

Functions and graphs 2C

$$\begin{aligned}
 1 \quad \mathbf{a} \quad pq(-8) &= p\left(\frac{-8}{4}\right) \\
 &= p(-2) \\
 &= 1 - 3(-2) \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad qr(5) &= q[(5 - 2)^2] \\
 &= q(9) \\
 &= \frac{9}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad rq(6) &= r\left(\frac{6}{4}\right) \\
 &= r\left(\frac{3}{2}\right) \\
 &= \left(\frac{3}{2} - 2\right)^2 \\
 &= \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad p^2(-5) &= p(1 - 3(-5)) \\
 &= p(16) \\
 &= 1 - 3(16) \\
 &= -47
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} \quad pqr(8) &= pq[(8 - 2)^2] \\
 &= pq(36) \\
 &= p\left(\frac{36}{4}\right) \\
 &= p(9) \\
 &= 1 - 3(9) \\
 &= -26
 \end{aligned}$$

$$\begin{aligned}
 2 \quad \mathbf{a} \quad fg(x) &= f(x^2 - 4) \\
 &= 4(x^2 - 4) + 1 \\
 &= 4x^2 - 15, \quad x \in \mathbb{R}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad gf(x) &= g(4x + 1) \\
 &= (4x + 1)^2 - 4 \\
 &= 16x^2 + 8x - 3, \quad x \in \mathbb{R}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad gh(x) &= g\left(\frac{1}{x}\right) \\
 &= \left(\frac{1}{x}\right)^2 - 4 \\
 &= \frac{1}{x^2} - 4, \quad x \in \mathbb{R}, x \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad fh(x) &= f\left(\frac{1}{x}\right) \\
 &= 4 \times \left(\frac{1}{x}\right) + 1 \\
 &= \frac{4}{x} + 1, \quad x \in \mathbb{R}, x \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} \quad f^2(x) &= ff(x) \\
 &= f(4x + 1) \\
 &= 4(4x + 1) + 1 \\
 &= 16x + 5, \quad x \in \mathbb{R}
 \end{aligned}$$

3 a $fg(x) = f(x^2)$
 $= 3x^2 - 2, \quad x \in \mathbb{R}$

b $gf(x) = g(3x - 2)$
 $= (3x - 2)^2$

When $fg(x) = gf(x)$ then

$$\begin{aligned} 3x^2 - 2 &= (3x - 2)^2 \\ 3x^2 - 2 &= 9x^2 - 12x + 4 \\ 0 &= 6x^2 - 12x + 6 \\ 0 &= x^2 - 2x + 1 \\ 0 &= (x - 1)^2 \end{aligned}$$

Hence $x = 1$

4 a $qp(x) = q\left(\frac{1}{x-2}\right)$
 $= 3 \times \left(\frac{1}{x-2}\right) + 4$
 $= \frac{3}{x-2} + \frac{4(x-2)}{x-2}$
 $= \frac{4x-5}{x-2}, \quad x \in \mathbb{R}, x \neq 2$

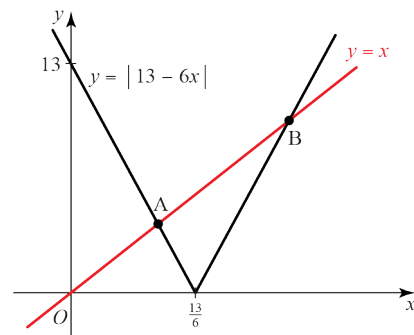
b If $qp(m) = 16$ then

$$\begin{aligned} 3\left(\frac{1}{m-2}\right) + 4 &= 16 \\ \frac{3}{m-2} &= 12 \\ 3 &= 12(m-2) \\ \frac{3}{12} &= m-2 \\ \frac{1}{4} &= m-2 \\ m &= \frac{9}{4} \end{aligned}$$

5 a $fg(6) = f\left(\frac{3(6)-2}{2}\right)$
 $= f(8)$
 $= |9 - 4(8)|$
 $= |-23|$
 $= 23$

b $fg(x) = f\left(\frac{3x-2}{2}\right)$
 $= \left|9 - 4\left(\frac{3x-2}{2}\right)\right|$
 $= |9 - 6x + 4|$
 $= |13 - 6x|$

Now $fg(x) = x$ when $|13 - 6x| = x$



At A: $13 - 6x = x$
 $13 = 7x$
 $x = \frac{13}{7}$

At B: $-(13 - 6x) = x$
 $5x = 13$
 $x = \frac{13}{5}$

The solutions are
 $x = \frac{13}{7}$ and $x = \frac{13}{5}$

$$\begin{aligned}
 \mathbf{6 \ a} \quad f^2(x) &= f\left(\frac{1}{x+1}\right) \\
 &= \left(\frac{1}{\left(\frac{1}{x+1}\right)+1}\right) \\
 &= \left(\frac{1}{\left(\frac{1+x+1}{x+1}\right)}\right) \\
 &= \left(\frac{x+1}{x+2}\right), \quad x \neq -1, x \neq -2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad f^3(x) &= f\left(\frac{x+1}{x+2}\right) \\
 &= \left(\frac{1}{\left(\frac{x+1}{x+2}\right)+1}\right) \\
 &= \left(\frac{1}{\left(\frac{x+1+x+2}{x+2}\right)}\right) \\
 &= \left(\frac{x+2}{2x+3}\right), \quad x \neq -1, x \neq -2, x \neq -\frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7 \ a} \quad st(x) &= s(x+3) \\
 &= 2^{x+3}, \quad x \in \mathbb{R}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad ts(x) &= t(2^x) \\
 &= 2^x + 3, \quad x \in \mathbb{R}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad 2^{x+3} &= 2^x + 3 \\
 2^{x+3} - 2^x &= 3 \\
 2^x \times 2^3 - 2^x &= 3 \\
 2^x(8-1) &= 3 \\
 2^x &= \frac{3}{7} \\
 x \ln 2 &= \ln\left(\frac{3}{7}\right) \\
 x &= \frac{\ln\left(\frac{3}{7}\right)}{\ln 2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{8 \ a} \quad gf(x) &= g(e^{5x}) \\
 &= 4 \ln(e^{5x}) \\
 &= 4(5x) \\
 &= 20x, \quad x \in \mathbb{R}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad fg(x) &= f(4 \ln x) \\
 &= e^{5(4 \ln x)} \\
 &= e^{\ln x^{20}} \\
 &= x^{20}, \quad x \in \mathbb{R}, x > 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{9 \ a} \quad qp(x) &= q(\ln(x+3)) \\
 &= e^{3(\ln(x+3))} - 1 \\
 &= e^{\ln(x+3)^3} - 1 \\
 &= (x+3)^3 - 1, \quad x \in \mathbb{R}, x > -3
 \end{aligned}$$

Since $x > -3$, so $qp(x) > -1$

$$\begin{aligned}
 \mathbf{b} \quad qp(7) &= (7+3)^3 - 1 \\
 &= 999
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad qp(x) &= (x+3)^3 - 1 = 124 \\
 (x+3)^3 &= 125 \\
 x+3 &= 5 \\
 x &= 2
 \end{aligned}$$

$$\begin{aligned}
 10 \quad t^2(x) &= t(5 - 2x) \\
 &= 5 - 2(5 - 2x) \\
 &= 5 - 10 + 4x \\
 &= -5 + 4x
 \end{aligned}$$

$$\begin{aligned}
 t^2(x) - (t(x))^2 &= 0 \\
 -5 + 4x - (5 - 2x)^2 &= 0 \\
 -5 + 4x - 25 + 20x - 4x^2 &= 0 \\
 -4x^2 + 24x - 30 &= 0 \\
 2x^2 - 12x + 15 &= 0
 \end{aligned}$$

Using the formula:

$$\begin{aligned}
 x &= \frac{12 \pm \sqrt{(-12)^2 - 4 \times 2 \times 15}}{2 \times 2} \\
 &= \frac{12 \pm \sqrt{24}}{4} \\
 &= \frac{12 \pm 2\sqrt{6}}{4} \\
 &= 3 \pm \frac{\sqrt{6}}{2}
 \end{aligned}$$

11 a Range of g is $-8 \leq x \leq 12$

b From the graph,

$$\begin{aligned}
 g(x) &= -\frac{1}{2}x + 12 \text{ for } 0 \leq x \leq 14 \\
 \text{and } g(0) &= 12
 \end{aligned}$$

$$\begin{aligned}
 \text{So } gg(0) &= g(12) \\
 &= -\frac{1}{2}(12) + 12 \\
 &= 6
 \end{aligned}$$

$$\begin{aligned}
 \text{c } gh(7) &= g\left(\frac{2(7) - 5}{10 - 7}\right) \\
 &= g(3) \\
 &= -\frac{1}{2}(3) + 12 \\
 &= 10.5
 \end{aligned}$$