Functions and graphs 2A

1 a $\left|\frac{3}{4}\right| = \frac{3}{4}$

b $|-0.28| = 0.28$

c $|3-11| = |-8|$
$= 8$

d $\left|\frac{5}{7} - \frac{3}{8}\right| = \left|\frac{40}{56} - \frac{21}{56}\right|$
$= \frac{19}{56}$

e $|20-6\times 4| = |20-24|$
$= |-4|$
$= 4$

f $|4^2 \times 2 - 3\times 7| = |32-21|$
$= 11$

2 a $f(1) = |7-5\times 1| + 3$
$= |7-5| + 3$
$= 5$

b $f(10) = |7-5\times 10| + 3$
$= |7-50| + 3$
$= |-43| + 3$
$= 46$

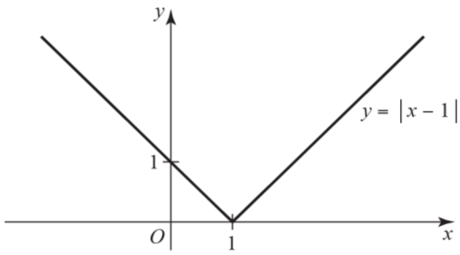
c $f(-6) = |7-5\times(-6)| + 3$
$= |7+30| + 3$
$= 40$

3 a $g(4) = |4^2 - 8\times 4|$
$= |16-32|$
$= |-16|$
$= 16$

b $g(-5) = |(-5)^2 - 8\times(-5)|$
$= |25+40|$
$= 65$

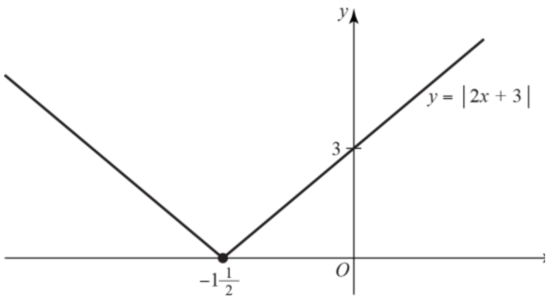
c $g(8) = |8^2 - 8\times 8|$
$= |64-64|$
$= 0$

4 a



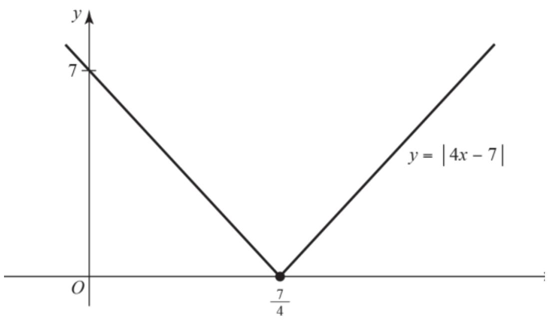
The graph meets the axes at (1, 0) and (0, 1)

4 b



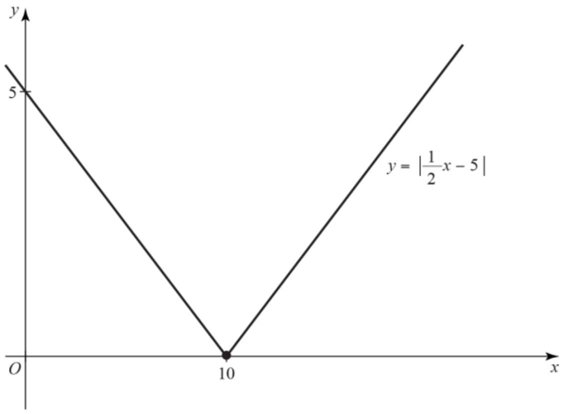
The graph meets the axes at $\left(-1\frac{1}{2},\ 0\right)$ and (0, 3)

c



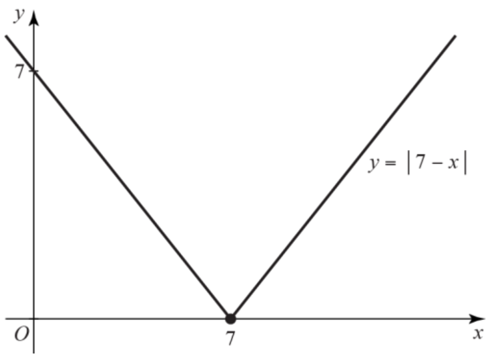
The graph meets the axes at $\left(\frac{7}{4},\ 0\right)$ and (0, 7)

d



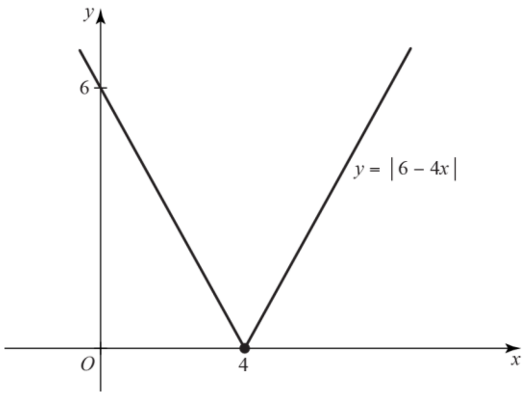
The graph meets the axes at (10, 0) and (0, 5)

e



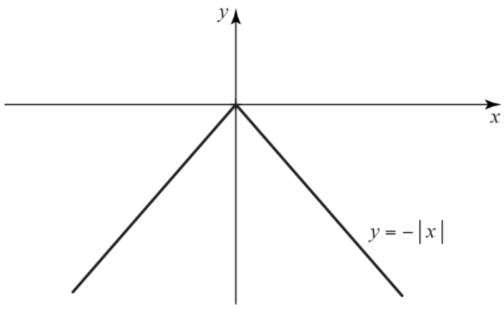
The graph meets the axes at (7, 0) and (0, 7)

4 f



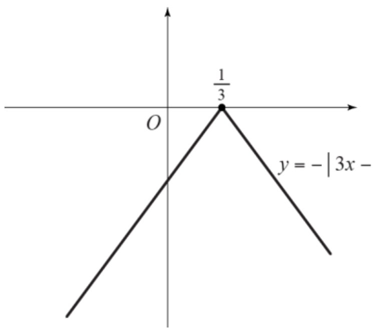
The graph meets the axes at $\left(\frac{3}{2}, 0\right)$ and (0, 6)

g



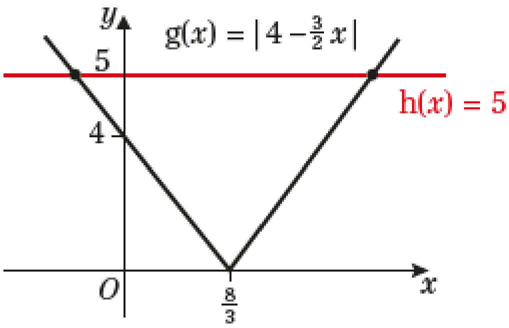
The graph meets the axes at (0, 0)

h



The graph meets the axes at $\left(\frac{1}{3}, 0\right)$ and (0, −1)

5 a



b At the left-hand point of intersection:

$$4-\frac{3}{2}x=5$$

$$\frac{3}{2}x=-1$$

$$x=-\frac{2}{3}$$

At the right-hand point of intersection:

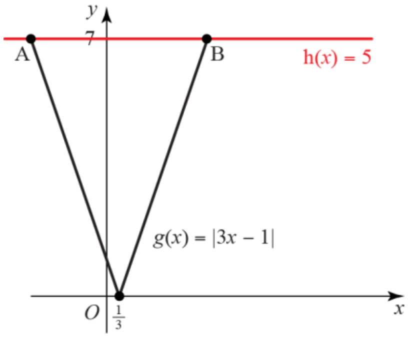
$$-(4-\frac{3}{2}x)=5$$

$$\frac{3}{2}x=9$$

$$x=6$$

The solutions are $x=-\frac{2}{3}$ and $x=6$

6 a



At A: $-(3x-1)=5$

$-3x=4$

$x=-\frac{4}{3}$

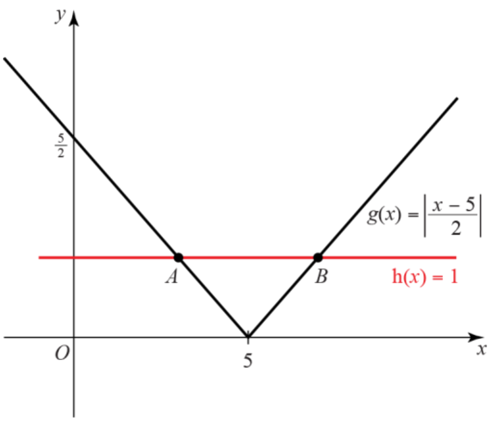
At B: $3x-1=5$

$3x=6$

$x=2$

The solutions are $x=-\frac{4}{3}$ and $x=2$

b



At A: $-\left(\frac{x-5}{2}\right)=1$

$x-5=-2$

$x=3$

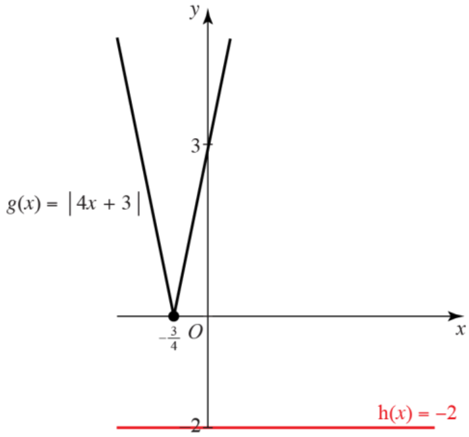
At B: $\frac{x-5}{2}=1$

$x-5=2$

$x=7$

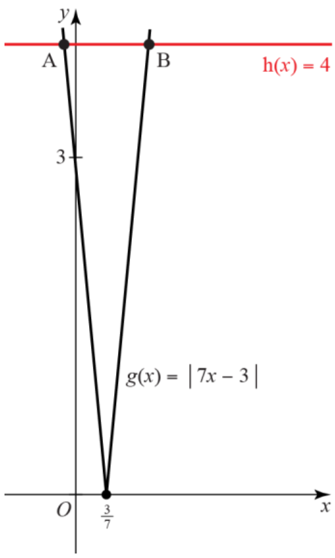
The solutions are $x=3$ and $x=7$

c



The graphs do not intersect so there are no solutions.

d



At A: $-(7x-3)=4$

$7x=-1$

$x=-\frac{1}{7}$

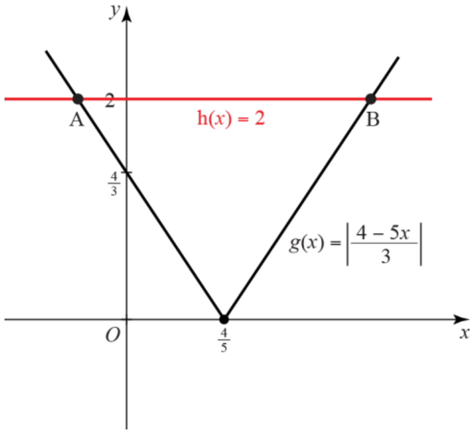
At B: $7x-3=4$

$7x=7$

$x=1$

The solutions are $x=-\frac{1}{7}$ and $x=1$

6 e



At A: $\frac{4-5x}{3}=2$

$-5x=2$

$x=-\frac{2}{5}$

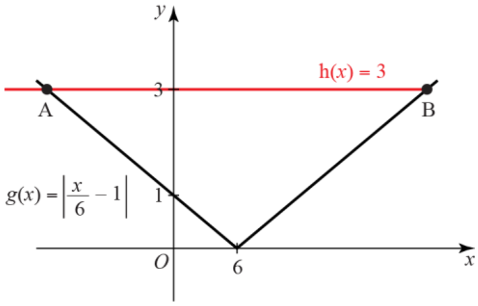
At B: $-\left(\frac{4-5x}{3}\right)=2$

$-5x=-10$

$x=2$

The solutions are $x=-\frac{2}{5}$ and $x=2$

f



At A: $-\left(\frac{x}{6}-1\right)=3$

$\frac{x}{6}=-2$

$x=-12$

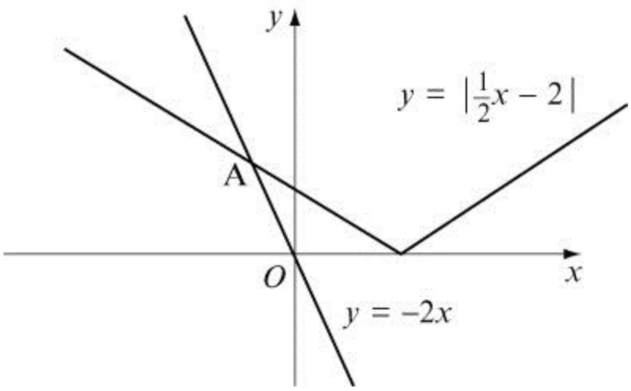
At B: $\frac{x}{6}-1=3$

$\frac{x}{6}=4$

$x=24$

The solutions are
$x=-12$ and $x=24$

7 a



7 b Intersection point A is

on the reflected part of $y = \frac{1}{2}x - 2$

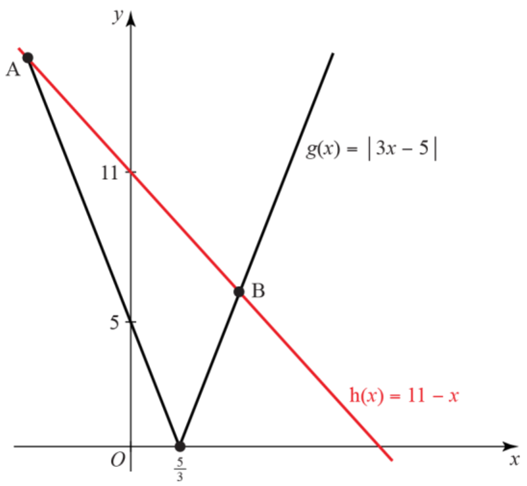
$$-\left(\frac{1}{2}x - 2\right) = -2x$$

$$2x - \frac{1}{2}x = -2$$

$$\frac{3}{2}x = -2$$

$$x = -\frac{4}{3}$$

8



At A: $-(3x-5) = 11 - x$

$$-6 = 2x$$

$$x = -3$$

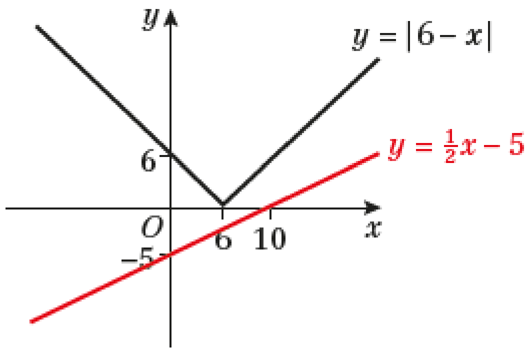
At B: $3x - 5 = 11 - x$

$$4x = 16$$

$$x = 4$$

The solutions are $x = -3$ and $x = 4$

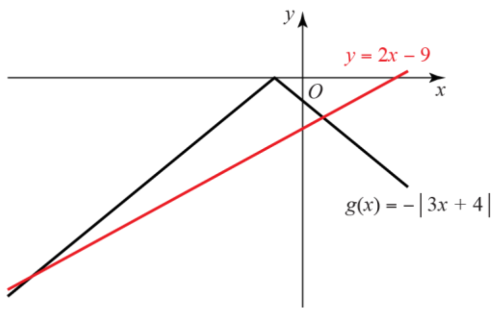
9 a



b The two graphs do not intersect, therefore there are no solutions to the equation $|6 - x| = \frac{1}{2}x - 5$

10 The value for x cannot be negative as it equals a modulus which is ≥ 0

11 a



11 b At the left-hand point of intersection:

$3x+4=2x-9$

$x=-13$

At the right-hand point of intersection:

$-(3x+4)=2x-9$

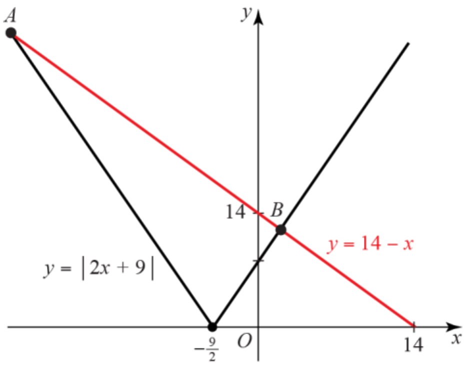
$-5x=-5$

$x=1$

The points of intersection are $x=-13$ and $x=1$

So the solution to $-|3x+4|<2x-9$ is $x<-13$ and $x>1$

12



At A: $-(2x+9)=14-x$

$-x=23$

$x=-23$

At B: $2x+9=14-x$

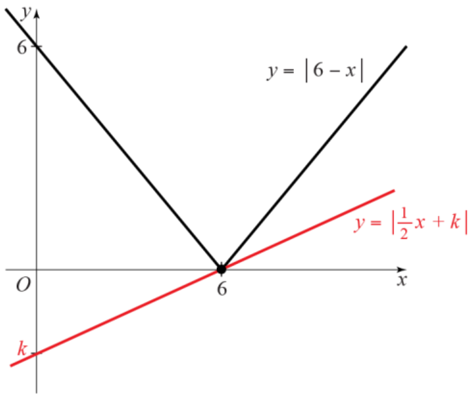
$3x=5$

$x=\frac{5}{3}$

The points of intersection are $x=-23$ and $x=\frac{5}{3}$

So the solution to $|2x+9|<14-x$ is $-23<x<\frac{5}{3}$

13 a For there to be one solution, the graphs $y=|6-x|$ and $y=\frac{1}{2}x+k$ must intersect once at the vertex of $y=|6-x|$



This vertex occurs at (6, 0)

Substituting (6, 0) into $y=\frac{1}{2}x+k$ gives:

$0=\frac{1}{2}\times 6+k$

$0=3+k$

$k=-3$

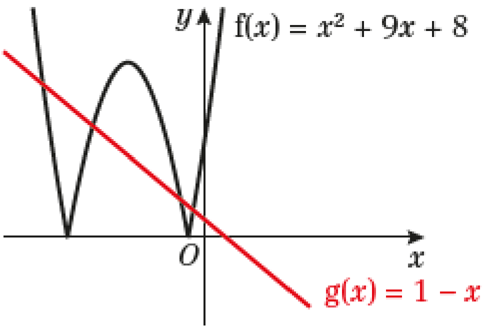
b $6-x=\frac{1}{2}x-3$

$9=\frac{3}{2}x$

$x=6$

Challenge

a



b At the far left-hand and far right-hand points of intersection:

$x^2 + 9x + 8 = 1 - x$
$x^2 + 10x + 7 = 0$
Using the formula:
$x = \frac{-10 \pm \sqrt{10^2 - 4 \times 1 \times 7}}{2 \times 1}$
$x = \frac{-10 \pm \sqrt{72}}{2}$
$x = \frac{-10 \pm 6\sqrt{2}}{2}$
$x = -5 \pm 3\sqrt{2}$

At the two inside points of intersection:

$-(x^2 + 9x + 8) = 1 - x$
$x^2 + 9x + 8 = x - 1$
$x^2 + 8x + 9 = 0$

Using the formula:

$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times 9}}{2 \times 1}$
$x = \frac{-8 \pm \sqrt{28}}{2}$
$x = \frac{-8 \pm 2\sqrt{7}}{2}$
$x = -4 \pm \sqrt{7}$

The four solutions are
$x = -5 \pm 3\sqrt{2}$ and $x = -4 \pm \sqrt{7}$