Variable acceleration 11E

1 $v = \int a\mathrm{d}t$

$= at + c$, where c is a constant of integration.

When $t = 0$, $v = 0$

$0 = a \times 0 + c \Rightarrow c = 0$

$v = at$

$s = \int v\mathrm{d}t$

$= \int at\mathrm{d}t$

$= \frac{1}{2}at^2 + k$, where k is a constant of integration.

When $t = 0$, $s = x$

$x = \frac{1}{2} \times a \times 0^2 + k \Rightarrow k = x$

$s = \frac{1}{2}at^2 + x$

2 **a** $v = \int a\mathrm{d}t$

$= \int 5\mathrm{d}t$

$= 5t + c$, where c is a constant of integration.

When $t = 0$, $v = 12$

$12 = 0 + c \Rightarrow c = 12$

$v = 12 + 5t$

b $s = \int v\mathrm{d}t$

$= \int (12 + 5t)\mathrm{d}t$

$= 12t + \frac{5}{2}t^2 + k$, where k is a constant of integration.

When $t = 0$, $s = 7$

$7 = 0 + 0 + k \Rightarrow k = 7$

$s = 12t + \frac{5}{2}at^2 + 7$

$= 12t + 2.5t^2 + 7$

3 $s = ut + \frac{1}{2}at^2$

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = u + at$

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = a$

So acceleration is constant.

4 A $s = 2t^2 - t^3$

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = 4t - 3t^2$

4 A $a = \frac{dv}{dt} = 4 - 6t$

Not constant

B $s = 4t + 7$

$v = \frac{ds}{dt} = 4$

$a = \frac{dv}{dt} = 0$

No acceleration

C $s = \frac{t^2}{4}$

$v = \frac{ds}{dt} = \frac{t}{2}$

$a = \frac{dv}{dt} = \frac{1}{2}$

Constant acceleration

D $s = 3t - \frac{2}{t^2}$

$v = \frac{ds}{dt} = 3 + \frac{4}{t^3}$

$a = \frac{dv}{dt} = -\frac{12}{t^4}$

Not constant

E $s = 6$

$v = \frac{ds}{dt} = 0$

Particle stationary

5 **a** $v = u + at$

$u = 5, v = 13, t = 2$

$13 = 5 + 2a$

$a = \frac{13-5}{2} = 4$

The acceleration of the particle is 4 m s^{-2}.

b $v = \int a\,dt$

$= \int 4\,dt$

$= 4t + c$, where c is a constant of integration.

5 b When $t = 0$, $v = 5$

$5 = 0 + c \Rightarrow c = 5$

$v = 4t + 5$

$s = \int v\,\mathrm{d}t$

$= \int (4t + 5)\,\mathrm{d}t$

$= 2t^2 + 5t + k$, where k is a constant of integration.

When $t = 0$, $s = 0$

$0 = 0 + 0 + k \Rightarrow k = 0$

$s = 2t^2 + 5t$

This is an equation of the required form with $p = 2$, $q = 5$ and $r = 0$.

6 a $s = 25t - 0.2t^2$

When $t = 40$, $s = 25 \times 40 - 0.2 \times 40^2$

$= 680$

The distance AB is 680 m.

b $v = \dfrac{\mathrm{d}s}{\mathrm{d}t} = 25 - 0.4t$

$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = -0.4$

The train has a constant acceleration (of $-0.4\ \mathrm{m\,s^{-2}}$).

c Taking the direction in which the train travels to be positive:

For the bird: $a = -0.6$, $u = -7$, initial displacement = 680

$v_B = \int a\,\mathrm{d}t$

$= \int -0.6\,\mathrm{d}t$

$= -0.6t + c$, where c is a constant of integration.

When $t = 0$, $v_B = -7$

$-7 = 0 + c \Rightarrow c = -7$

$v = -0.6t - 7$

$s_B = \int v_B\,\mathrm{d}t$

$= \int (-0.6t - 7)\,\mathrm{d}t$

$= -0.3t^2 - 7t + k$, where k is a constant of integration.

When $t = 0$, $s_B = 680$

$680 = 0 - 0 + k \Rightarrow k = 680$

$s_B = -0.3t^2 - 7t + 680$

When the bird is directly above the train, the displacement of both train and bird are the same.

$25t - 0.2t^2 = -0.3t^2 - 7t + 680$

$0.1t^2 + 32t - 680 = 0$

$t^2 + 320t - 6800 = 0$

$(t - 20)(t + 340) = 0$

6 c $t > 0$, so $t = 20$

When $t = 20$,
$s = 25 \times 20 - 0.2 \times 20^2$
$= 420$

The bird is directly above the train 420 m from *A*.