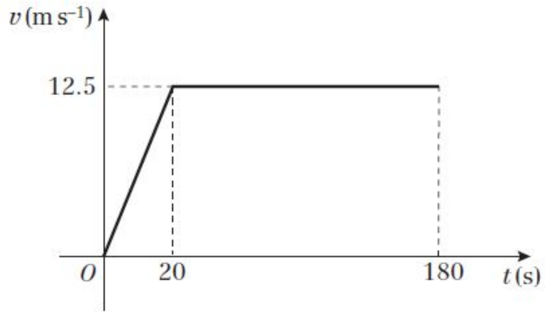
Constant acceleration, Mixed Exercise 9

1 a $45 \text{ km h}^{-1} = \dfrac{45 \times 1000}{3600} \text{ m s}^{-1}$
$= 12.5 \text{ m s}^{-1}$

$3 \text{ min} = 180 \text{ s}$

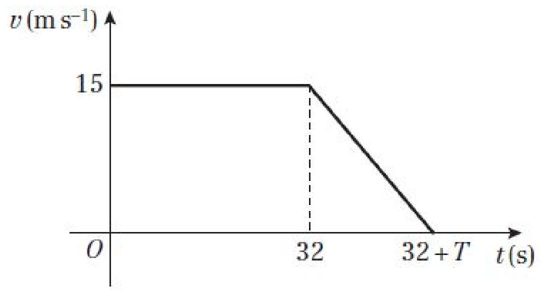


b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(160+180) \times 12.5 = 2125$

The distance from A to B is 2125 m.

2 a



b $s = \frac{1}{2}(a+b)h$

$570 = \frac{1}{2}(32+32+T) \times 15$

$\frac{15}{2}(T+64) = 570$

$T + 64 = \dfrac{570 \times 2}{15} = 76$

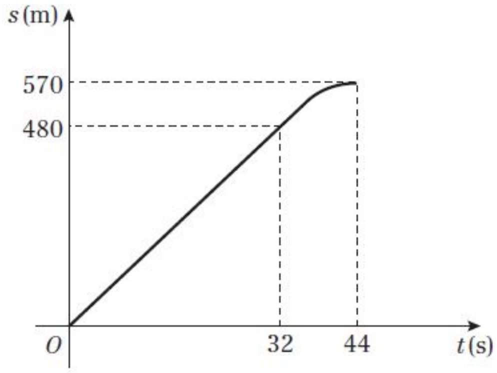
$T = 76 - 64 = 12$

c At $t = 32$, $s = 32 \times 15 = 480$

At $t = 44$, $s = 480 + \text{area of the triangle}$

$= 480 + \frac{1}{2} \times 12 \times 15 = 570$

2 c



3 a i Gradient of line = $\dfrac{v-u}{t}$

$a = \dfrac{v-u}{t}$

Rearranging: $v = u + at$

ii Shaded area is a trapezium

area = $\left(\dfrac{u+v}{2}\right)t$

$s = \left(\dfrac{u+v}{2}\right)t$

b i Rearrange $v = u + at$

$t = \dfrac{v-u}{a}$

Substitute into $s = \left(\dfrac{u+v}{2}\right)t$

$s = \left(\dfrac{u+v}{2}\right)\left(\dfrac{v-u}{a}\right)$

$2as = v^2 - u^2$
$v^2 = u^2 + 2as$

ii Substitute $v = u + at$ into $s = \left(\dfrac{u+v}{2}\right)t$

$s = \left(\dfrac{u+u+at}{2}\right)t$

$s = \left(\dfrac{2u}{2} + \dfrac{at}{2}\right)t$

$s = ut + \frac{1}{2}at^2$

3 b iii Substitute $u = v - at$ into $s = ut + \frac{1}{2}at^2$

$$s = (v - at)t + \frac{1}{2}at^2$$
$$s = vt - \frac{1}{2}at^2$$

4 $s = \frac{1}{2}(a+b)h$

$$152 = \frac{1}{2}(15+23)u = 19u$$
$$u = \frac{152}{19} = 8$$

5 $40 \text{ km h}^{-1} = \dfrac{40\times1000}{3600} \text{ m s}^{-1} = \dfrac{100}{9} \text{ m s}^{-1}$

$24 \text{ km h}^{-1} = \dfrac{24\times1000}{3600} \text{ m s}^{-1} = \dfrac{20}{3} \text{ m s}^{-1}$

$u = \frac{100}{9},\ v = \frac{20}{3},\ s = 240,\ a = ?$

$$v^2 = u^2 + 2as$$
$$\left(\tfrac{20}{3}\right)^2 = \left(\tfrac{100}{9}\right)^2 + 2\times a\times 240$$
$$a = \frac{\left(\frac{20}{3}\right)^2 - \left(\frac{100}{9}\right)^2}{2\times240} = -0.165 \text{ (to 2 s.f.)}$$

The deceleration of the car is 0.165 m s^{-2}.

6 a $a = -2.5,\ u = 20,\ t = 12,\ s = ?$

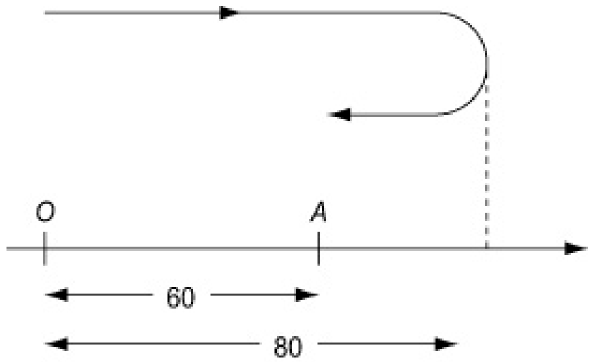
$$s = ut + \tfrac{1}{2}at^2$$
$$= 20\times12 - \tfrac{1}{2}\times2.5\times12^2$$
$$= 240 - 180 = 60$$

$OA = 60$ m

b The particle will turn round when $v = 0$

$a = -2.5,\ u = 20,\ v = 0,\ s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 20^2 - 5s \Rightarrow s = 80$$



The total distance P travels is $(80+20)$ m $= 100$ m

7 $u = 6,\ v = 25,\ a = 9.8,\ t = ?$

$$v = u + at$$
$$25 = 6 + 9.8t$$
$$t = \frac{25-6}{9.8} = 1.9 \text{ (to 2 s.f.)}$$

The ball takes 1.9 s to move from the top of the tower to the ground.

8 Take downwards as the positive direction.

a $u = 0,\ s = 82,\ a = 9.8,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$82 = 0 + 4.9t^2$$
$$t = \sqrt{\frac{82}{4.9}} = 4.1 \text{ (to 2 s.f.)}$$

The time taken for the ball to reach the sea is 4.1 s.

b $u = 0$, $s = 82$, $a = 9.8$, $v = ?$

$$v^2 = u^2 + 2as$$
$$= 0 + 2 \times 9.8 \times 82 = 1607.2$$
$$v = \sqrt{1607.2} = 40 \text{ (to 2 s.f.)}$$

The speed at which the ball hits the sea is 40 m s^{-1}.

c Air resistance/wind/turbulence

9 **a** distance = area of triangle + area of rectangle + area of trapezium

$$451 = \tfrac{1}{2} \times 8 \times 2u + 12 \times 2u + \tfrac{1}{2} \times (u + 2u) \times 6$$
$$= 8u + 24u + 9u = 41u$$
$$u = \tfrac{451}{41} = 11$$

b The particle is moving with speed less than u m s^{-1} for the first 4 s

$$s = \tfrac{1}{2} \times 4 \times 11 = 22$$

The distance moved with speed less than u m s^{-1} is 22 m.

10 a From O to P, $u=18,\ t=12,\ v=24,\ a=?$

$u=18,\ t=12,\ v=24,\ a=?$

$v=u+at$

$24=18+12a$

$$a=\frac{24-18}{12}=\frac{1}{2}$$

From O to Q, $u=18,\ t=20,\ a=\tfrac{1}{2},\ v=?$

$v=u+at$

$=18+\tfrac{1}{2}\times 20=28$

The speed of the train at Q is 28 m s^{-1}.

b From P to Q

$u=24,\ v=28,\ t=8,\ s=?$

$$s=\left(\frac{u+v}{2}\right)t=\left(\frac{24+28}{2}\right)\times 8=208$$

The distance from P to Q is 208 m.

11 a $s=104,\ t=8,\ v=18,\ u=?$

$$s=\left(\frac{u+v}{2}\right)t$$

$$104=\left(\frac{u+18}{2}\right)\times 8=(u+18)\times 4=4u+72$$

$$u=\frac{104-72}{4}=8$$

The speed of the particle at X is 8 m s^{-1}

b $s=104,\ t=8,\ v=18,\ a=?$

$s=vt-\tfrac{1}{2}at^2$

$104=18\times 8-\tfrac{1}{2}a\times 8^2=144-32a$

$$a=\frac{144-104}{32}=1.25$$

The acceleration of the particle is 1.25 m s^{-2}.

11 c From X to Z, $u = 8$, $v = 24$, $a = 1.25$, $s = ?$

$$v^2 = u^2 + 2as$$
$$24^2 = 8^2 + 2 \times 1.25 \times s$$
$$s = \frac{24^2 - 8^2}{2.5} = 204.8$$

$XZ = 204.8$ m

12 a Take upwards as the positive direction.

$u = 21$, $s = -32$, $a = -9.8$, $v = ?$

$$v^2 = u^2 + 2as$$
$$= 21^2 + 2 \times (-9.8) \times (-32) = 441 + 627.2 = 1068.2$$
$$v = \sqrt{1068.2} = \pm 33 \text{ (to 2 s.f.)}$$

The velocity with which the pebble strikes the ground is -33 m s^{-1}.
The speed is 33 m s^{-1}.

b 40 m above the ground is 8 m above the point of projection.

$u = 21$, $s = 8$, $a = -9.8$, $t = ?$
$s = ut + \frac{1}{2}at^2$
$8 = 21t - 4.9t^2$

$0 = 4.9t^2 - 21t + 8$, so using the quadratic formula,

$$t = \frac{21 \pm \sqrt{21^2 - 4 \times 4.9 \times 8}}{9.8} = \frac{21 \pm \sqrt{284.2}}{9.8} = 3.86, 0.423 \text{ (to 3 s.f.)}$$

The pebble is above 40 m between these times: $3.863... - 0.423... = 3.4$ (to 2 s.f.)
The pebble is more than 40 m above the ground for 3.4 s.

12 c Take upwards as the positive direction.

$u = 21$, $a = -9.8$

$v = u + at = 21 - 9.8t \Rightarrow t = \dfrac{21 - v}{9.8}$

From part **a**, the pebble hits the ground when $v = -33$.

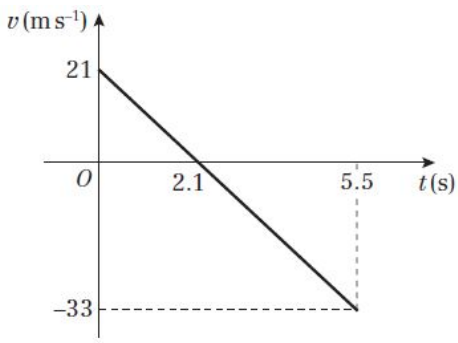
$t = \dfrac{21 - v}{9.8} = \dfrac{21 - (-33)}{9.8} = \dfrac{54}{9.8} = 5.5$ (to 2 s.f.)

This is shown on the graph at point $(5.5, -33)$

The graph crosses the t-axis when $v = 0$.

$t = \dfrac{21 - v}{9.8} = \dfrac{21 - 0}{9.8} = \dfrac{21}{9.8} = 2.1$ (to 2 s.f.)

So the graph passes through point $(2.1,\ 0)$



13 a $u = 12,\ v = 32,\ s = 1100,\ t = ?$

$s = \left(\dfrac{u + v}{2}\right)t$

$1100 = \left(\dfrac{12 + 32}{2}\right)t = 22t \Rightarrow t = \dfrac{1100}{22} = 50$

The time taken by the car to move from A to C is 50 s.

13 b Find a first.

From A to C, $u = 12,\ v = 32,\ t = 50,\ a = ?$

$$v = u + at$$
$$32 = 12 + a \times 50$$
$$a = \frac{32 - 12}{50} = 0.4$$

From A to B, $u = 12,\ s = 550,\ a = 0.4,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 12^2 + 2 \times 0.4 \times 550 = 584 \Rightarrow v = 24.2 \text{ (to 3 s.f.)}$$

The car passes B with speed 24.2 m s^{-1}.

14 Take upwards as the positive direction.

At the top:

$u = 30,\ v = 0,\ a = -9.8,\ t = ?$

$$v = u + at$$
$$0 = 30 - 9.8t \Rightarrow t = \frac{30}{9.8}$$

The ball spends 2.4 seconds above h, thus (by symmetry) 1.2 seconds rising between h and the top.
So it passes h 1.2 seconds earlier, at $t = \frac{30}{9.8} - 1.2 = 1.86$ (to 3 s.f.)

At h, $u = 30,\ a = -9.8,\ t \approx 1.86, s = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$= 30 \times 1.86 + \tfrac{1}{2}(-9.8) \times 1.86^2 = 39 \text{ (to 2 s.f.)}$$

15 a $u = 20,\ a = 4,\ s = 78,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 20^2 + 2 \times 4 \times 78 = 1024$$
$$v = \sqrt{1024} = 32$$

The speed of B when it has travelled 78 m is 32 m s^{-1}.

15 b Find time for B to reach the point 78 m from O.

$v = 32,\ u = 20,\ a = 4,\ t = ?$

$v = u + at$

$32 = 20 + 4t \Rightarrow t = \dfrac{32 - 20}{4} = 3$

For A, distance = speed × time

$s = 30 \times 3 = 90$

The distance from O of A when B is 78 m from O is 90 m.

c At time t seconds, for A, $s = 30t$

for B, $s = ut + \frac{1}{2}at^2 = 20t + 2t^2$

On overtaking the distances are the same.

$20t + 2t^2 = 30t$

$t^2 - 5t = t(t - 5) = 0$

$t = 5$ (at $t = 0$, A overtakes B)

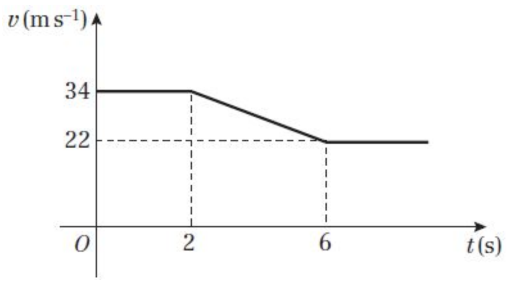
B overtakes A 5 s after passing O.

16 a To find time decelerating:

$u = 34,\ v = 22,\ a = -3,\ t = ?$

$v = u + at$

$22 = 34 - 3t \Rightarrow t = \dfrac{34 - 22}{3} = 4$

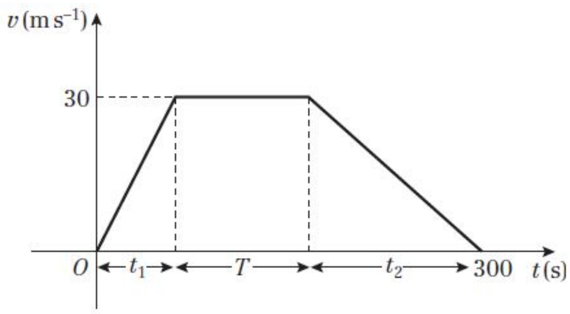


16 b distance = rectangle + trapezium

$$s = 34 \times 2 + \tfrac{1}{2}(22 + 34) \times 4$$
$$= 68 + 112 = 180$$

Distance required is 180 m.

17 a



b Acceleration is the gradient of a line.

For the first part of the journey, $3x = \dfrac{30}{t_1} \Rightarrow t_1 = \dfrac{30}{3x} = \dfrac{10}{x}$

For the last part of the journey, $-x = -\dfrac{30}{t_2} \Rightarrow t_2 = \dfrac{30}{x}$

$t_1 + T + t_2 = 300$

$\frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300,$ as required

c $s = \tfrac{1}{2}(a + b)h$

$6000 = \tfrac{1}{2}(T + 300) \times 30 = 15T + 4500$

$T = \dfrac{6000 - 4500}{15} = 100$

Substitute into the result in part **b**:

$\frac{40}{x} + 100 = 300 \Rightarrow \frac{40}{x} = 200$

$x = \frac{40}{200} = 0.2$

d From part **c**, $T = 100$

At constant velocity, distance = velocity × time $= 30 \times 100 = 3000$ (m)

The distance travelled at a constant speed is 3 km.

17 e From part **b**, $t_1 = \frac{10}{x} = \frac{10}{0.2} = 50$

Total distance travelled = 6 km (given) so halfway = 3 km = 3000 m

While accelerating, distance travelled is $\left(\frac{1}{2} \times 50 \times 30\right)$ m = 750 m.

At constant velocity, the train must travel a further 2250 m.

At constant velocity, time $= \frac{\text{distance}}{\text{velocity}} = \frac{2250}{30}$ s = 75 s

Time for train to reach halfway is $(50 + 75)$ s $= 125$ s

Challenge

Find the time taken by the first ball to reach 25 m below its point of projection (25 m above the ground). Take upwards as the positive direction.

$u = 10,\ s = -25,\ a = -9.8,\ t = ?$

$$\begin{aligned}
s &= ut + \tfrac{1}{2}at^2 \\
-25 &= 10t - 4.9t^2 \\
0 &= 4.9t^2 - 10t - 25 \\
t &= 10 \pm \frac{\sqrt{102 + 4 \times 4.9 \times 25}}{9.8} \\
&= 3.5 \text{ (to 2 s.f.)}
\end{aligned}$$

As we discard the negative solution. Find the time taken by the second ball to reach 25 m below its point of projection. Take downwards as the positive direction.

$u = 0,\ s = 25,\ a = 9.8,\ t = ?$

$$\begin{aligned}
s &= ut + \tfrac{1}{2}at^2 \\
25 &= 4.9t^2 \\
t &= 2.3 \text{ (to 2 s.f.)}
\end{aligned}$$

Combining the two results:

$T = 3.4989\ldots - 2.2587\ldots = 1.2$ (to 2 s.f. using exact figures)