

**Binomial expansion 4B**

**1 a i**  $\sqrt{(4+2x)}$  Write in index form.

$$= (4+2x)^{\frac{1}{2}} \quad \text{Take out a factor of 4}$$

$$= \left(4\left(1+\frac{2x}{4}\right)\right)^{\frac{1}{2}} \quad \text{Remember to put the 4 to the power } \frac{1}{2}$$

$$= 4^{\frac{1}{2}} \left(1+\frac{x}{2}\right)^{\frac{1}{2}} \quad 4^{\frac{1}{2}} = 2$$

$$= 2 \left(1+\frac{x}{2}\right)^{\frac{1}{2}} \quad \text{Use the expansion with } n = \frac{1}{2} \text{ and } x = \frac{x}{2}$$

$$= 2 \left(1 + \frac{\left(\frac{1}{2}\right)\left(\frac{x}{2}\right)}{\left(\frac{2}{2}\right)} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{x}{2}\right)^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{x}{2}\right)^3}{3!} + \dots\right)$$

$$= 2 \left(1 + \frac{x}{4} - \frac{x^2}{32} + \frac{x^3}{128} + \dots\right) \quad \text{Multiply by the 2}$$

$$= 2 + \frac{x}{2} - \frac{x^2}{16} + \frac{x^3}{64} + \dots$$

**ii** Valid if  $\left|\frac{x}{2}\right| < 1 \Rightarrow |x| < 2$

**b i**  $\frac{1}{2+x}$  Write in index form

$$= (2+x)^{-1} \quad \text{Take out a factor of 2}$$

$$= \left(2\left(1+\frac{x}{2}\right)\right)^{-1} \quad \text{Remember to put 2 to the power } -1$$

$$= 2^{-1} \left(1+\frac{x}{2}\right)^{-1}, \quad 2^{-1} = \frac{1}{2}. \quad \text{Use the binomial expansion with } n = -1 \text{ and } x = \frac{x}{2}$$

$$= \frac{1}{2} \left(1 + (-1)\left(\frac{x}{2}\right) + \frac{(-1)(-2)\left(\frac{x}{2}\right)^2}{2!} + \frac{(-1)(-2)(-3)\left(\frac{x}{2}\right)^3}{3!} + \dots\right)$$

$$= \frac{1}{2} \left(1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \dots\right) \quad \text{Multiply by the } \frac{1}{2}$$

$$= \frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16} + \dots$$

**ii** Valid if  $\left|\frac{x}{2}\right| < 1 \Rightarrow |x| < 2$

**1 c i**  $\frac{1}{(4-x)^2}$  Write in index form  
 $= (4-x)^{-2}$  Take 4 out as a factor  
 $= \left(4\left(1-\frac{x}{4}\right)\right)^{-2}$   
 $= 4^{-2}\left(1-\frac{x}{4}\right)^{-2}$ ,  $4^{-2} = \frac{1}{16}$ . Use the binomial expansion with  $n = -2$  and  $x = -\frac{x}{4}$   
 $= \frac{1}{16}\left(1+(-2)\left(-\frac{x}{4}\right) + \frac{(-2)(-3)}{2!}\left(-\frac{x}{4}\right)^2 + \frac{(-2)(-3)(-4)}{3!}\left(-\frac{x}{4}\right)^3 + \dots\right)$   
 $= \frac{1}{16}\left(1+\frac{x}{2} + \frac{3x^2}{16} + \frac{x^3}{16} + \dots\right)$  Multiply by  $\frac{1}{16}$   
 $= \frac{1}{16} + \frac{x}{32} + \frac{3x^2}{256} + \frac{x^3}{256} + \dots$

**ii** Valid if  $\left|\frac{x}{4}\right| < 1 \Rightarrow |x| < 4$

**d i**  $\sqrt{9+x}$  Write in index form  
 $= (9+x)^{\frac{1}{2}}$  Take 9 out as a factor  
 $= \left(9\left(1+\frac{x}{9}\right)\right)^{\frac{1}{2}}$   
 $= 9^{\frac{1}{2}}\left(1+\frac{x}{9}\right)^{\frac{1}{2}}$ ,  $9^{\frac{1}{2}} = 3$ . Use binomial expansion with  $n = \frac{1}{2}$  and  $x = \frac{x}{9}$   
 $= 3\left(1+\left(\frac{1}{2}\right)\left(\frac{x}{9}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{9}\right)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{x}{9}\right)^3 + \dots\right)$   
 $= 3\left(1+\frac{x}{18} - \frac{x^2}{648} + \frac{x^3}{11664} + \dots\right)$  Multiply by 3  
 $= 3 + \frac{x}{6} - \frac{x^2}{216} + \frac{x^3}{3888} + \dots$

**ii** Valid for  $\left|\frac{x}{9}\right| < 1 \Rightarrow |x| < 9$

**1 e i**  $\frac{1}{\sqrt{2+x}}$  Write in index form  
 $= (2+x)^{-\frac{1}{2}}$  Take out a factor of 2  
 $= \left(2\left(1+\frac{x}{2}\right)\right)^{-\frac{1}{2}}$   
 $= 2^{-\frac{1}{2}}\left(1+\frac{x}{2}\right)^{-\frac{1}{2}}$ ,  $2^{-\frac{1}{2}} = \frac{1}{2^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}$ . Use binomial expansion with  $n = -\frac{1}{2}$  and  $x = \frac{x}{2}$

$$= \frac{1}{\sqrt{2}} \left( 1 + \left(-\frac{1}{2}\right)\left(\frac{x}{2}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(\frac{x}{2}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(\frac{x}{2}\right)^3 + \dots \right)$$

$$= \frac{1}{\sqrt{2}} \left( 1 - \frac{x}{4} + \frac{3x^2}{32} - \frac{5x^3}{128} + \dots \right) \text{ Multiply by } \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} - \frac{x}{4\sqrt{2}} + \frac{3x^2}{32\sqrt{2}} - \frac{5x^3}{128\sqrt{2}} + \dots \text{ Rationalise surds}$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}x}{8} + \frac{3\sqrt{2}x^2}{64} - \frac{5\sqrt{2}x^3}{256} + \dots$$

**ii** Valid if  $\left|\frac{x}{2}\right| < 1 \Rightarrow |x| < 2$

**f i**  $\frac{5}{3+2x}$  Write in index form  
 $= 5(3+2x)^{-1}$  Take out a factor of 3  
 $= 5\left(3\left(1+\frac{2x}{3}\right)\right)^{-1}$   
 $= 5 \times 3^{-1} \left(1+\frac{2x}{3}\right)^{-1}$ ,  $3^{-1} = \frac{1}{3}$ . Use binomial expansion with  $n = -1$  and  $x = \frac{2x}{3}$

$$= \frac{5}{3} \left( 1 + (-1)\left(\frac{2x}{3}\right) + \frac{(-1)(-2)}{2!}\left(\frac{2x}{3}\right)^2 + \frac{(-1)(-2)(-3)}{3!}\left(\frac{2x}{3}\right)^3 + \dots \right)$$

$$= \frac{5}{3} \left( 1 - \frac{2x}{3} + \frac{4x^2}{9} - \frac{8x^3}{27} + \dots \right) \text{ Multiply by } \frac{5}{3}$$

$$= \frac{5}{3} - \frac{10x}{9} + \frac{20x^2}{27} - \frac{40x^3}{81} + \dots$$

**ii** Valid if  $\left|\frac{2x}{3}\right| < 1 \Rightarrow |x| < \frac{3}{2}$

1 g i

$$\begin{aligned}
\frac{1+x}{2+x} &= 1 - \frac{1}{2+x} \quad \text{Write } \frac{1}{2+x} \text{ in index form} \\
&= 1 - (2+x)^{-1} \quad \text{Take out a factor of 2} \\
&= 1 - \left( 2 \left( 1 + \frac{x}{2} \right) \right)^{-1} \\
&= 1 - \left( 2^{-1} \left( 1 + \frac{x}{2} \right)^{-1} \right) \quad \text{Expand } \left( 1 + \frac{x}{2} \right)^{-1} \text{ using the binomial expansion} \\
&= 1 - \left( \frac{1}{2} \left( 1 + (-1) \left( \frac{x}{2} \right) + \frac{(-1)(-2)}{2!} \left( \frac{x}{2} \right)^2 + \frac{(-1)(-2)(-3)}{3!} \left( \frac{x}{2} \right)^3 + \dots \right) \right) \\
&= 1 - \left( \frac{1}{2} \left( 1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \dots \right) \right) \quad \text{Multiply } \left( 1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \dots \right) \text{ by } \frac{1}{2} \\
&= 1 - \left( \frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16} + \dots \right) \\
&= \frac{1}{2} + \frac{1}{4}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots
\end{aligned}$$

ii Valid for  $\left| \frac{x}{2} \right| < 1 \Rightarrow |x| < 2$

$$\begin{aligned}
 \mathbf{1 \ h \ i} \quad & \sqrt{\frac{2+x}{1-x}} \\
 & = (2+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}} \quad \text{Put both in index form} \\
 & = 2^{\frac{1}{2}}\left(1+\frac{x}{2}\right)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}} \quad \text{Expand both using the binomial expansion} \\
 & = \sqrt{2} \left( 1 + \frac{\binom{1}{2}\binom{x}{2}}{2!} + \frac{\binom{1}{2}\binom{-1}{2}\binom{x}{2}^2}{2!} + \frac{\binom{1}{2}\binom{-1}{2}\binom{-3}{2}\binom{x}{2}^3}{3!} + \dots \right) \\
 & \times \left( 1 + \binom{-1}{2}(-x) + \frac{\binom{-1}{2}\binom{-3}{2}(-x)^2}{2!} + \frac{\binom{-1}{2}\binom{-3}{2}\binom{-5}{2}(-x)^3}{3!} + \dots \right) \\
 & = \sqrt{2} \left( 1 + \frac{1}{4}x - \frac{1}{32}x^2 + \frac{1}{128}x^3 + \dots \right) \left( 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \dots \right) \quad \text{Multiply out} \\
 & = \sqrt{2} \left( 1 \left( 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{15}{16}x^3 + \dots \right) + \frac{1}{4}x \left( 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \dots \right) \right. \\
 & \quad \left. - \frac{1}{32}x^2 \left( 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \dots \right) + \frac{1}{128}x^3 \left( 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \dots \right) + \dots \right) \\
 & = \sqrt{2} \left( 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{1}{4}x + \frac{1}{8}x^2 \right. \\
 & \quad \left. + \frac{3}{32}x^3 - \frac{1}{32}x^2 - \frac{1}{64}x^3 + \frac{1}{128}x^3 + \dots \right) \\
 & \text{Collect like terms} \\
 & = \sqrt{2} \left( 1 + \frac{3}{4}x + \frac{15}{32}x^2 + \frac{51}{128}x^3 + \dots \right) \quad \text{Multiply by } \sqrt{2} \\
 & = \sqrt{2} + \frac{3\sqrt{2}}{4}x + \frac{15\sqrt{2}}{32}x^2 + \frac{51\sqrt{2}}{128}x^3 + \dots
 \end{aligned}$$

**ii** Valid if  $\left| \frac{x}{2} \right| < 1$  and  $|-x| < 1 \Rightarrow |x| < 1$  for both to be valid.

$$\begin{aligned}
 2 \quad (5+4x)^{-2} &= \left(5\left(1+\frac{4}{5}x\right)\right)^{-2} = 5^{-2}\left(1+\frac{4}{5}x\right)^{-2} = \frac{1}{25}\left(1+\frac{4}{5}x\right)^{-2} \\
 &= \frac{1}{25}\left(1+(-2)\left(\frac{4}{5}x\right) + \frac{(-2)(-3)}{2!}\left(\frac{4}{5}x\right)^2 + \frac{(-2)(-3)(-4)}{3!}\left(\frac{4}{5}x\right)^3 + \dots\right) \\
 &= \frac{1}{25}\left(1+(-2)\left(\frac{4}{5}x\right) + \frac{(-2)(-3)}{2} \frac{16}{25}x^2 + \frac{(-2)(-3)(-4)}{6} \frac{64}{125}x^3 + \dots\right) \\
 &= \frac{1}{25}\left(1 - \frac{8}{5}x + \frac{48}{25}x^2 - \frac{256}{125}x^3 + \dots\right) \\
 &= \frac{1}{25} - \frac{8}{125}x + \frac{48}{625}x^2 - \frac{256}{3125}x^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 3 \quad \mathbf{a} \quad \sqrt{(4-x)} &= (4-x)^{\frac{1}{2}} \\
 &= \left[4\left(1-\frac{x}{4}\right)\right]^{\frac{1}{2}} \\
 &= 4^{\frac{1}{2}}\left(1-\frac{x}{4}\right)^{\frac{1}{2}} \\
 &= 2\left[1 + \left(\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2 + \dots\right] \\
 &= 2\left(1 - \frac{x}{8} - \frac{x^2}{128} + \dots\right) \\
 &= 2 - \frac{x}{4} - \frac{x^2}{64} + \dots
 \end{aligned}$$

Valid for  $\left|-\frac{x}{4}\right| < 1 \Rightarrow |x| < 4$

**b** Substitute  $x = \frac{1}{9}$  into both sides of the expansion:

$$\begin{aligned}
 \sqrt{\left(4-\frac{1}{9}\right)} &\approx 2 - \frac{\frac{1}{9}}{4} - \frac{\left(\frac{1}{9}\right)^2}{64} - \frac{\left(\frac{1}{9}\right)^3}{512} \\
 \sqrt{\frac{35}{9}} &\approx 2 - \frac{1}{36} - \frac{1}{5184} - \frac{1}{373248} \\
 \frac{\sqrt{35}}{3} &\approx \frac{736055}{373248}
 \end{aligned}$$

$$3 \text{ c } m(x) \approx 2 - \frac{1}{4}x - \frac{1}{64}x^2$$

$$m\left(\frac{1}{9}\right) = \frac{\sqrt{35}}{3}$$

$$\sqrt{35} = 3m\left(\frac{1}{9}\right)$$

$$\approx 3\left(2 - \frac{1}{4}\left(\frac{1}{9}\right) - \frac{1}{64}\left(\frac{1}{9}\right)^2\right)$$

$$\approx 3\left(2 - \frac{1}{36} - \frac{1}{5184}\right)$$

$$\approx 5.916087963$$

$$\sqrt{35} = 5.916079783$$

$$\text{Percentage error} = \frac{5.916087963 - 5.916079783}{5.916079783} \times 100 = 0.000138\%$$

$$4 \text{ a } \frac{1}{\sqrt{a+bx}} = (a+bx)^{-\frac{1}{2}} = \left(a\left(1+\frac{b}{a}x\right)\right)^{-\frac{1}{2}} = a^{-\frac{1}{2}}\left(1+\frac{b}{a}x\right)^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}}\left(1+\frac{b}{a}x\right)^{-\frac{1}{2}}$$

$$= \frac{1}{a^{\frac{1}{2}}}\left(1 + \left(-\frac{1}{2}\right)\left(\frac{b}{a}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{b}{a}x\right)^2 + \dots\right)$$

$$= \frac{1}{a^{\frac{1}{2}}}\left(1 + \left(-\frac{1}{2}\right)\left(\frac{b}{a}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\frac{b^2}{a^2}x^2 + \dots\right)$$

$$= \frac{1}{a^{\frac{1}{2}}}\left(1 - \frac{b}{2a}x + \frac{3b^2}{8a^2}x^2 + \dots\right)$$

$$= \frac{1}{a^{\frac{1}{2}}} - \frac{b}{2a^{\frac{3}{2}}}x + \frac{3b^2}{8a^{\frac{5}{2}}}x^2 + \dots = 3 + \frac{1}{3}x + \frac{1}{18}x^2 + \dots$$

Equating coefficients gives  $\frac{1}{a^{\frac{1}{2}}} = 3$ , so  $a = \frac{1}{9}$

and  $-\frac{b}{2\left(\frac{1}{9}\right)^{\frac{3}{2}}} = \frac{1}{3}$

$$-\frac{b}{27} = \frac{1}{3}$$

$$b = -\frac{2}{81}$$

$$a = \frac{1}{9}, b = -\frac{2}{81}$$

$$\begin{aligned}
 \mathbf{4\ b} \quad x^3 \text{ term of } 3\left(1 - \frac{2}{9}x\right)^{-\frac{1}{2}} &= 3 \left( \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!} \left(-\frac{2}{9}x\right)^3 \right) \\
 &= -3 \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{6} \frac{8}{729} x^3 \\
 &= \frac{5}{486} x^3 \\
 \text{Coefficient} &= \frac{5}{486}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{5} \quad \frac{3+2x-x^2}{4-x} &\equiv (3+2x-x^2)(4-x)^{-1} \quad \text{Write } \frac{1}{4-x} \text{ as } (4-x)^{-1} \\
 &= (3+2x-x^2) \left( 4 \left( 1 - \frac{x}{4} \right) \right)^{-1} \quad \text{Take out a factor of 4} \\
 &= (3+2x-x^2) \frac{1}{4} \left( 1 - \frac{x}{4} \right)^{-1} \quad \text{Expand } \left( 1 - \frac{x}{4} \right)^{-1} \text{ using the binomial expansion} \\
 &= (3+2x-x^2) \frac{1}{4} \left( 1 + (-1) \left( -\frac{x}{4} \right) + \frac{(-1)(-2)}{2!} \left( -\frac{x}{4} \right)^2 + \dots \right) \quad \text{Ignore terms higher than } x^2 \\
 &= (3+2x-x^2) \frac{1}{4} \left( 1 + \frac{x}{4} + \frac{x^2}{16} + \dots \right) \quad \text{Multiply expansion by } \frac{1}{4} \\
 &= (3+2x-x^2) \left( \frac{1}{4} + \frac{x}{16} + \frac{x^2}{64} + \dots \right) \quad \text{Multiply result by } (3+2x-x^2) \\
 &= 3 \left( \frac{1}{4} + \frac{x}{16} + \frac{x^2}{64} + \dots \right) + 2x \left( \frac{1}{4} + \frac{x}{16} + \frac{x^2}{64} + \dots \right) - x^2 \left( \frac{1}{4} + \frac{x}{16} + \frac{x^2}{64} + \dots \right) \\
 &= \frac{3}{4} + \frac{3}{16}x + \frac{3}{64}x^2 + \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{4}x^2 + \dots \quad \text{Ignore any terms bigger than } x^2 \\
 &= \frac{3}{4} + \frac{11}{16}x - \frac{5}{64}x^2
 \end{aligned}$$

Expansion is valid if  $\left| \frac{-x}{4} \right| < 1 \Rightarrow |x| < 4$

$$\begin{aligned}
 \mathbf{6 \ a} \quad \frac{1}{\sqrt{5+2x}} &= (5+2x)^{-\frac{1}{2}} = 5^{-\frac{1}{2}} \left(1 + \frac{2}{5}x\right)^{-\frac{1}{2}} = \frac{1}{\sqrt{5}} \left(1 + \frac{2}{5}x\right)^{-\frac{1}{2}} \\
 &= \frac{1}{\sqrt{5}} \left( 1 + \left(-\frac{1}{2}\right) \left(\frac{2}{5}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!} \left(\frac{2}{5}x\right)^2 + \dots \right) \\
 &= \frac{1}{\sqrt{5}} \left( 1 + \left(-\frac{1}{2}\right) \left(\frac{2}{5}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2} \frac{4}{25}x^2 + \dots \right) \\
 &= \frac{1}{\sqrt{5}} \left( 1 - \frac{1}{5}x + \frac{3}{50}x^2 + \dots \right) \\
 &= \frac{1}{\sqrt{5}} - \frac{1}{5\sqrt{5}}x + \frac{3}{50\sqrt{5}}x^2 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \frac{2x-1}{\sqrt{5+2x}} &= \frac{2x-1}{\sqrt{5}} \left(1 + \frac{2}{5}x\right)^{-\frac{1}{2}} \\
 &= (2x-1) \left( \frac{1}{\sqrt{5}} - \frac{1}{5\sqrt{5}}x + \frac{3}{50\sqrt{5}}x^2 + \dots \right) \\
 &= \frac{2}{\sqrt{5}}x - \frac{2}{5\sqrt{5}}x^2 - \frac{1}{\sqrt{5}} + \frac{1}{5\sqrt{5}}x - \frac{3}{50\sqrt{5}}x^2 + \dots \\
 &= -\frac{1}{\sqrt{5}} + \frac{11}{5\sqrt{5}}x - \frac{23}{50\sqrt{5}}x^2 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7 \ a} \quad (16-3x)^{\frac{1}{4}} &= \left(16 \left(1 - \frac{3}{16}x\right)\right)^{\frac{1}{4}} = 2 \left(1 - \frac{3}{16}x\right)^{\frac{1}{4}} \\
 &= 2 \left( 1 + \left(\frac{1}{4}\right) \left(-\frac{3}{16}x\right) + \frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)}{2!} \left(-\frac{3}{16}x\right)^2 + \dots \right) \\
 &= 2 \left( 1 + \left(\frac{1}{4}\right) \left(-\frac{3}{16}x\right) + \frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)}{2} \frac{9}{256}x^2 + \dots \right) \\
 &= 2 \left( 1 - \frac{3}{64}x - \frac{27}{8192}x^2 + \dots \right) \\
 &= 2 - \frac{3}{32}x - \frac{27}{4096}x^2 + \dots
 \end{aligned}$$

7 b Let  $x = 0.1$

$$\sqrt[4]{15.7} \approx 2 - \frac{3}{32}(0.1) - \frac{27}{4096}(0.1)^2$$

$$= 1.991$$

8 a  $\frac{3}{4-2x} = 3(4-2x)^{-1} = 3\left(4\left(1-\frac{1}{2}x\right)\right)^{-1} = \frac{3}{4}\left(1-\frac{1}{2}x\right)^{-1}$

$$= \frac{3}{4}\left(1 + (-1)\left(-\frac{1}{2}x\right) + \frac{(-1)(-2)}{2!}\left(-\frac{1}{2}x\right)^2 + \dots\right)$$

$$= \frac{3}{4}\left(1 - \left(-\frac{1}{2}x\right) + \frac{1}{4}x^2 + \dots\right)$$

$$= \frac{3}{4} + \frac{3}{8}x + \frac{3}{16}x^2 + \dots$$

$\frac{2}{3+5x} = 2(3+5x)^{-1} = 2\left(3\left(1+\frac{5}{3}x\right)\right)^{-1} = \frac{2}{3}\left(1+\frac{5}{3}x\right)^{-1}$

$$= \frac{2}{3}\left(1 + (-1)\left(\frac{5}{3}x\right) + \frac{(-1)(-2)}{2!}\left(\frac{5}{3}x\right)^2 + \dots\right)$$

$$= \frac{2}{3}\left(1 - \frac{5}{3}x + \frac{25}{9}x^2 + \dots\right)$$

$$= \frac{2}{3} - \frac{10}{9}x + \frac{50}{27}x^2 + \dots$$

$$\frac{3}{4-2x} - \frac{2}{3+5x}$$

$$= \frac{3}{4} + \frac{3}{8}x + \frac{3}{16}x^2 + \dots - \left(\frac{2}{3} - \frac{10}{9}x + \frac{50}{27}x^2 + \dots\right)$$

$$= \frac{1}{12} + \frac{107}{72}x - \frac{719}{432}x^2 + \dots$$

b  $g(0.01) = \frac{3}{4-2(0.01)} - \frac{2}{3+5(0.01)} = 0.0980311$

c Using the series expansion:

$$g(0.01) \approx \frac{1}{12} + \frac{107}{72}(0.01) - \frac{719}{432}(0.01)^2 = 0.098028009$$

$$\text{Percentage error} = \frac{0.0980311 - 0.098028009}{0.098028009} \times 100 = 0.0032\%$$