## Forces and motion 10B

**1 a** $(-\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) = (3\mathbf{i} + 2\mathbf{j})$
The resultant force is $(3\mathbf{i} + 2\mathbf{j})$ N.

**b** $\begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} -3 \\ -6 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

The resultant force is $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ N.

**c** $(\mathbf{i} + \mathbf{j}) + (5\mathbf{i} - 3\mathbf{j}) + (-2\mathbf{i} - \mathbf{j}) = (4\mathbf{i} - 3\mathbf{j})$
The resultant force is $(4\mathbf{i} - 3\mathbf{j})$ N.

**d** $\begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ -7 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$

The resultant force is $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$ N.

**2 a**
$$\begin{aligned} &\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0 \\ &\Rightarrow (2\mathbf{i} + 7\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) + \mathbf{F}_3 = 0 \\ &\Rightarrow \mathbf{F}_3 = -(2\mathbf{i} + 7\mathbf{j}) - (-3\mathbf{i} + \mathbf{j}) \\ &\qquad = -2\mathbf{i} - 7\mathbf{j} + 3\mathbf{i} - \mathbf{j} \\ &\qquad = \mathbf{i} - 8\mathbf{j} \end{aligned}$$

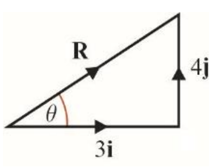
**b**
$$\begin{aligned} &\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0 \\ &\Rightarrow (3\mathbf{i} - 4\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) + \mathbf{F}_3 = 0 \\ &\Rightarrow \mathbf{F}_3 = -(3\mathbf{i} - 4\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) \\ &\qquad = -3\mathbf{i} + 4\mathbf{j} - 2\mathbf{i} - 3\mathbf{j} \\ &\qquad = -5\mathbf{i} + \mathbf{j} \end{aligned}$$

**3** Since object is in equilibrium:

$$\begin{pmatrix} a \\ 2b \end{pmatrix} + \begin{pmatrix} -2a \\ -b \end{pmatrix} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\begin{pmatrix} a \\ 2b \end{pmatrix} + \begin{pmatrix} -2a \\ -b \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$
$$\begin{pmatrix} -a \\ b \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$a = 3$ and $b = 4$

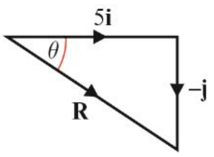
**4 a** $(3\mathbf{i} + 4\mathbf{j})$



**4** **a** **i** $\sqrt{3^2+4^2}=\sqrt{25}$
The resultant force is 5 N.

**ii** $\tan\theta=\frac{4}{3}$
The force makes an angle of 53.1° with **i**.

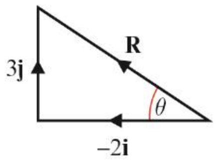
**b** (5**i** – **j**)




**i** $\sqrt{5^2+1^2}=\sqrt{26}$
The resultant force is $\sqrt{26}$ N.

**ii** $\tan\theta=\frac{1}{5}$
The force makes an angle of 11.3° with **i**.

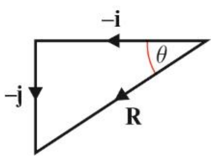
**c** (−2**i** + 3**j**)




**i** $\sqrt{2^2+3^2}=\sqrt{13}$
The resultant force is $\sqrt{13}$ N.

**ii** $\tan\theta=\frac{3}{2}$
$\theta$ = 56.3° This is the angle made with the negative **i** vector
Angle made with the positive **i** vector = 180 − $\theta$
The force makes an angle of 123.7° with **i**.

**d**



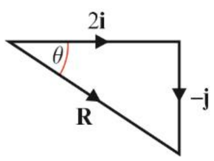

**i** $\sqrt{1^2+1^2}=\sqrt{2}$
The resultant force is $\sqrt{2}$ N.

**4 d ii** $\tan\theta = \frac{1}{1}$

$\theta = 45°$. This is the angle made with the negative **i** vector.
The obtuse angle made with the positive **i** vector = $180 - \theta$
The force makes an angle of 135° with **i**.

**5 a i** $(-2\mathbf{i} + \mathbf{j}) + (5\mathbf{i} + 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = (2\mathbf{i} - \mathbf{j})$
The resultant vector is $(2\mathbf{i} - \mathbf{j})$ N.

**ii** $\sqrt{2^2 + 1^2} = \sqrt{5}$
The magnitude of the resultant vector is $\sqrt{5}$ N.

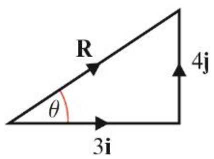



**iii** $\tan\theta = \frac{1}{2}$

$\theta = -26.6°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 116.6°.

**b i** $(-2\mathbf{i} + \mathbf{j}) + (2\mathbf{i} - 3\mathbf{j}) + (3\mathbf{i} + 6\mathbf{j}) = (3\mathbf{i} + 4\mathbf{j})$
The resultant vector is $(3\mathbf{i} + 4\mathbf{j})$ N

**ii** $\sqrt{3^2 + 4^2} = \sqrt{25}$
The resultant force is 5 N.




**iii** $\tan\theta = \frac{4}{3}$

$\theta = 53.1°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 036.9°.

**6** Since the object is in equilibrium:
$(a\mathbf{i} - b\mathbf{j}) + (b\mathbf{i} + a\mathbf{j}) + (-4\mathbf{i} - 2\mathbf{j}) = 0$
Considering **i** components:
$a + b - 4 = 0$
so $b = 4 - a$ **(1)**
Considering **j** components:
$-b + a - 2 = 0$
Substituting $b = 4 - a$ from **(1)**:
$-(4 - a) + a - 2 = 0$
$2a = 2 + 4 = 6$
$a = 3$ **(2)**

**6** Substituting **(2)** into **(1)**:

$b = 4 – 3 = 1$
The values of $a$ and $b$ are 3 and 1, respectively.

**7** Since the object is in equilibrium:
$(2a\mathbf{i} + 2b\mathbf{j}) + (-5b\mathbf{i} + 3a\mathbf{j}) + (-11\mathbf{i} - 7\mathbf{j}) = 0$

Considering **i** components:
$2a - 5b - 11 = 0$ **(1)**

Considering **j** components:
$2b + 3a - 7 = 0$ **(2)**

equation **(1)** × 3 → $6a - 15b - 33 = 0$ **(3)**
equation **(2)** × 2 → $6a + 4b - 14 = 0$ **(4)**

Subtracting (**4**) from (**3**):
$-15b - 33 - 4b - (-14) = 0$
$-19b = 33 - 14$
$b = -1$
Substituting this value into equation (**1**):
$2a – 5(-1) - 11 = 0$
$2a = 11 - 5 = 6$
The values of $a$ and $b$ are 3 and −1, respectively.

**8 a** $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$\Rightarrow (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = 0$

$(-3 + 1 + p)\mathbf{i} + (7 - 1 + q)\mathbf{j} = 0$

$p = 2,\ q = -6$

**b** $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$

$= (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j})$

$= -2\mathbf{i} + 6\mathbf{j}$

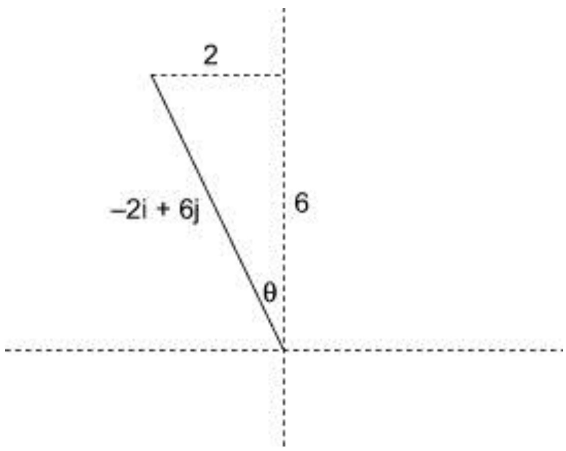
$|\mathbf{R}| = \sqrt{(-2)^2 + 6^2}$

$= \sqrt{4 + 36}$

$= \sqrt{40}$

$= 6.32$ N

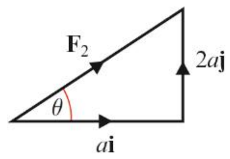
**c**




$\tan\theta = \frac{2}{6}$

$\theta = 18^{\circ}$

**9 a** $\mathbf{F}_2 = (a\mathbf{i} + 2a\mathbf{j})$




$\tan\theta = \frac{2a}{a} = 2$

$\mathbf{F}_2$ makes an angle of 63.4° with **i**.

**b** $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (3\mathbf{i} - 2\mathbf{j}) + (a\mathbf{i} + 2a\mathbf{j})$

**i** vector = $3 + a$  **j** vector = $-2 + 2a$

In the vector $(13\mathbf{i} + 10\mathbf{j})$:
**i** vector = 13  **j** vector = 10

Let $\theta_1$ = the angle of vector **R** and $\theta_2$ = the angle of vector $(13\mathbf{i} + 10\mathbf{j})$

Since the vectors are parallel, $\theta_1 = \theta_2$ so $\tan\theta_1 = \tan\theta_2$:

$\tan\theta_1 = \frac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \frac{-2+2a}{3+a}$  $\tan\theta_2 = \frac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \frac{10}{13}$

$\Rightarrow \frac{-2+2a}{3+a} = \frac{10}{13}$

$(-2 + 2a) \times 13 = (3 + a) \times 10$

$16a = 56$

$a = 3.5$

**10 a** Since the particle $P$ is in equilibrium:

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$$\begin{pmatrix}-7\\-4\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

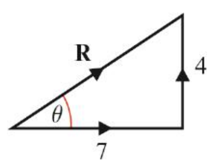
$$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix} + \begin{pmatrix}-4\\-2\end{pmatrix}$$

$$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}$$

The values are $a = 3$, $b = 2$

**b** $\mathbf{R} = \mathbf{F}_2 + \mathbf{F}_3$

$$\mathbf{R} = \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix}$$



**10 b i** $|\mathbf{R}| = \sqrt{7^2 + 4^2} = \sqrt{65}$

The magnitude of **R** is $\sqrt{65}$ N.

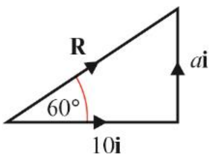
**ii** $\tan\theta = \frac{4}{7}$

$\theta = 29.7\ldots°$

**R** acts at 30° above the horizontal (to 2 s.f.)

**Challenge**

Redrawing the diagram as a closed triangle:




$\tan 60 = \frac{a}{10}$

$a = 10\tan 60 = 10 \times \sqrt{3}$

$\mathbf{R} = \begin{pmatrix} 10 \\ a \end{pmatrix} = \begin{pmatrix} 10 \\ 10\sqrt{3} \end{pmatrix}$

$|\mathbf{R}| = \sqrt{10^2 + \left(10\sqrt{3}\right)^2} = \sqrt{100 + 300} = \sqrt{400}$

The value of $a$ is 17.3 (to 3 s.f.), and the magnitude of the resultant force is 20 N.