Radians 5C

1 a Using $l = r\theta$:

i $l = 6 \times 0.45 = 2.7$

ii $l = 4.5 \times 0.45 = 2.025$

iii $l = 20 \times \frac{3}{8}\pi = 7.5\pi$ (23.6 to 3 s.f.)

b Using $r = \frac{l}{\theta}$:

i $r = \frac{10}{0.6} = \frac{50}{3}$

ii $r = \frac{1.26}{0.7} = 1.8$

iii $r = \frac{1.5\pi}{\frac{5}{12}\pi} = 1.5 \times \frac{12}{5} = \frac{18}{5} = 3.6$

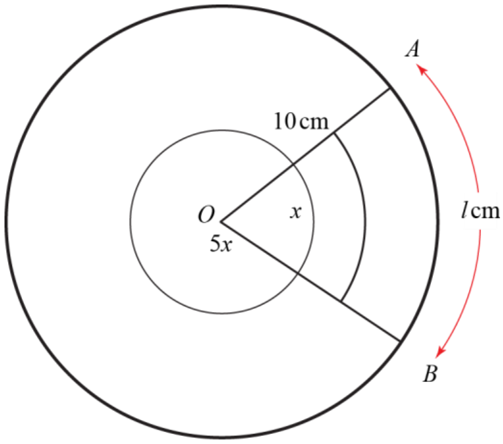
c Using $\theta = \frac{l}{r}$:

i $\theta = \frac{10}{7.5} = \frac{4}{3}$

ii $\theta = \frac{4.5}{5.625} = 0.8$

iii $\theta = \frac{\sqrt{12}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$

2



The total angle at the centre is $6x$ so

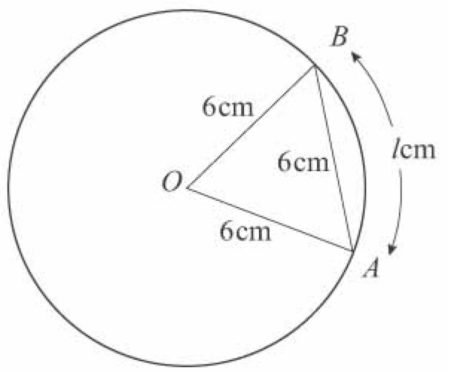
$6x = 2\pi$

$x = \frac{\pi}{3}$

Using $l = r\theta$ to find the minor arc AB:

$l = 10 \times \frac{\pi}{3} = \frac{10\pi}{3}$ cm

3



Triangle OAB is equilateral, so $\angle AOB = \frac{\pi}{3}$

Using $l = r\theta$:

$l = 6 \times \frac{\pi}{3} = 2\pi$

4 $r = \sqrt{10}$ cm and $\theta = \sqrt{5}$ rad

Using $l = r\theta$:

$l = \sqrt{10} \times \sqrt{5} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$

5 a Using $l = r\theta$:

length of shorter arc $= 3 \times 0.8 = 2.4$ cm

length of longer arc $= (3+2) \times 0.8 = 4$ cm

Perimeter $= 2.4$ cm $+ 2$ cm $+ 4$ cm $+ 2$ cm

$= 10.4$ cm

b Length of shorter arc $= 3\theta$ cm

Length of longer arc $= 5\theta$ cm

So perimeter $= (3\theta + 5\theta + 2 + 2)$ cm

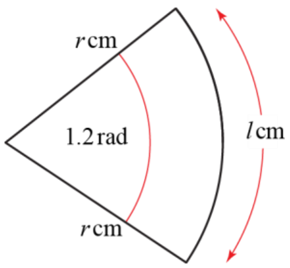
As the perimeter $= 14$ cm,

$8\theta + 4 = 14$

$8\theta = 10$

$\theta = \frac{10}{8} = 1.25$ rad

6



Using $l = r\theta$, the arc length $= 1.2r$ cm.

The area of the square $= 36\text{ cm}^2$, so each side $= 6$ cm and the perimeter is, therefore, 24 cm.

The perimeter of the sector

$=$ arc length $+ 2r$ cm

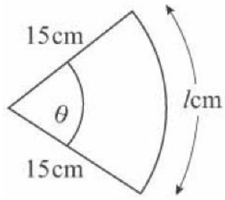
$= (1.2r + 2r)$ cm $= 3.2r$ cm

Perimeter of square $=$ perimeter of sector, so

$24 = 3.2r$

$r = \frac{24}{3.2} = 7.5$

7



Using $l = r\theta$:

the arc length of the sector $= 15\theta$ cm

So the perimeter $= (15\theta + 30)$ cm

As the perimeter $= 42$ cm,

$15\theta + 30 = 42$

$15\theta = 12$

$\theta = \frac{12}{15} = 0.8$

8 a $\angle COA = \pi - \frac{2}{3}\pi = \frac{\pi}{3}$

b The perimeter of the brooch

$= AB +$ arc $BC +$ chord AC

$AB = 4$ cm

$l = r\theta$ with $r = 2$ cm and $\theta = \frac{2}{3}\pi$

So length of arc $BC = 2 \times \frac{2}{3}\pi = \frac{4}{3}\pi$ cm

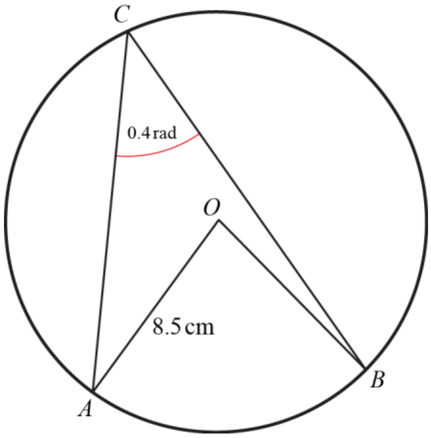
As $\angle COA = \frac{\pi}{3}$ (60°), triangle COA is equilateral.

So length of chord $AC = 2$ cm

So perimeter $= 4\text{ cm} + \frac{4}{3}\pi\text{ cm} + 2\text{ cm}$

$= \left(6 + \frac{4}{3}\pi\right)$ cm

9



Using the circle theorem, that angle subtended at the centre of a circle = 2 × angle subtended at the circumference:

$\angle AOB = 2\angle ACB = 0.8\,\text{rad}$

Using $l = r\theta$:

length of minor arc $AB = 8.5 \times 0.8\,\text{cm}$

$= 6.8\,\text{cm}$

10 a $OC = R - r$

b
$$OC = R - r$$
$$\sin\theta = \frac{r}{R-r}$$
$$(R-r)\sin\theta = r$$
$$R\sin\theta - r\sin\theta = r$$
$$R\sin\theta = r + r\sin\theta$$
$$= r(1+\sin\theta)$$

c $R\sin\theta = r(1 + \sin\theta)$
$$\frac{3}{4}R = r\left(1+\frac{3}{4}\right)$$
$$r = \frac{3}{7}R$$
$$\sin\theta = \frac{3}{4} \Rightarrow \theta = 0.848...$$
$$2R + 2R\theta = 21$$
$$2R + 1.696R = 21$$
$$3.696R = 21$$
$$R = 5.681\,\text{cm}$$
$$r = \frac{3}{7} \times R = 2.43\,\text{cm}$$

11 Length of arc $= r\theta$
$$\text{Perimeter} = 2r + r\theta$$
$$2r + r\theta = 2r\theta$$
$$2r = r\theta$$
$$\theta = 2\text{ rad}$$

12 a $\theta = \frac{2\pi}{24} = \frac{\pi}{12}$
$$r\theta = \frac{3\pi}{2}$$
$$r = \frac{3\pi}{2} \div \frac{\pi}{12} = 18\,\text{m}$$
$$d = 36\,\text{m}$$

b $C = \pi d = 36\pi$
$$\text{Speed} = \frac{36\pi \times 60 \times 60}{30 \times 1000}$$
$$= 13.6\,\text{km/h}$$

13 a $SR = 7 \times 0.5 = 3.5\text{ m}$

b Using the cosine rule:
$$QR^2 = 7^2 + 12^2 - 2 \times 7 \times 12 \times \cos 0.5$$
$$QR = 6.75\,\text{m}$$
$$SQ = PQ - PS = 12 - 7 = 5\,\text{m}$$
$$\text{Perimeter} = 6.75 + 5 + 3.5$$
$$= 15.3\,\text{m (3 s.f.)}$$

14 a $\angle XOZ = \frac{2\pi - 1.1}{2} = 2.59\text{ rad}$

b Using the cosine rule:
$$XZ^2 = 5^2 + 15^2 - 2 \times 5 \times 15 \times \cos 2.59$$
$$XZ = 19.44\,\text{mm}$$
$$\text{Arc length } YZ = 5 \times 1.1 = 5.5\,\text{mm}$$
$$\text{Perimeter} = 19.44 \times 2 + 5.5 \approx 44\,\text{mm}$$