Algebraic Methods 1F

1 $$\frac{x^3+2x^2+3x-4}{x+1} \equiv Ax^2+Bx+C+\frac{D}{x+1}$$

$$\begin{array}{r} x^2+x+2 \\ x+1\overline{)x^3+2x^2+3x-4} \\ \underline{x^3+x^2} \\ x^2+3x \\ \underline{x^2+x} \\ 2x-4 \\ \underline{2x+2} \\ -6 \end{array}$$

$$\frac{x^3+2x^2+3x-4}{x+1} \equiv x^2+x+2-\frac{6}{x+1}$$

So $A=1, B=1, C=2, D=-6$

2 Using algebraic long division:

$$\begin{array}{r} 2x^2-3x+5 \\ x+3\overline{)2x^3+3x^2-4x+5} \\ \underline{2x^3+6x^2} \\ -3x^2-4x \\ \underline{-3x^2-9x} \\ 5x+5 \\ \underline{5x+15} \\ -10 \end{array}$$

$$\frac{2x^3+3x^2-4x+5}{x+3} = 2x^2-3x+5-\frac{10}{x+3}$$

So $a = 2$, $b = -3$, $c = 5$ and $d = -10$

3 Using algebraic long division:

$$\begin{array}{r}
x^2 + 2x + 4 \\
x - 2 \overline{\smash{\big)} x^3 + 0x^2 + 0x - 8} \\
\underline{x^3 - 2x^2} \quad\quad\quad\quad \\
2x^2 + 0x \quad\quad \\
\underline{2x^2 - 4x} \quad\quad \\
4x - 8 \\
\underline{4x - 8} \\
0
\end{array}$$

So $\dfrac{x^3-8}{x-2} = x^2 + 2x + 4$

$p = 1$, $q = 2$ and $r = 4$

4 Using algebraic long division:

$$\begin{array}{r}
2 \\
x^2 - 1 \overline{\smash{\big)} 2x^2 + 4x + 5} \\
\underline{2x^2 + 0x - 2} \\
4x + 7
\end{array}$$

$$\frac{2x^2+4x+5}{x^2-1} \equiv 2 + \frac{4x+7}{x^2-1}$$

So $m = 2$, $n = 4$ and $p = 7$

5 Using algebraic long division:

$$\begin{array}{r}
4x + 1 \\
2x^2 + 2 \overline{\smash{\big)} 8x^3 + 2x^2 + 0x + 5} \\
\underline{8x^3 \quad\quad + 8x} \quad\quad \\
2x^2 - 8x + 5 \\
\underline{2x^2 \quad\quad + 2} \\
-8x + 3
\end{array}$$

$8x^2 + 2x^2 + 5 \equiv (4x + 1)(2x^2 + 2) - 8x + 3$

So $A = 4$, $B = 1$, $C = -8$ and $D = 3$.

6 Using algebraic long division:

$$\begin{array}{r} 4x-13 \\ x^2+2x-1\overline{\smash{)}4x^3-5x^2+3x-14} \\ \underline{4x^3+8x^2-4x} \\ -13x^2+7x-14 \\ \underline{-13x^2-26x+13} \\ 33x-27 \end{array}$$

$$\frac{4x^3-5x^2+3x-14}{x^2+2x-1} \equiv 4x-13+\frac{33x-27}{x^2+2x-1}$$

So $A = 4$, $B = -13$, $C = 33$ and $D = -27$.

7 Using algebraic long division:

$$\begin{array}{r} x^2+2 \\ x^2+1\overline{\smash{)}x^4+3x^2-4} \\ \underline{x^4+x^2} \\ 2x^2-4 \\ \underline{2x^2+2} \\ -6 \end{array}$$

$$\frac{x^4+3x^2-4}{x^2+1} \equiv x^2+2-\frac{6}{x^2+1}$$

So $p = 1$, $q = 0$, $r = 2$, $s = 0$ and $t = -6$.

8 Using algebraic long division:

$$\begin{array}{r} 2x^2+x+1 \\ x^2+x-2\overline{\smash{)}2x^4+3x^3-2x^2+4x-6} \\ \underline{2x^4+2x^3-4x^2} \\ x^3+2x^2+4x \\ \underline{x^3+x^2-2x} \\ x^2+6x-6 \\ \underline{x^2+x-2} \\ 5x-4 \end{array}$$

$$\frac{2x^4+3x^3-2x^2+4x-6}{x^2+x-2} \equiv 2x^2+x+1+\frac{5x-4}{x^2+x-2}$$

So $a = 2$, $b = 1$, $c = 1$, $d = 5$ and $e = -4$.

9 $3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (Ax^2 + Bx + C)(x^2 - 3) + Dx + E$

Compare coefficients of x^4 :

$$A = 3$$

Compare coefficients of x^3:

$$B = -4$$

Compare coefficients of x^2:

$$-8 = -3A + C$$
$$-8 = -9 + C \qquad \text{(substituting } A = 3\text{)}$$
$$C = 1$$

Compare coefficients of x :

$$16 = -3B + D$$
$$16 = 12 + D \qquad \text{(substituting } B = -4\text{)}$$
$$D = 4$$

Equate constant terms:

$$-2 = -3C + E$$
$$-2 = -3 + E \qquad \text{(substituting } C = 1\text{)}$$
$$E = 1$$

Hence $3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (3x^2 - 4x + 1)(x^2 - 3) + 4x + 1$

Note: After lots of comparing coefficients, it is a good idea to check your answer by substituting a simple value of $\boldsymbol{x}$ into both sides of the identity to check that your answers are correct. For example,

Substitute $x = 1$ into LHS

$$\Rightarrow \; 3 - 4 - 8 + 16 - 2 = 5$$

Substitute $x = 1$ into RHS

$$\Rightarrow \; (3 - 4 + 1) \times (1 - 3) + 4 + 1$$
$$= 0 \times -2 + 4 + 1 = 5$$

LHS = RHS, so you can be fairly sure the identity is correct.

10 a $x^4 - 1 \equiv (x^2 - 1)(x^2 + 1)$

$$\equiv (x - 1)(x + 1)(x^2 + 1)$$

b $\dfrac{x^4 - 1}{x + 1} \equiv \dfrac{(x - 1)(x + 1)(x^2 + 1)}{(x + 1)}$

$$\equiv (x - 1)(x^2 + 1)$$

So a = 1, b = −1, c = 1, d = 0 and e = 1.