Sequences and series 3A

1 a i $u_n = 5n + 2$

$n = 1 \rightarrow \quad u_1 = 5(1) + 2 = 7$
$n = 2 \rightarrow \quad u_2 = 5(2) + 2 = 12$
$n = 3 \rightarrow \quad u_3 = 5(3) + 2 = 17$
$n = 4 \rightarrow \quad u_4 = 5(4) + 2 = 22$

ii $a = 7$ and $d = 5$

b i $u_n = 9 - 2n$

$n = 1 \rightarrow \quad u_1 = 9 - 2(1) = 7$
$n = 2 \rightarrow \quad u_2 = 9 - 2(2) = 5$
$n = 3 \rightarrow \quad u_3 = 9 - 2(3) = 3$
$n = 4 \rightarrow \quad u_4 = 9 - 2(4) = 1$

ii $a = 7$ and $d = -2$

c i $u_n = 7 + 0.5n$

$n = 1 \rightarrow \quad u_1 = 7 + 0.5(1) = 7.5$
$n = 2 \rightarrow \quad u_2 = 7 + 0.5(2) = 8$
$n = 3 \rightarrow \quad u_3 = 7 + 0.5(3) = 8.5$
$n = 4 \rightarrow \quad u_4 = 7 + 0.5(4) = 9$

ii $a = 7.5$ and $d = 0.5$

d i $u_n = n - 10$

$n = 1 \rightarrow \quad u_1 = 1 - 10 = -9$
$n = 2 \rightarrow \quad u_2 = 2 - 10 = -8$
$n = 3 \rightarrow \quad u_3 = 3 - 10 = -7$
$n = 4 \rightarrow \quad u_4 = 4 - 10 = -6$

ii $a = -9$ and $d = 1$

2 a $5 \underset{+2}{\rightarrow} 7 \underset{+2}{\rightarrow} 9 \underset{+2}{\rightarrow} 11$

10th term $= 5 + 9 \times 2 = 5 + 18 = 23$
nth term $= 5 + (n - 1) \times 2$
$= 5 + 2n - 2$
$= 2n + 3$

b $5 \underset{+3}{\rightarrow} 8 \underset{+3}{\rightarrow} 11 \underset{+3}{\rightarrow} 14$

10th term $= 5 + 9 \times 3 = 5 + 27 = 32$
nth term $= 5 + (n - 1) \times 3$
$= 5 + 3n - 3$
$= 3n + 2$

c $24 \underset{-3}{\rightarrow} 21 \underset{-3}{\rightarrow} 18 \underset{-3}{\rightarrow} 15$

10th term $= 24 + 9 \times (-3)$
$= 24 - 27 = -3$
nth term $= 24 + (n - 1) \times (-3)$
$= 24 - 3n + 3$
$= 27 - 3n$

d $-1 \underset{+4}{\rightarrow} 3 \underset{+4}{\rightarrow} 7 \underset{+4}{\rightarrow} 11$

10th term $= -1 + 9 \times 4$
$= -1 + 36 = 35$
nth term $= -1 + (n - 1) \times 4$
$= -1 + 4n - 4$
$= 4n - 5$

e $x \underset{+x}{\rightarrow} 2x \underset{+x}{\rightarrow} 3x \underset{+x}{\rightarrow} 4x$

10th term $= x + 9 \times x = 10x$
nth term $= x + (n - 1)x = nx$

f $a \underset{+d}{\rightarrow} a + d \underset{+d}{\rightarrow} a + 2d \underset{+d}{\rightarrow} a + 3d$

10th term $= a + 9d$
nth term $= a + (n - 1)d$

3 a $3\underset{+4}{\to}7\underset{+4}{\to}11\ldots83\underset{+4}{\to}87$

number of jumps $= \dfrac{87-3}{4} = 21$

so number of terms = 21 + 1 = 22

3 b $5\underset{+3}{\to}8\underset{+3}{\to}11\ldots119\underset{+3}{\to}122$

number of jumps $= \dfrac{122-5}{3} = 39$

therefore number of terms = 40

c $90\underset{-2}{\to}88\underset{-2}{\to}86\ldots16\underset{-2}{\to}14$

number of jumps $= \dfrac{90-14}{2} = 38$

therefore number of terms = 39

d $4\underset{+5}{\to}9\underset{+5}{\to}14\ldots224\underset{+5}{\to}229$

number of jumps $= \dfrac{229-4}{5} = 45$

therefore number of terms = 46

e $x\underset{+2x}{\to}3x\underset{+2x}{\to}5x\ldots35x$

number of jumps $= \dfrac{35x-x}{2x} = 17$

therefore number of terms = 18

f $a\underset{+d}{\to}a+d\underset{+d}{\to}a+2d\ldots a+(n-1)d$

number of jumps $= \dfrac{a+(n-1)d-a}{d}$

$= \dfrac{(n-1)d}{d} = n-1$

therefore number of terms $= n$

4 $u_1 = 14$ and $u_4 = 32$
$d = (32 - 14) \div 3$
$d = 6$

5 $u_n = pn + q$
$u_6 = 9$, so $6p + q = 9$ **(1)**
$u_9 = 11$, so $9p + q = 11$ **(2)**
(2) − **(1)** gives:
$3p = 2$
$p = \dfrac{2}{3}$

Substitute $p = \dfrac{2}{3}$ in **(1)**:

$6\left(\dfrac{2}{3}\right) + q = 9$

$q = 5$

Constants are $p = \dfrac{2}{3}$ and $q = 5$

6 $u_3 = 30$ and $u_9 = 9$
$d = (9 - 30) \div 6 = -3.5$
$u_{10} = 5.5$, $u_{11} = 2$, $u_{12} = -1.5$
The first negative term is −1.5

7 $u_{20} = 14$ and $u_{40} = -6$
$d = (-6 - 14) \div 20 = -1$
$u_{10} = 14 - 10(-1) = 24$

8 $u_1 = 5p$, $u_2 = 20$ and $u_3 = 3p$
$d = 20 - 5p$ and $d = 3p - 20$
$20 - 5p = 3p - 20$
$8p = 40$
$p = 5$
$d = 20 - 5 \times 5 = -5$
$u_{20} = 5 \times 5 - 5(20 - 1) = -70$

9 $u_1 = -8$, $u_2 = k^2$ and $u_3 = 17k$
$d = k^2 + 8$ and $d = 17k - k^2$
$k^2 + 8 = 17k - k^2$
$2k^2 - 17k + 8 = 0$
$(2k - 1)(k - 8) = 0$
$k = \dfrac{1}{2}$ or $k = 8$

10 $a = k^2$, $d = k$, $u_5 = 41$

$u_5 = k^2 + (5-1)k = 41$

$k^2 + 4k - 41 = 0$

Using the formula:

$$k = \frac{-4 \pm \sqrt{4^2 - 4 \times (1) \times (-41)}}{2 \times 1}$$

$$k = \frac{-4 \pm \sqrt{180}}{2}$$

$$k = \frac{-4 \pm 6\sqrt{5}}{2}$$

$$k = -2 \pm 3\sqrt{5}$$

As $k > 0$, $k = -2 + 3\sqrt{5}$

Challenge

$u_n = \ln a + (n - 1)\ln b$

$u_3 = \ln 16$ and $u_7 = \ln 256$

$d = \ln b$

$d = \frac{1}{4}(\ln 256 - \ln 16)$

$\ln b = \frac{1}{4}(\ln 256 - \ln 16)$

$\ln b = \ln 256^{\frac{1}{4}} - \ln 16^{\frac{1}{4}}$

$\ln b = \ln 4 - \ln 2$

$\ln b = \ln\left(\frac{4}{2}\right)$

$\ln b = \ln 2$

$b = 2$

$u_3 = \ln 16$

$= \ln a + (3 - 1)\ln 2$

$= \ln a + \ln 2^2$

So $\ln 16 = \ln a + \ln 4 = \ln 4a$

$a = 4$, $b = 2$