Sequences and series 3C

1 a $1 \underset{\times 2}{\rightarrow} 2 \underset{\times 2}{\rightarrow} 4 \underset{\times 2}{\rightarrow} 8 \underset{\times 2}{\rightarrow} 16 \underset{\times 2}{\rightarrow} 32$

Geometric, $r = 2$

b $2 \underset{+3}{\rightarrow} 5 \underset{+3}{\rightarrow} 8 \underset{+3}{\rightarrow} 11 \underset{+3}{\rightarrow} 14$

Not geometric
(this is an arithmetic sequence)

c $40 \underset{-4}{\rightarrow} 36 \underset{-4}{\rightarrow} 32 \underset{-4}{\rightarrow} 28$

Not geometric (arithmetic)

d $2 \underset{\times 3}{\rightarrow} 6 \underset{\times 3}{\rightarrow} 18 \underset{\times 3}{\rightarrow} 54$

Geometric, $r = 3$

e $10 \underset{\times \frac{1}{2}}{\rightarrow} 5 \underset{\times \frac{1}{2}}{\rightarrow} 2.5 \underset{\times \frac{1}{2}}{\rightarrow} 1.25$

Geometric, $r = \frac{1}{2}$

f $5 \underset{\times(-1)}{\rightarrow} -5 \underset{\times(-1)}{\rightarrow} 5 \underset{\times(-1)}{\rightarrow} -5$

Geometric, $r = -1$

g $3 \underset{\times 1}{\rightarrow} 3 \underset{\times 1}{\rightarrow} 3 \underset{\times 1}{\rightarrow} 3 \underset{\times 1}{\rightarrow} 3$

Geometric, $r = 1$

h $4 \underset{\times\left(-\frac{1}{4}\right)}{\rightarrow} -1 \underset{\times\left(-\frac{1}{4}\right)}{\rightarrow} 0.25 \underset{\times\left(-\frac{1}{4}\right)}{\rightarrow} -0.0625$

Geometric, $r = -\frac{1}{4}$

2 a $5 \underset{\times 3}{\rightarrow} 15 \underset{\times 3}{\rightarrow} 45 \underset{\times 3}{\rightarrow} 135 \underset{\times 3}{\rightarrow} 405 \underset{\times 3}{\rightarrow} 1215$

b $4 \underset{\times(-2)}{\rightarrow} -8 \underset{\times(-2)}{\rightarrow} 16 \underset{\times(-2)}{\rightarrow} -32 \underset{\times(-2)}{\rightarrow} 64 \underset{\times(-2)}{\rightarrow} -128$

c $60 \underset{\times \frac{1}{2}}{\rightarrow} 30 \underset{\times \frac{1}{2}}{\rightarrow} 15 \underset{\times \frac{1}{2}}{\rightarrow} 7.5 \underset{\times \frac{1}{2}}{\rightarrow} 3.75 \underset{\times \frac{1}{2}}{\rightarrow} 1.875$

d $1 \underset{\times \frac{1}{4}}{\rightarrow} \frac{1}{4} \underset{\times \frac{1}{4}}{\rightarrow} \frac{1}{16} \underset{\times \frac{1}{4}}{\rightarrow} \frac{1}{64} \underset{\times \frac{1}{4}}{\rightarrow} \frac{1}{256} \underset{\times \frac{1}{4}}{\rightarrow} \frac{1}{1024}$

e $1 \underset{\times p}{\rightarrow} p \underset{\times p}{\rightarrow} p^2 \underset{\times p}{\rightarrow} p^3 \underset{\times p}{\rightarrow} p^4 \underset{\times p}{\rightarrow} p^5$

f

$x \underset{\times(-2x)}{\rightarrow} -2x^2 \underset{\times(-2x)}{\rightarrow} 4x^3 \underset{\times(-2x)}{\rightarrow} -8x^4 \underset{\times(-2x)}{\rightarrow} 16x^5 \underset{\times(-2x)}{\rightarrow} -32x^6$

3 a $3 \quad x \quad 9$

Common ratio $= \frac{\text{term } 2}{\text{term } 1}$ or $\frac{\text{term } 3}{\text{term } 2}$ $\frac{x}{3}$ or $\frac{9}{x}$

Therefore,

$$\frac{x}{3} = \frac{9}{x} \quad \text{(cross multiply)}$$
$$x^2 = 27$$
$$x = \sqrt{27}$$
$$x = \sqrt{9 \times 3}$$
$$x = 3\sqrt{3}$$

3 b Term 4 = term 3 × r

Term 3 = 9 and

$r = \frac{\text{term }2}{\text{term }1} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

So term $4 = 9\sqrt{3}$

4 a 2, 6, 18, 54, …
6th term = 2×3^5
= 2 × 243
= 486
nth term = $2 \times 3^{n-1}$

b 100, 50, 25, 12.5, …

6th term = $100 \times \left(\frac{1}{2}\right)^5$

$= 100 \times \frac{1}{32}$

$= \frac{25}{8}$

nth term = $100 \times \left(\frac{1}{2}\right)^{n-1}$

c 1, −2, 4, −8, ...
6th term = $1 \times (-2)^5$
= 1 × −32
= −32
nth term = $(-2)^{n-1}$

d 1, 1.1, 1.21, 1.331, ...
6th term = $1 \times (1.1)^5$
= 1 × 1.61051
= 1.61051
nth term = $(1.1)^{n-1}$

5 nth term = 2×5^n
1st term = $2 \times 5^1 = 10$
5th term = $2 \times 5^5 = 6250$

6 Let the first term be a and the common ratio = r
6th term is 32
$\Rightarrow ar^{6-1} = 32$
$\Rightarrow ar^5 = 32$ **(1)**

3rd term is 4
$\Rightarrow ar^{3-1} = 4$
$\Rightarrow ar^2 = 4$ **(2)**

(1) ÷ **(2)**:

$\frac{\cancel{a}r^5}{\cancel{a}r^2} = \frac{32}{4}$

$r^3 = 8$

$r = 2$

Common ratio is 2.

Substitute $r = 2$ into equation **(2)**
$a \times 2^2 = 4$
$a \times 4 = 4$
$a = 1$

First term is 1.

7 First term is 4.

$\Rightarrow a = 4$ **(1)**

Third term is 1 $\Rightarrow ar^{3-1} = 1$

$\Rightarrow ar^2 = 1$ **(2)**

Substitute $a = 4$ into **(2)**

$4r^2 = 1$

$r^2 = \frac{1}{4}$

$r = \pm\frac{1}{2}$

The sixth term = $ar^{6-1} = ar^5$

7 (continued)

If $r = \frac{1}{2}$ then sixth term $= 4 \times \left(\frac{1}{2}\right)^5 = \frac{1}{8}$

If $r = -\frac{1}{2}$ then sixth term $= 4 \times \left(-\frac{1}{2}\right)^5$

$= -\frac{1}{8}$

Possible values for sixth term: $\frac{1}{8}, -\frac{1}{8}$.

8 a $\frac{u_2}{u_1} = \frac{u_3}{u_2}$

$\frac{2x}{8 - x} = \frac{x^2}{2x}$

$4x^2 = 8x^2 - x^3$

$x^3 - 4x^2 = 0$

b $x^2(x - 4) = 0$

$x = 0$ or 4

As $x > 0$, $x = 4$

$a = 4$, $r = 2$

20th term $= ar^{19}$

$= 4 \times 2^{19}$

$= 4 \times 524\,288$

$= 2\,097\,152$

c If 4096 in the sequence then,

for some n, $ar^{n-1} = 4096$

$4 \times 2^{n-1} = 4096$

$2^{n-1} = 1024$

$n - 1 = 10$

$n = 11$

Yes, 4096 is in the sequence as n is an integer.

9 a $a = 200$, $r = p$

$u_6 = 200p^5 = 40$

$p^5 = \frac{1}{5}$

$\log p^5 = \log \frac{1}{5}$

$5\log p = \log 1 - \log 5$

$5\log p + \log 5 = 0$

b $\log p = \frac{-\log 5}{5}$

$p = 10^{\frac{-\log 5}{5}}$

$p = 0.725$

10 $a = 4$, $u_4 = 108 = 4r^3$

$r^3 = 27$

$r = 3$

We want kth term $> 500\,000$

So $4 \times 3^{k-1} > 500\,000$

$3^{k-1} > 125\,000$

$\log 3^{k-1} > \log 125\,000$

$(k - 1)\log 3 > \log 125\,000$

$k - 1 > \frac{\log 125\,000}{\log 3}$

$k - 1 > 10.68$

$k > 11.68$

So $k = 12$

11 $a = 9$, $r = 4$

$u_n = 9 \times 4^{n-1} = 383\,616$

$4^{n-1} = 42\,624$

$\log 4^{n-1} = \log 42\,624$

$(n - 1) \log 4 = \log 42\,624$

$n - 1 = \frac{\log 42\,624}{\log 4}$

$n - 1 = 7.69$

$n = 8.69$

n is not an integer so 383 616 is not in the sequence.

12 $a = 3$, $r = -4$

3, −12, 48, −192, 768, −3072, 12 288, −49 152

So 49 152 is not in the sequence, but −49 152 is.

13 $3 \underset{\times 4}{\rightarrow} 12 \underset{\times 4}{\rightarrow} 48 \ldots$

This is a geometric series with $a = 3$ and $r = 4$.
If a term exceeds 1 000 000 then

$ar^{n-1} > 1\,000\,000$

Substitute $a = 3$, $r = 4$:

$$3 \times 4^{n-1} > 1000\,000$$
$$4^{n-1} > \frac{1000\,000}{3}$$
$$\log 4^{n-1} > \log\left(\frac{1000\,000}{3}\right)$$
$$(n-1)\log 4 > \log\left(\frac{1000\,000}{3}\right)$$
$$n-1 > \frac{\log\left(\frac{1000\,000}{3}\right)}{\log 4}$$
$$n-1 > 9.173\ldots$$
$$n > 10.173\ldots$$
$$\text{So } n = 11$$

Term is $3 \times 4^{10} = 3\,145\,728$