Variable acceleration 11A

1 a $s = 9(1) - 1^3 = 8$ m

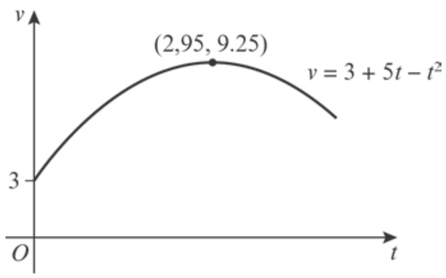
b $9t - t^3 = 0$
$t(9 - t^2) = 0$
Either $t = 0$ or $t^2 = 9$
$\Rightarrow t = 0$ or $t = \pm 3$

2 a At $t = 2$,
$s = 5(2)^2 - 2^3 = 12$
At $t = 4$,
$s = 5(4)^2 - 4^3 = 16$
Change in displacement = 16 − 12 = 4 m

b At $t = 3$,
$s = 5(3)^2 - 3^3 = 18$
Change in displacement in the third second = 18 − 12 = 6 m

3 a $v = 3 + 5(1) - 1^2 = 7$ m s^{-1}

b



At $t = 0$, $v = 3$
$v = 3$ again when $5t - t^2 = 0 \Rightarrow t = 5$

Using symmetry, turning point is when $t = 2.5$.
When $t = 2.5$,
$v = 3 + 5(2.5) - 2.5^2 = 9.25$
So in $0 \le t \le 4$, range of v is $3 \le v \le 9.25$
Greatest speed is 9.25 m s^{-1}.

c $v = 3 + 5(7) - 7^2$
$= 3 + 35 - 49$
$= -11$
When $t = 7$, the velocity of the particle is −11 m s^{-1}. This means it is moving in the opposite direction to that in which it was initially travelling.

4 a $s = 0$ when
$\frac{1}{5}(4t - t^2) = 0$
$\frac{1}{5}t(4 - t) = 0$
$\Rightarrow t = 0$ or $t = 4$
By symmetry, maximum distance is when $t = 2$.
When $t = 2$, $s = \frac{1}{5}(4(2) - 2^2)$
$= \frac{4}{5}$

4 a The maximum displacement is 0.8 m.

b When the toy car returns to P, $s = 0$

$\frac{1}{5}(4t - t^2) = 0$

$\frac{1}{5}t(4 - t) = 0$

$\Rightarrow t = 0$ or $t = 4$

The toy car returns to P after 4 s.

c The toy car travels to maximum distance and back again.

So total distance = 0.8 + 0.8 = 1.6 m

d The model is valid for $0 \le t \le 4$.

5 a When $t = 0$,

$v = 0 - 0 + 8 = 8$

The initial velocity is 8 m s^{-1}.

b $3t^2 - 10t + 8 = 0$

$(3t - 4)(t - 2) = 0$

The body is at rest when $t = \frac{4}{3}$ and $t = 2$.

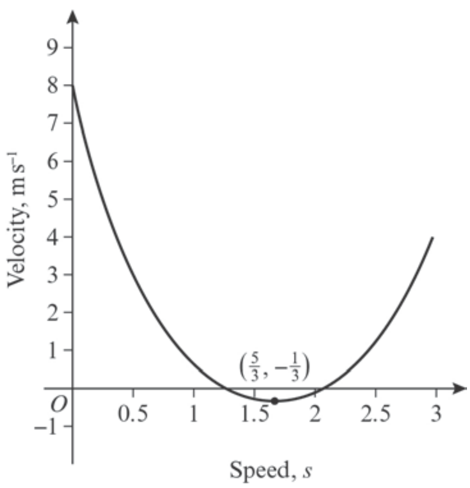
c $3t^2 - 10t + 8 = 5$

$3t^2 - 10t + 3 = 0$

$(3t - 1)(t - 3) = 0$

Velocity = 5 m s^{-1} when $t = \frac{1}{3}$ and $t = 3$.

d



Using the answer to part **b** and symmetry, the body has its maximum/minimum velocity when $t = \frac{5}{3}$ s.

When $t = \frac{5}{3}$,

$$v = 3\left(\frac{5}{3}\right)^2 - 10\left(\frac{5}{3}\right) + 8$$

$$= \frac{25}{3} - \frac{50}{3} + 8$$

$$= -\frac{25}{3} + 8$$

5 d $v = -\frac{1}{3}$

So in $0 \le t \le 2$, range of v is $-\frac{1}{3} \le v \le 8$.

Greatest speed is $8\ \text{m s}^{-1}$.

6 a $8t - 2t^2 = 0$

$2t(4 - t) = 0$

The particle is next at rest after 4 s.

b By symmetry, minimum/maximum velocity is when $t = 2$.

When $t = 2$,

$v = 8(2) - 2(2)^2$

$= 8$

So in $0 \le t \le 4$, greatest speed is $8\ \text{m s}^{-1}$.

7 $s = 3t^2 - t^3$

Model is valid until particle returns to starting point, i.e. until next point at which $s = 0$.

After this it would have a negative displacement, i.e. be beyond O.

$s = 0$ when

$3T^2 - T^3 = 0$

$T^2(3 - T) = 0$

$T \neq 0$ so $T = 3$

8 a $\frac{1}{5}\left(3t^2 - 10t + 3\right) = 0$

$3t^2 - 10t + 3 = 0$

$(3t - 1)(t - 3) = 0$

Particle is at rest when $t = \frac{1}{3}$ and $t = 3$.

b Using answer to part **a** and symmetry, the body has its maximum/minimum velocity when $t = \frac{5}{3}$.

When $t = \frac{5}{3}$,

$$v = \frac{1}{5}\left(3\left(\frac{5}{3}\right)^2 - 10\left(\frac{5}{3}\right) + 3\right)$$

$$= \frac{1}{5}\left(\frac{25}{3} - \frac{50}{3} + \frac{9}{3}\right)$$

$$= \frac{1}{5}\left(-\frac{16}{3}\right)$$

$$= -\frac{16}{15}$$

So in $0 \le t \le 3$, greatest speed is $\frac{16}{15}\ \text{m s}^{-1}$.