Functions and graphs 2C

1 a $pq(-8) = p\left(\frac{-8}{4}\right)$
$= p(-2)$
$= 1 - 3(-2)$
$= 7$

b $qr(5) = q[(5-2)^2]$
$= q(9)$
$= \frac{9}{4}$

c $rq(6) = r\left(\frac{6}{4}\right)$
$= r\left(\frac{3}{2}\right)$
$= \left(\frac{3}{2} - 2\right)^2$
$= \frac{1}{4}$

d $p^2(-5) = p(1 - 3(-5))$
$= p(16)$
$= 1 - 3(16)$
$= -47$

e $pqr(8) = pq[(8-2)^2]$
$= pq(36)$
$= p\left(\frac{36}{4}\right)$
$= p(9)$
$= 1 - 3(9)$
$= -26$

2 a $fg(x) = f(x^2 - 4)$
$= 4(x^2 - 4) + 1$
$= 4x^2 - 15, \quad x \in \mathbb{R}$

b $gf(x) = g(4x+1)$
$= (4x+1)^2 - 4$
$= 16x^2 + 8x - 3, \quad x \in \mathbb{R}$

c $gh(x) = g\left(\frac{1}{x}\right)$
$= \left(\frac{1}{x}\right)^2 - 4$
$= \frac{1}{x^2} - 4, \quad x \in \mathbb{R}, x \neq 0$

d $fh(x) = f\left(\frac{1}{x}\right)$
$= 4 \times \left(\frac{1}{x}\right) + 1$
$= \frac{4}{x} + 1, \quad x \in \mathbb{R}, x \neq 0$

e $f^2(x) = ff(x)$
$= f(4x+1)$
$= 4(4x+1) + 1$
$= 16x + 5, \quad x \in \mathbb{R}$

3 a $\text{fg}(x) = \text{f}(x^2)$
$= 3x^2 - 2, \quad x \in \mathbb{R}$

b $\text{gf}(x) = \text{g}(3x-2)$
$= (3x-2)^2$

When $\text{fg}(x) = \text{gf}(x)$ then

$$3x^2 - 2 = (3x-2)^2$$
$$3x^2 - 2 = 9x^2 - 12x + 4$$
$$0 = 6x^2 - 12x + 6$$
$$0 = x^2 - 2x + 1$$
$$0 = (x-1)^2$$

Hence $x = 1$

4 a $\text{qp}(x) = \text{q}\left(\frac{1}{x-2}\right)$

$$= 3 \times \left(\frac{1}{x-2}\right) + 4$$
$$= \frac{3}{x-2} + \frac{4(x-2)}{x-2}$$
$$= \frac{4x-5}{x-2}, \quad x \in \mathbb{R}, x \neq 2$$

b If $\text{qp}(m) = 16$ then

$$3\left(\frac{1}{m-2}\right) + 4 = 16$$
$$\frac{3}{m-2} = 12$$
$$3 = 12(m-2)$$
$$\frac{3}{12} = m-2$$
$$\frac{1}{4} = m-2$$
$$m = \frac{9}{4}$$

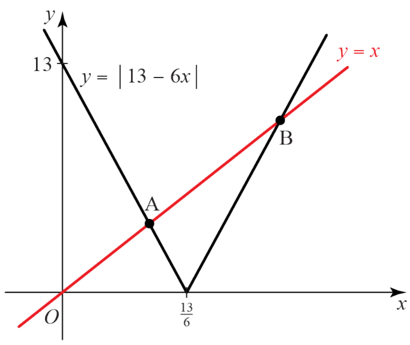
5 a $\text{fg}(6) = \text{f}\left(\frac{3(6)-2}{2}\right)$

$$= \text{f}(8)$$
$$= |9 - 4(8)|$$
$$= |-23|$$
$$= 23$$

b $\text{fg}(x) = \text{f}\left(\frac{3x-2}{2}\right)$

$$= \left|9 - 4\left(\frac{3x-2}{2}\right)\right|$$
$$= |9 - 6x + 4|$$
$$= |13 - 6x|$$

Now $\text{fg}(x) = x$ when $|13 - 6x| = x$



At A: $13 - 6x = x$
$13 = 7x$
$x = \frac{13}{7}$

At B: $-(13 - 6x) = x$
$5x = 13$
$x = \frac{13}{5}$

The solutions are
$x = \frac{13}{7}$ and $x = \frac{13}{5}$

6 **a** $f^2(x) = f\left(\frac{1}{x+1}\right)$

$$= \left(\frac{1}{\left(\frac{1}{x+1}\right)+1}\right)$$

$$= \left(\frac{1}{\left(\frac{1+x+1}{x+1}\right)}\right)$$

$$= \left(\frac{x+1}{x+2}\right), \quad x \neq -1, x \neq -2$$

b $f^3(x) = f\left(\frac{x+1}{x+2}\right)$

$$= \left(\frac{1}{\left(\frac{x+1}{x+2}\right)+1}\right)$$

$$= \left(\frac{1}{\left(\frac{x+1+x+2}{x+2}\right)}\right)$$

$$= \left(\frac{x+2}{2x+3}\right), x \neq -1, x \neq -2, x \neq -\tfrac{3}{2}$$

7 **a** $st(x) = s(x + 3)$

$$= 2^{x+3}, \quad x \in \mathbb{R}$$

b $ts(x) = t(2^x)$

$$= 2^x + 3, \quad x \in \mathbb{R}$$

c $2^{x+3} = 2^x + 3$

$$2^{x+3} - 2^x = 3$$

$$2^x \times 2^3 - 2^x = 3$$

$$2^x(8 - 1) = 3$$

$$2^x = \frac{3}{7}$$

$$x \ln 2 = \ln\left(\frac{3}{7}\right)$$

$$x = \frac{\ln\left(\frac{3}{7}\right)}{\ln 2}$$

8 **a** $gf(x) = g(e^{5x})$

$$= 4\ln(e^{5x})$$

$$= 4(5x)$$

$$= 20x, \; x \in \mathbb{R}$$

b $fg(x) = f(4 \ln x)$

$$= e^{5(4 \ln x)}$$

$$= e^{\ln x^{20}}$$

$$= x^{20}, \; x \in \mathbb{R}, x > 0$$

9 **a** $qp(x) = q(\ln(x + 3))$

$$= e^{3(\ln(x+3))} - 1$$

$$= e^{\ln(x+3)^3} - 1$$

$$= (x + 3)^3 - 1, \; x \in \mathbb{R}, x > -3$$

Since $x > -3$, so $qp(x) > -1$

b $qp(7) = (7 + 3)^3 - 1$

$$= 999$$

c $qp(x) = (x + 3)^3 - 1 = 124$

$$(x + 3)^3 = 125$$

$$x + 3 = 5$$

$$x = 2$$

10 $t^2(x) = t(5 - 2x)$
$= 5 - 2(5 - 2x)$
$= 5 - 10 + 4x$
$= -5 + 4x$

$$t^2(x) - (t(x))^2 = 0$$
$$-5 + 4x - (5 - 2x)^2 = 0$$
$$-5 + 4x - 25 + 20x - 4x^2 = 0$$
$$-4x^2 + 24x - 30 = 0$$
$$2x^2 - 12x + 15 = 0$$

Using the formula:

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4 \times 2 \times 15}}{2 \times 2}$$
$$= \frac{12 \pm \sqrt{24}}{4}$$
$$= \frac{12 \pm 2\sqrt{6}}{4}$$
$$= 3 \pm \frac{\sqrt{6}}{2}$$

11 a Range of g is $-8 \le x \le 12$

b From the graph,

$$g(x) = -\frac{1}{2}x + 12 \text{ for } 0 \le x \le 14$$

and $g(0) = 12$

So $gg(0) = g(12)$
$$= -\frac{1}{2}(12) + 12$$
$$= 6$$

c $gh(7) = g\left(\frac{2(7) - 5}{10 - 7}\right)$
$$= g(3)$$
$$= -\frac{1}{2}(3) + 12$$
$$= 10.5$$