int \left( \frac{1}{2} \times \frac{1}{x} + 2 \operatorname{cosec}^2 x \right) dx \\
 &= \frac{1}{2} \ln |x| - 2 \cot x + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{g} \quad & \int \left( \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} \right) dx \\
 &= \int \left( \frac{1}{x} + x^{-2} + x^{-3} \right) dx \\
 &= \ln |x| + \frac{x^{-1}}{-1} + \frac{x^{-2}}{-2} + c \\
 &= \ln |x| - \frac{1}{x} - \frac{1}{2x^2} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{h} \quad & \int (e^x + \sin x + \cos x) dx \\
 &= e^x - \cos x + \sin x + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{i} \quad & \int (2 \operatorname{cosec} x \cot x - \sec^2 x) dx \\
 &= -2 \operatorname{cosec} x - \tan x + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{j} \quad & \int \left( e^x + \frac{1}{x} - \operatorname{cosec}^2 x \right) dx \\
 &= e^x + \ln |x| + \cot x + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{2 \ a} \quad & \int \left( \frac{1}{\cos^2 x} + \frac{1}{x^2} \right) dx \\
 &= \int (\sec^2 x + x^{-2}) dx \\
 &= \tan x - \frac{1}{x} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & \int \left( \frac{\sin x}{\cos^2 x} + 2e^x \right) dx \\
 &= \int (\tan x \sec x + 2e^x) dx \\
 &= \sec x + 2e^x + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad & \int \left( \frac{1 + \cos x}{\sin^2 x} + \frac{1+x}{x^2} \right) dx \\
 &= \int (\operatorname{cosec}^2 x + \cot x \operatorname{cosec} x + x^{-2} + x^{-1}) dx \\
 &= -\cot x - \operatorname{cosec} x - \frac{1}{x} + \ln |x| + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{2 \ d} \quad & \int \left( \frac{1}{\sin^2 x} + \frac{1}{x} \right) dx \\
 &= \int \left( \operatorname{cosec}^2 x + \frac{1}{x} \right) dx \\
 &= -\cot x + \ln |x| + c
 \end{aligned}$$

$$\begin{aligned}
 2 \text{ e } \int \sin x(1 + \sec^2 x) dx &= \int (\sin x + \sin x \sec^2 x) dx \\
 &= \int (\sin x + \tan x \sec x) dx \\
 &= -\cos x + \sec x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{f } \int \cos x(1 + \operatorname{cosec}^2 x) dx &= \int (\cos x + \cos x \operatorname{cosec}^2 x) dx \\
 &= \int (\cos x + \cot x \operatorname{cosec} x) dx \\
 &= \sin x - \operatorname{cosec} x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{g } \int \operatorname{cosec}^2 x(1 + \tan^2 x) dx &= \int (\operatorname{cosec}^2 x + \operatorname{cosec}^2 x \tan^2 x) dx \\
 &= \int (\operatorname{cosec}^2 x + \sec^2 x) dx \\
 &= -\cot x + \tan x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{h } \int \sec^2 x(1 - \cot^2 x) dx &= \int (\sec^2 x - \sec^2 x \cot^2 x) dx \\
 &= \int (\sec^2 x - \operatorname{cosec}^2 x) dx \\
 &= \tan x + \cot x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{i } \int \sec^2 x(1 + e^x \cos^2 x) dx &= \int (\sec^2 x + e^x \cos^2 x \sec^2 x) dx \\
 &= \int (\sec^2 x + e^x) dx \\
 &= \tan x + e^x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{j } \int \left( \frac{1 + \sin x}{\cos^2 x} + \cos^2 x \sec x \right) dx &= \int (\sec^2 x + \tan x \sec x + \cos x) dx \\
 &= \tan x + \sec x + \sin x + c
 \end{aligned}$$

$$3 \text{ a } \int_3^7 2e^x dx = [2e^x]_3^7 = 2e^7 - 2e^3$$

$$\begin{aligned}
 \text{b } \int_1^6 \frac{1+x}{x^3} dx &= \int_1^6 \left( \frac{1}{x^3} + \frac{1}{x^2} \right) dx \\
 &= \left[ -\frac{1}{2x^2} - \frac{1}{x} \right]_1^6 = \left( -\frac{1}{72} - \frac{1}{6} \right) - \left( -\frac{1}{2} - 1 \right) \\
 &= -\frac{13}{72} + \frac{108}{72} = \frac{95}{72}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \int_{\frac{\pi}{2}}^{\pi} -5 \sin x dx &= [5 \cos x]_{\frac{\pi}{2}}^{\pi} \\
 &= 5 \cos \pi - 5 \cos \frac{\pi}{2} = -5 - 0 = -5
 \end{aligned}$$

$$\begin{aligned}
 \text{d } \int_{-\frac{\pi}{4}}^0 \sec x(\sec x + \tan x) dx &= \int_{-\frac{\pi}{4}}^0 (\sec^2 x + \sec x \tan x) dx \\
 &= [\tan x + \sec x]_{-\frac{\pi}{4}}^0 = (0 + 1) - (-1 + \sqrt{2}) \\
 &= 2 - \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 4 \int_a^{2a} \frac{3x-1}{x} dx &= \int_a^{2a} 3 - \frac{1}{x} dx = [3x - \ln|x|]_a^{2a} \\
 &= (6a - \ln 2a) - (3a - \ln a) \quad (a \text{ is positive}) \\
 &= 3a - \ln 2a + \ln a \\
 &= 3a - (\ln 2 + \ln a) + \ln a \\
 &= 3a - \ln 2
 \end{aligned}$$

$$\begin{aligned}
 &= 3a + \ln\left(\frac{1}{2}\right) = 6 + \ln\left(\frac{1}{2}\right) \\
 \text{so } a &= 2.
 \end{aligned}$$

$$\begin{aligned}
 5 \int_{\ln 1}^{\ln a} e^x + e^{-x} dx &= [e^x - e^{-x}]_{\ln 1}^{\ln a} \\
 &= (e^{\ln a} - e^{-\ln a}) - (e^{\ln 1} - e^{-\ln 1}) \\
 &= \left(a - \frac{1}{a}\right) - (1 - 1) \\
 \text{So } a - \frac{1}{a} &= \frac{48}{7} \\
 7a^2 - 48a - 7 &= 0 \\
 (7a+1)(a-7) &= 0 \\
 a &= 7 \text{ since } a > 0.
 \end{aligned}$$

$$\begin{aligned}
 6 \int_2^b (3e^x + 6e^{-2x}) dx &= [3e^x - 3e^{-2x}]_2^b \\
 &= 3((e^b - e^{-2b}) - (e^2 - e^{-4})) = 0 \\
 (e^b - e^{-2b}) - (e^2 - e^{-4}) &= 0 \\
 \text{so } b &= 2.
 \end{aligned}$$

$$\begin{aligned}
 7 \text{ a } f(x) &= \frac{1}{8} x^{\frac{3}{2}} - \frac{4}{x} = 0 \\
 \frac{1}{8} x^{\frac{5}{2}} - 4 &= 0 \text{ since } x \neq 0 \\
 x^{\frac{5}{2}} &= 32 \Rightarrow x = 4
 \end{aligned}$$

$$7 \text{ b } \int \left( \frac{1}{8}x^{\frac{3}{2}} - \frac{4}{x} \right) dx = \frac{x^{\frac{5}{2}}}{20} - 4 \ln|x| + c$$

$$\begin{aligned} \text{c } \int_1^4 f(x) dx &= \left[ \frac{x^{\frac{5}{2}}}{20} - 4 \ln|x| \right]_1^4 \\ &= \left( \frac{32}{20} - 4 \ln 4 \right) - \left( \frac{1}{20} - 4 \ln 1 \right) \\ &= \frac{31}{20} - 4 \ln 4 \end{aligned}$$