## Variable acceleration 11C

**1** **a** $s = 0.4t^3 - 0.3t^2 - 1.8t + 5$

$v = \frac{ds}{dt} = 1.2t^2 - 0.6t - 1.8$

$\frac{dv}{dt} = 2.4t - 0.6$

$\frac{dv}{dt} = 0$ when $2.4t = 0.6$

$t = 0.25$

$P$ is moving with minimum velocity at $t = 0.25$ s.

**b** When $t = 0.25$
$s = 0.4(0.25)^3 - 0.3(0.25)^2 - 1.8(0.25) + 5$
$= 4.54$ (3 s.f.)

When $P$ is moving with minimum velocity, the displacement is 4.54 m.

**c** $v = \frac{ds}{dt} = 1.2t^2 - 0.6t - 1.8$
When $t = 0.25$, $v = 1.2 \times 0.25^2 - 0.6 \times 0.25 - 1.8$
$= -1.88$ (3 s.f.)

**2** **a** $s = 4t^3 - t^4$
When $t = 4$,
$s = 4(4)^3 - 4^4 = 0$

The body returns to its starting position 4 s after leaving it.

**b** $s = 4t^3 - t^4 = s = t^3(4 - t)$
Since $t \geq 0$, $t^3$ is always positive.
Since $t \leq 4$, $(4 - t)$ is always positive.
So for $0 \leq t \leq 4$, $s$ is always non-negative.

**c** $\frac{ds}{dt} = 12t^2 - 4t^3$

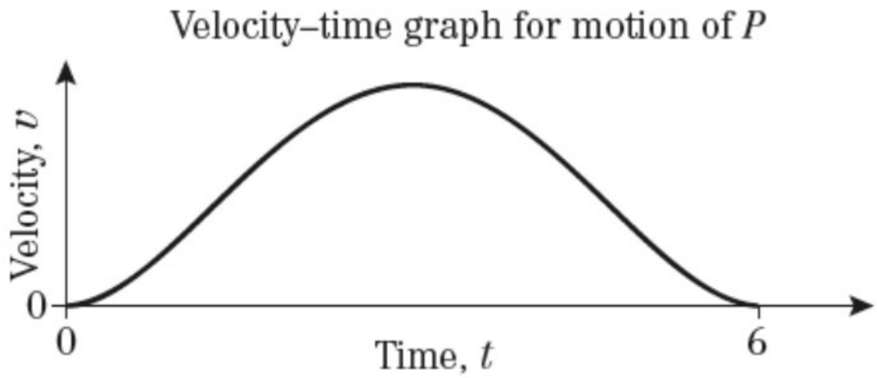
$\frac{ds}{dt} = 0$ when
$12t^2 - 4t^3 = 0$
$4t^2(3 - t) = 0$
$t = 0$ or $3$

At $t = 0$, the body is at $s = 0$, so maximum displacement occurs when $t = 3$.

When $t = 3$, using factorised form of the equation of motion:
$s = 3^3(4 - 3) = 27$
The maximum displacement of the body from its starting point is 27 m.

**3** **a** $v = t^2(6 - t)^2$

Velocity is zero when $t = 0$ and $t = 6$.
The graph touches the time axis at $t = 0$ and $t = 6$.
Graph only shown for $0 \le t \le 6$, as this is the range over which equation is valid.




**b** $v = t^2(6 - t)^2$
$= t^2(36 - 12t + t^2)$
$= 36t^2 - 12t^3 + t^4$

$\frac{dv}{dt} = 72t - 36t^2 + 4t^3$

$\frac{dv}{dt} = 0$ when
$72t - 36t^2 + 4t^3 = 0$
$4t(18 - 9t + t^2) = 0$
$4t(3 - t)(6 - t) = 0$

The turning points are at $t = 0$, $t = 3$ and $t = 6$.
$v = 0$ when $t = 0$ and $t = 6$, therefore the maximum velocity occurs when $t = 3$.

When $t = 3$,
$v = 3^2(6 - 3)^2 = 9 \times 9 = 81$

The maximum velocity is 81 m s$^{-1}$ and the body reaches this 3 s after leaving $O$.

**4** **a** $v = 2t^2 - 3t + 5$

For this particle to come to rest, $v$ must be 0 for some positive value of $t$.

$2t^2 - 3t + 5 = 0$ must have real, positive roots.
$b^2 - 4ac = (-3)^2 - 4(2)(5)$
$= 9 - 40$
$= -31 < 0$

The equation therefore has no real roots, so $v$ is never zero.

**4 b** $v = 2t^2 - 3t + 5$

$\frac{dv}{dt} = 4t - 3$

$\frac{dv}{dt} = 0$ when $4t = 3$

$t = 0.75$

Minimum velocity is when $t = 0.75$.

When $t = 0.75$, $v = 2(0.75)^2 - 3(0.75) + 5$

$= 1.125 - 2.25 + 5$

$= 3.875$

$= 3.88$ (3 s.f.)

The minimum velocity of the particle is 3.88 m $s^{-1}$.

**5 a** $s = \frac{9t^2}{2} - t^3$

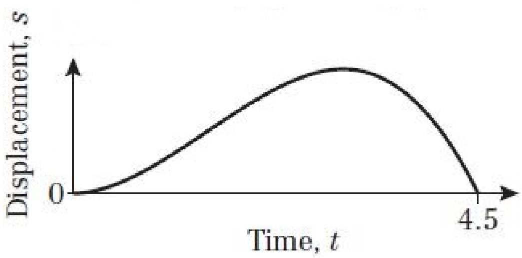
$= t^2(4.5 - t)$

Displacement is zero when $t = 0$ and $t = 4.5$.

The graph touches the time axis at $t = 0$ and crosses it at $t = 4.5$.

Graph only shown for $0 \leq t \leq 4.5$, as this is range over which equation is valid.

The curve is cubic, so not symmetrical.




**b** For values of $t > 4.5$, $s$ is negative. However $s$ is a distance and can only be positive.

**c** $s = \frac{9t^2}{2} - t^3$

$\frac{ds}{dt} = 9t - 3t^2$

$\frac{ds}{dt} = 0$ when

$9t - 3t^2 = 0$

$3t(3 - t) = 0$

The turning points are at $t = 0$ and $t = 3$.

$s = 0$ when $t = 0$, so maximum distance occurs when $t = 3$.

When $t = 3$, using factorised form of the equation of motion:

$s = 3^2(4.5 - 3) = 9 \times 1.5 = 13.5$

The maximum distance of $P$ from $O$ is 13.5 m.

**5 d** $v = \frac{ds}{dt} = 9t - 3t^2$

$a = \frac{dv}{dt} = 9 - 6t$

When $t = 3$,

$a = 9 - 6 \times 3 = -9$

The magnitude of the acceleration of $P$ at the maximum distance is 9 m s$^{-2}$.

**6** $s = 3.6t + 1.76t^2 - 0.02t^3$

$\frac{ds}{dt} = 3.6 + 3.52t - 0.06t^2$

Maximum distance occurs when $\frac{ds}{dt} = 0$.

$\frac{ds}{dt} = 0$ when

$3.6 + 3.52t - 0.06t^2 = 0$

$3t^2 - 176t - 180 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{176 \pm \sqrt{(-176)^2 + (4)(3)(180)}}{2 \times 3}$$

$$= \frac{176 \pm \sqrt{33\,136}}{6}$$

$$= -1.005 \text{ or } 59.67$$

$t > 0$, so maximum distance occurs when $t = 59.67$.

When $t = 59.67$, $s = 3.6(59.67) + 1.76(59.67)^2 - 0.02(59.67)^3$
$= 2230$ (3 s.f.)

The maximum distance from the start of the track is 2230 m or 2.23 km.
Since this is less than 4 km, the train never reaches the end of the track.