Constant acceleration 9E

1 a Take downwards as the positive direction.

\[ s = 28, u = 0, a = 9.8, t = {?} \]

\[ s = ut + \frac{1}{2}at^2 \]
\[ 28 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2 \]
\[ t = \sqrt{\frac{28}{4.9}} = 2.4 \text{ (to 2 s.f.)} \]

The time taken for the diver to hit the water is 2.4 s.

b \[ v^2 = u^2 + 2as \]
\[ v^2 = 0 + 2 \times 9.8 \times 28 = 548.8 \]
\[ v = \sqrt{548.8} = 23 \text{ (to 2 s.f.)} \]

When the diver hits the water, he is travelling at 23 m s\(^{-1}\).

2 Take upwards as the positive direction.

\[ u = 20, \ a = -9.8, \ s = 0, \ t = {?} \]

\[ s = ut + \frac{1}{2}at^2 \]
\[ 0 = 20t - 4.9t^2 = t(20 - 4.9t), \ t \neq 0 \]
\[ t = \frac{20}{4.9} = 4.1 \text{ (to 2 s.f.)} \]

The time of flight of the particle is 4.1 s.

3 Take downwards as the positive direction.

\[ u = 18, \ a = 9.8, \ t = 1.6, \ s = {?} \]

\[ s = ut + \frac{1}{2}at^2 = 18 \times 1.6 + 4.9 \times 1.6^2 = 41 \text{ (to 2 s.f.)} \]

The height of the tower is 41 m.

4 a Take upwards as the positive direction.

\[ u = 24, \ a = -9.8, \ v = 0, \ s = {?} \]

\[ v^2 = u^2 + 2as \]
\[ 0^2 = 24^2 - 2 \times 9.8 \times s \]
\[ s = \frac{24^2}{2 \times 9.8} = 29 \text{ (to 2 s.f.)} \]

The greatest height reached by the pebble above the point of projection is 29 m.
4 b  \( u = 24, \ a = -9.8, \ v = 0, \ t = ? \)

\[
v = u + at \\
0 = 24 - 9.8t \\
t = \frac{24}{9.8} = 2.4 \text{ (to 2 s.f.)}
\]

The time taken to reach the greatest height is 2.4 s.

5 a  Take upwards as the positive direction.

\( u = 18, \ a = -9.8, \ s = 15, \ v = ? \)

\[
v^2 = u^2 + 2as = 18^2 - 2 \times 9.8 \times 15 = 30 \\
v = \sqrt{30} = \pm 5.5 \text{ (to 2 s.f.)}
\]

The speed of the ball when it is 15 m above its point of projection is 5.5 m s\(^{-1}\).

b  \( u = 18, \ a = -9.8, \ s = -4, \ v = ? \)

\[
v^2 = u^2 + 2as = 18^2 + 2 \times (-9.8) \times (-4) = 324 + 78.4 = 402.4 \\
v = -\sqrt{402.2} = -20 \text{ (to 2 s.f.)}
\]

The speed with which the ball hits the ground is 20 m s\(^{-1}\).

6 a  Take downwards as the positive direction.

\( s = 80, \ u = 4, \ a = 9.8, \ v = ? \)

\[
v^2 = u^2 + 2as \\
= 4^2 + 2 \times 9.8 \times 80 = 1584 \\
v = \sqrt{1584} = 40 \text{ (to 2 s.f.)}
\]

The speed with which \( P \) hits the ground is 40 m s\(^{-1}\).

b  \( u = 4, \ a = 9.8, \ v = \sqrt{1584}, \ t = ? \)

\[
v = u + at \\
\sqrt{1584} = 4 + 9.8t \\
t = \frac{\sqrt{1584} - 4}{9.8} = 3.7 \text{ (to 2 s.f.)}
\]

The time \( P \) takes to reach the ground is 3.7 s.
7 a Take upwards as the positive direction.

\[ v = -10, \ a = -9.8, \ t = 5, \ u = ? \]

\[ v = u + at \]
\[ -10 = u - 9.8 \times 5 \]
\[ u = 9.8 \times 5 - 10 = 39 \]

The speed of projection of \( P \) is \( 39 \text{ m s}^{-1} \).

b \quad u = 39, \ v = 0, \ a = -9.8, \ s = ?

\[ v^2 = u^2 + 2as \]
\[ 0^2 = 39^2 - 2 \times 9.8 \times s \]
\[ s = \frac{1521}{2 \times 9.8} = 78 \text{ (to 2 s.f.)} \]

The greatest height above \( X \) attained by \( P \) during its motion is 78 m.

8 Take upwards as the positive direction.

\[ u = 21, \ t = 4.5, \ a = -9.8, \ s = ? \]

\[ s = ut + \frac{1}{2}at^2 = 21 \times 4.5 - 4.9 \times 4.5^2 = -4.7 \text{ (to 2 s.f.)} \]

The height above the ground from which the ball was thrown is 4.7 m.

9 a Take upwards as the positive direction.

\[ s = -3, \ u = 16, \ a = -9.8, \ t = ? \]

\[ s = ut + \frac{1}{2}at^2 \]
\[ -3 = 16t - 4.9t^2 \]

\[ 4.9t^2 - 16t - 3 = 0, \ \text{so using the quadratic formula,} \]

\[ t = \frac{-(16) \pm \sqrt{(-16)^2 - 4 \times 4.9 \times (-3)}}{2 \times 4.9} \]

\[ t = 3.4 \text{ (to 2 s.f.) as we may discount the negative answer.} \]

The time of flight of the stone is 3.4 s.
9 b  \( u = 16, \ v = 0, \ a = -9.8, \ s = ? \)

\[ v^2 = u^2 + 2as \]
\[ 0^2 = 16^2 - 2 \times 9.8 \times s \]
\[ s = \frac{16^2}{2 \times 4.9} = 13 \text{ (to 2 s.f.)} \]

The total distance travelled by the stone is \((2 \times 13 + 3) \text{ m} = 29 \text{ m.}\)

10 Take upwards as the positive direction.

\( u = 24.5, \ a = -9.8, \ s = 21, \ t = ? \)

\[ s = ut + \frac{1}{2}at^2 \]
\[ 21 = 24.5t - 4.9t^2 \]

\[ 4.9t^2 - 24.5t + 21 = 0 \]

Using the quadratic formula,

\[ t = \frac{-(-24.5) \pm \sqrt{(-24.5)^2 - 4 \times 4.9 \times 21}}{2 \times 4.9} \]

\[ = 1.1 \text{ or } 3.9 \]

The difference between these times is

\((3.9 - 1.1) \text{ s} = 2.8 \text{ s} \)

The total time for which the particle is 21 m or more above its point of projection is 2.8 s.

11 a Take upwards as the positive direction.

\( v = \frac{1}{2}u, \ a = -9.8, \ t = 2, \ u = ? \)

\[ v = u + at \]
\[ \frac{1}{2}u = u - 9.8 \times 2 \]
\[ \frac{2}{3}u = 19.6 \Rightarrow u = \frac{3}{2} \times 19.6 = 29.4 \]

\[ u = 29 \text{ (to 2 s.f.)} \]
11 b \( u = 29.4, \ s = 0, \ a = -9.8, \ t = ? \)

\[
s = ut + \frac{1}{2}at^2
\]

\[
0 = 29.4t - 4.9t^2 = t(29.4 - 4.9t), \ t \neq 0
\]

\[
t = \frac{29.4}{4.9} = 6
\]

The time from the instant that the particle leaves O to the instant that it returns to O is 6 s.

12 For A, take downwards as the positive direction, \( s_A = ut + \frac{1}{2}at^2 = 5t + 4.9t^2 \)

For B, take upwards as the positive direction, \( s_B = ut + \frac{1}{2}at^2 = 18t - 4.9t^2 \)

\[
s_A + s_B = 46
\]

\[
(5t + 4.9t^2) + (18t - 4.9t^2) = 46
\]

\[
23t = 46 \Rightarrow t = 2
\]

Substitute \( t = 2 \) into \( s_A = 5t + 4.9t^2 \)

\[
s_A = 5 \times 2 + 4.9 \times 2^2 = 29.6 = 30 \text{ (to 2 s.f.)}
\]

The distance of the point where A and B collide from the point where A was thrown is 30 m.

13 a Find the speed, \( u_1 \) say, immediately before the ball strikes the floor.

\[
u = 0, \ a = 9.8, \ s = 10, \ v = u_1
\]

\[
v^2 = u^2 + 2as
\]

\[
u_1^2 = 0^2 + 2 \times 9.8 \times 10 = 196
\]

\[
u_1 = \sqrt{196} = 14
\]

The speed of the first rebound, \( u_2 \) say, is given by

\[
u_2 = \frac{1}{2}u_1 = \frac{1}{2} \times 14 = 10.5
\]

Find the maximum height, \( h_1 \) say, reached after the first rebound.

\[
u = 10.5, \ v = 0, \ a = -9.8, \ s = h_1
\]

\[
v^2 = u^2 + 2as
\]

\[
0^2 = 10.5^2 - 2 \times 9.8 \times h_1
\]
13 a  \[ h_1 = \frac{10.5^2}{2 \times 9.8} = 5.6 \text{ (to 2 s.f.)} \]

The greatest height above the floor reached by the ball the first time it rebounds is 5.6 m.

b Immediately before the ball strikes the floor for the second time, its speed is again \( u_2 = 10.5 \) by symmetry. The speed of the second rebound, \( u_3 \) say, is given by

\[ u_3 = \frac{1}{4} u_2 = \frac{1}{4} \times 10.5 = 7.875 \]

Find the maximum height, \( h_2 \) say, reached after the second rebound.

\[ u = 7.875, \ v = 0, \ a = -9.8, \ s = h_2 \]

\[ v^2 = u^2 + 2as \]
\[ 0^2 = 7.875^2 - 2 \times 9.8 \times h_2 \]
\[ h_2 = \frac{7.875^2}{2 \times 9.8} = 3.2 \text{ (to 2 s.f.)} \]

The greatest height above the floor reached by the ball the second time it rebounds is 3.2 m.

Challenge

1 a Take upwards as the positive direction.

For \( P \), \( s = ut + \frac{1}{2}at^2 \) gives \( s_p = 12t - 4.9t^2 \)

For \( Q \), \( s = ut + \frac{1}{2}at^2 \)

\( Q \) has been moving for 1 less second than \( P \), so

\[ s_Q = 20(t - 1) - 4.9(t - 1)^2 \]

At the point of collision \( s_p = s_Q \)

\[ 12t - 4.9t^2 = 20(t - 1) - 4.9(t - 1)^2 \]
\[ = 20t - 20 - 4.9t^2 + 9.8t - 4.9 \]
\[ 24.9 = 17.8t \Rightarrow t = \frac{24.9}{17.8} = 1.4 \text{ (to 2 s.f.)} \]

The time between the instant when \( P \) is projected and the instant when \( P \) and \( Q \) collide is 1.4 s.
1 b Substitute $t$ into $s_p = 12t - 4.9t^2$ from part a

$$s_p = 12t - 4.9t^2 \approx 12 \times 1.4 - 4.9 \times 1.4^2 = 7.2 \text{ (to 2 s.f.)}$$

The distance of the point where $P$ and $Q$ collide from $O$ is 7.2 m.

2 Take downwards as positive.

For 1st stone: $u = 0, t = t_1, a = 9.8, s = h$

$$s = ut + \frac{1}{2}at^2$$

$$h = 0 \times t_1 + \frac{1}{2} \times 9.8 \times t_1^2 = 4.9t_1^2$$

For 2nd stone: $u = 25, t = t_1 - 2, a = 9.8, s = h$

$$s = ut + \frac{1}{2}at^2$$

$$h = 25(t_1 - 2) + \frac{1}{2}(9.8 \times (t_1 - 2)^2)$$

$$= 25t_1 - 50 + 4.9 \times (t_1^2 - 4t_1 + 4)$$

$$= 25t_1 - 50 + 4.9t_1^2 - 19.6t_1 + 19.6$$

$$= 4.9t_1^2 + 5.4t_1 - 30.4$$

Substituting for $h$ from information for first stone:

$$4.9t_1^2 = 4.9t_1^2 + 5.4t_1 - 30.4$$

$$30.4 = 5.4t_1$$

$$t_1 = \frac{30.4}{5.4} = 5.629$$

Putting this value into equation for first stone:

$$h = 4.9 \times 5.629^2 = 4.9 \times 31.69 = 155 \text{ (to 3 s.f.)}$$

The height of the building is 155 m.