Constant acceleration, Mixed Exercise 9

1 a \[45 \text{ km h}^{-1} = \frac{45 \times 1000}{3600} \text{ m s}^{-1} = 12.5 \text{ m s}^{-1}\]
\[3 \text{ min} = 180 \text{ s}\]

b \[s = \frac{1}{2} (a + b)h\]
\[= \frac{1}{2} (160 + 180) \times 12.5 = 2125\]

The distance from A to B is 2125 m.

2 a

b \[s = \frac{1}{2} (a + b)h\]

\[570 = \frac{1}{2} (32 + 32 + T) \times 15\]
\[\frac{570}{15} (T + 64) = 570\]
\[T + 64 = \frac{570 \times 2}{15} = 76\]
\[T = 76 - 64 = 12\]

c At \( t = 32\), \( s = 32 \times 15 = 480\)

At \( t = 44\), \( s = 480 + \text{area of the triangle}\)
\[= 480 + \frac{1}{2} \times 12 \times 15 = 570\]
2 c

![Graph showing s(m) vs t(s)]

3 a i Gradient of line $= \frac{v-u}{t}$

$$a = \frac{v-u}{t}$$

Rearranging: $v = u + at$

ii Shaded area is a trapezium

Area $= \frac{(u+v)t}{2}$

$$s = \left(\frac{u+v}{2}\right)t$$

3 b i Rearrange $v = u + at$

$$t = \frac{v-u}{a}$$

Substitute into $s = \left(\frac{u+v}{2}\right)t$

$$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

ii Substitute $v = u + at$ into $s = \left(\frac{u+v}{2}\right)t$

$$s = \left(\frac{u+u+at}{2}\right)t$$

$$s = \left(\frac{2u + at}{2}\right)t$$

$$s = ut + \frac{1}{2}at^2$$
3 b iii Substitute \( u = v - at \) into \( s = ut + \frac{1}{2}at^2 \)

\[
s = (v - at)t + \frac{1}{2}at^2
\]

\[
s = vt - \frac{1}{2}at^2
\]

4 \( s = \frac{1}{2}(a + b)h \)

\[152 = \frac{1}{2}(15 + 23)u = 19u\]

\[u = \frac{152}{19} = 8\]

5 \( 40 \text{ km h}^{-1} = \frac{40 \times 1000}{3600} \text{ m s}^{-1} = \frac{100}{9} \text{ m s}^{-1} \)

\( 24 \text{ km h}^{-1} = \frac{24 \times 1000}{3600} \text{ m s}^{-1} = \frac{20}{3} \text{ m s}^{-1} \)

\[u = \frac{100}{9}, \quad v = \frac{20}{3}, \quad s = 240, \quad a = ? \]

\[v^2 = u^2 + 2as\]

\[\left(\frac{20}{3}\right)^2 = \left(\frac{100}{9}\right)^2 + 2 \times a \times 240\]

\[a = \frac{\left(\frac{20}{3}\right)^2 - \left(\frac{100}{9}\right)^2}{2 \times 240} = -0.165 \text{ (to 2 s.f.)} \]

The deceleration of the car is 0.165 m s\(^{-2}\).

6 a \( a = -2.5, \quad u = 20, \quad t = 12, \quad s = ? \)

\[s = ut + \frac{1}{2}at^2\]

\[= 20 \times 12 - \frac{1}{2} \times 2.5 \times 12^2\]

\[= 240 - 180 = 60\]

\(OA = 60 \text{ m}\)

b The particle will turn round when \( v = 0 \)

\[a = -2.5, \quad u = 20, \quad v = 0, \quad s = ? \]

\[v^2 = u^2 + 2as\]

\[0^2 = 20^2 - 5s \Rightarrow s = 80\]

The total distance \( P \) travels is \((80 + 20) \text{ m} = 100 \text{ m}\)
7 \[ u = 6, \quad v = 25, \quad a = 9.8, \quad t = ? \]
\[ v = u + at \]
\[ 25 = 6 + 9.8t \]
\[ t = \frac{25 - 6}{9.8} = 1.9 \text{ (to 2 s.f.)} \]

The ball takes 1.9 s to move from the top of the tower to the ground.

8 \[ \text{Take downwards as the positive direction.} \]

a \[ u = 0, \quad s = 82, \quad a = 9.8, \quad t = ? \]
\[ s = ut + \frac{1}{2}at^2 \]
\[ 82 = 0 + 4.9t^2 \]
\[ t = \sqrt{\frac{82}{4.9}} = 4.1 \text{ (to 2 s.f.)} \]

The time taken for the ball to reach the sea is 4.1 s.

b \[ u = 0, \quad s = 82, \quad a = 9.8, \quad v = ? \]
\[ v^2 = u^2 + 2as \]
\[ = 0 + 2 \times 9.8 \times 82 = 1607.2 \]
\[ v = \sqrt{1607.2} = 40 \text{ (to 2 s.f.)} \]

The speed at which the ball hits the sea is 40 m s\(^{-1}\).

c \[ \text{Air resistance/wind/turbulence} \]

9 a \[ \text{distance} = \text{area of triangle} + \text{area of rectangle} + \text{area of trapezium} \]
\[ 451 = \frac{1}{2} \times 8 \times 2u + 12 \times 2u + \frac{1}{2} \times (u + 2u) \times 6 \]
\[ = 8u + 24u + 9u = 41u \]
\[ u = \frac{451}{41} = 11 \]

b \[ \text{The particle is moving with speed less than} \ u \text{ m s}^{-1} \text{ for the first 4 s} \]
\[ s = \frac{1}{2} \times 4 \times 11 = 22 \]

The distance moved with speed less than \( u \) m s\(^{-1}\) is 22 m.

10 a \[ \text{From} \ O \text{ to} \ P, \ u = 18, \quad t = 12, \quad v = 24, \quad a = ? \]
\[ u = 18, \quad t = 12, \quad v = 24, \quad a = ? \]
\[ v = u + at \]
\[ 24 = 18 + 12a \]
10 a \[ a = \frac{24 - 18}{12} = \frac{1}{2} \]

From \( O \) to \( Q \), \( u = 18, \ t = 20, \ a = \frac{1}{2}, \ v = ? \)

\[ v = u + at \]
\[ = 18 + \frac{1}{2} \times 20 = 28 \]

The speed of the train at \( Q \) is 28 m s\(^{-1}\).

b From \( P \) to \( Q \)

\( u = 24, \ v = 28, \ t = 8, \ s = ? \)

\[ s = \left( \frac{u + v}{2} \right) t = \left( \frac{24 + 28}{2} \right) \times 8 = 208 \]

The distance from \( P \) to \( Q \) is 208 m.

11 a \[ s = 104, \ t = 8, \ v = 18, \ u = ? \]

\[ s = \left( \frac{u + v}{2} \right) t \]
\[ 104 = \left( \frac{u + 18}{2} \right) \times 8 = (u + 18) \times 4 = 4u + 72 \]
\[ u = \frac{104 - 72}{4} = 8 \]

The speed of the particle at \( X \) is 8 m s\(^{-1}\).

b \[ s = 104, \ t = 8, \ v = 18, \ a = ? \]

\[ s = vt - \frac{1}{2}at^2 \]
\[ 104 = 18 \times 8 - \frac{1}{2}a \times 8^2 = 144 - 32a \]
\[ a = \frac{144 - 104}{32} = 1.25 \]

The acceleration of the particle is 1.25 m s\(^{-2}\).

c From \( X \) to \( Z \), \( u = 8, \ v = 24, \ a = 1.25, \ s = ? \)

\[ v^2 = u^2 + 2as \]
\[ 24^2 = 8^2 + 2 \times 1.25 \times s \]
\[ s = \frac{24^2 - 8^2}{2.5} = 204.8 \]

\( XZ = 204.8 \) m
12 a  Take upwards as the positive direction.

\[ u = 21, \ s = -32, \ a = -9.8, \ v = ? \]

\[ v^2 = u^2 + 2as \]
\[ = 21^2 + 2 \times (-9.8) \times (-32) = 441 + 627.2 = 1068.2 \]
\[ v = \sqrt{1068.2} = \pm 33 \text{ (to 2 s.f.)} \]

The velocity with which the pebble strikes the ground is \(-33 \text{ m s}^{-1}\).
The speed is \(33 \text{ m s}^{-1}\).

b  40 m above the ground is 8 m above the point of projection.

\[ u = 21, \ s = 8, \ a = -9.8, \ t = ? \]
\[ s = ut + \frac{1}{2}at^2 \]
\[ 8 = 21t - 4.9t^2 \]
\[ 0 = 4.9t^2 - 21t + 8, \text{ so using the quadratic formula,} \]
\[ t = \frac{21 \pm \sqrt{21^2 - 4 \times 4.9 \times 8}}{9.8} = \frac{21 \pm \sqrt{284.2}}{9.8} = 3.86, 0.423 \text{ (to 3 s.f.)} \]

The pebble is above 40 m between these times: 3.86, 0.423, 3.44 (to 3 s.f.)
The pebble is more than 40 m above the ground for 3.44 s.

c  Take upwards as the positive direction.

\[ u = 21, \ a = -9.8 \]
\[ v = u + at = 21 - 9.8t \Rightarrow t = \frac{21 - v}{9.8} \]

From part a, the pebble hits the ground when \(v = -33\).
\[ t = \frac{21 - v}{9.8} = \frac{21 - (-33)}{9.8} = \frac{54}{9.8} = 5.5 \text{ (to 2 s.f.)} \]

This is shown on the graph at point (5.5, 33).

The graph crosses the \(t\)-axis when \(v = 0\).
\[ t = \frac{21 - v}{9.8} = \frac{21 - 0}{9.8} = \frac{21}{9.8} = 2.1 \text{ (to 2 s.f.)} \]

So the graph passes through point (2.1, 0)
13 a \(u = 12, \ v = 32, \ s = 1100, \ t = ?\)

\[s = \left(\frac{u + v}{2}\right)t\]

\[1100 = \left(\frac{12 + 32}{2}\right)t \Rightarrow t = \frac{1100}{22} = 50\]

The time taken by the car to move from A to C is 50 s.

b Find \(a\) first.

From A to C, \(u = 12, \ v = 32, \ t = 50, \ a = ?\)

\[v = u + at\]

\[32 = 12 + a \times 50\]

\[a = \frac{32 - 12}{50} = 0.4\]

From A to B, \(u = 12, \ s = 550, \ a = 0.4, \ v = ?\)

\[v^2 = u^2 + 2as\]

\[= 12^2 + 2 \times 0.4 \times 550 = 584 \Rightarrow v = 24.2 \text{ (to 3 s.f.)} \]

The car passes B with speed 24.2 m s\(^{-1}\).

14 Take upwards as the positive direction.

At the top:

\(u = 30, \ v = 0, \ a = -9.8, \ t = ?\)

\[v = u + at\]

\[0 = 30 - 9.8t \Rightarrow t = \frac{30}{9.8}\]
The ball spends 2.4 seconds above \( h \), thus (by symmetry) 1.2 seconds rising between \( h \) and the top. So it passes \( h \) 1.2 seconds earlier, at \( t = \frac{30}{9.8} - 1.2 = 1.86 \) (to 3 s.f.)

At \( h \), \( u = 30 \), \( a = -9.8 \), \( t \approx 1.86 \), \( s = ? \)

\[
s = ut + \frac{1}{2}at^2
\]
\[
= 30 \times 1.86 + \frac{1}{2}(-9.8) \times 1.86^2 = 39 \text{ (to 2 s.f.)}
\]

15a \( u = 20 \), \( a = 4 \), \( s = 78 \), \( v = ? \)

\[
v^2 = u^2 + 2as
\]
\[
= 20^2 + 2 \times 4 \times 78 = 1024
\]
\[
v = \sqrt{1024} = 32
\]

The speed of \( B \) when it has travelled 78 m is 32 m s\(^{-1}\).

b Find time for \( B \) to reach the point 78 m from \( O \).

\[
v = 32, \ u = 20, \ a = 4, \ t = ?
\]

\[
v = u + at
\]
\[
32 = 20 + 4t \Rightarrow t = \frac{32 - 20}{4} = 3
\]

For \( A \), distance = speed \times time

\[
s = 30 \times 3 = 90
\]

The distance from \( O \) of \( A \) when \( B \) is 78 m from \( O \) is 90 m.

c At time \( t \) seconds, for \( A \), \( s = 30t \)

for \( B \), \( s = ut + \frac{1}{2}at^2 = 20t + 2t^2 \)

On overtaking the distances are the same.

\[
20t + 2t^2 = 30t
\]
\[
t^2 - 5t = t(t - 5) = 0
\]
\[
t = 5 \text{ (at } t = 0, \ A \text{ overtakes } B)
\]

\( B \) overtakes \( A \) 5 s after passing \( O \).
16 a To find time decelerating:

\[ u = 34, \ v = 22, \ a = -3, \ t = ? \]

\[ v = u + at \]
\[ 22 = 34 - 3t \Rightarrow t = \frac{34 - 22}{3} = 4 \]

\[ v(m/s^2) \]

\[ 34 \]
\[ 22 \]

\[ 0 \]
\[ 2 \]
\[ 6 \]
\[ t(s) \]

b distance = rectangle + trapezium

\[ s = 34 \times 2 + \frac{1}{2} (22 + 34) \times 4 \]
\[ = 68 + 112 = 180 \]

Distance required is 180 m.

17 a

\[ v(m/s^2) \]

For the first part of the journey, \( 3x = \frac{30}{t_1} \Rightarrow t_1 = \frac{30}{3x} = \frac{10}{x} \)

For the last part of the journey, \( -x = -\frac{30}{t_2} \Rightarrow t_2 = \frac{30}{x} \)

\[ t_1 + T + t_2 = 300 \]

\[ \frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300, \text{ as required} \]

c \[ s = \frac{1}{2} (a + b)h \]
\[ 6000 = \frac{1}{2} (T + 300) \times 30 = 15T + 4500 \]
17c \[ T = \frac{6000 - 4500}{15} = 100 \]

Substitute into the result in part b:

\[ \frac{40}{x} + 100 = 300 \Rightarrow \frac{40}{x} = 200 \]

\[ x = \frac{40}{200} = 0.2 \]

d From part c, \( T = 100 \)

At constant velocity, distance = velocity \( \times \) time \( = 30 \times 100 = 3000 \) (m)

The distance travelled at a constant speed is 3 km.

e From part b, \( t_1 = \frac{10}{x} = \frac{10}{0.2} = 50 \)

Total distance travelled = 6 km (given) so halfway = 3 km = 3000 m

While accelerating, distance travelled is \( \left( \frac{1}{2} \times 50 \times 30 \right) \) m = 750 m.

At constant velocity, the train must travel a further 2250 m.

At constant velocity, time = \( \frac{\text{distance}}{\text{velocity}} = \frac{2250}{30} \) s = 75 s

Time for train to reach halfway is \( (50 + 75) \) s = 125 s
Challenge

Find the time taken by the first ball to reach 25 m below its point of projection (25 m above the ground). Take upwards as the positive direction.

\[ u = 10, \ s = -25, \ a = -9.8, \ t = ? \]

\[ s = ut + \frac{1}{2}at^2 \]
\[-25 = 10t - 4.9t^2\]
\[0 = 4.9t^2 - 10t - 25\]
\[t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 4.9 \times 25}}{2 \times 4.9}\]
\[t = \frac{10 \pm \sqrt{100 + 4 \times 4.9 \times 25}}{9.8}\]
\[t = 10 \pm \frac{\sqrt{102 + 4 \times 4.9 \times 25}}{9.8}\]
\[t = 3.5 \text{ (to 2 s.f.)}\]

As we discard the negative solution. Find the time taken by the second ball to reach 25 m below its point of projection. Take downwards as the positive direction.

\[ u = 0, \ s = 25, \ a = 9.8, \ t = ? \]

\[s = ut + \frac{1}{2}at^2\]
\[25 = 4.9t^2\]
\[t = 2.3 \text{ (to 2 s.f.)}\]

Combining the two results:

\[T = 3.4989 \ldots - 2.2587 \ldots = 1.2 \text{ (to 2 s.f. using exact figures)}\]