Constant acceleration 9D

1 $a = 2.5,\ u = 3,\ s = 8,\ v = ?$

$$v^2 = u^2 + 2as = 3^2 + 2\times 2.5\times 8 = 9 + 40 = 49$$
$$v = \sqrt{49} = 7$$

The velocity of the particle as it passes through B is $7\,\text{m s}^{-1}$.

2 $u = 8,\ t = 6,\ s = 60,\ a = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$60 = 8\times 6 + \tfrac{1}{2}\times a\times 6^2 = 48 + 18a$$
$$a = \frac{60-48}{18} = \frac{2}{3}$$

The acceleration of the car is $0.667\,\text{m s}^{-2}$ (to 3 s.f.)

3 $u = 12,\ v = 0,\ s = 36,\ a = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 + 2\times a\times 36 = 144 + 72a$$
$$a = -\tfrac{144}{72} = -2$$

The deceleration is $2\ \text{m s}^{-2}$.

4 $u = 15,\ v = 20,\ s = 500,\ a = ?$ $\quad 54\,\text{km h}^{-1} = \dfrac{54\times 1000}{3600}\text{m s}^{-1} = 15\,\text{m s}^{-1}$

$$72\,\text{km h}^{-1} = \frac{72\times 1000}{3600}\text{m s}^{-1} = 20\,\text{m s}^{-1}$$

$$v^2 = u^2 + 2as$$
$$20^2 = 15^2 + 2\times a\times 500$$
$$400 = 225 + 1000a$$
$$a = \frac{400-225}{1000} = 0.175$$

The acceleration of the train is $0.175\,\text{m s}^{-2}$.

5 a $s = 48, \; u = 4, \; v = 16, \; a = ?$

$v^2 = u^2 + 2as$

$16^2 = 4^2 + 2 \times a \times 48$

$256 = 16 + 96a$

$a = \frac{256 - 16}{96} = 2.5$

The acceleration of the particle is $2.5\,\text{m s}^{-2}$.

b $u = 4, \; v = 16, \; a = 2.5, \; t = ?$

$v = u + at$

$16 = 4 + 2.5t$

$t = \frac{16 - 4}{2.5} = 4.8$

The time taken to move from A to B is 4.8 s.

6 a $a = 3, s = 38, t = 4, u = ?$

$s = ut + \frac{1}{2}at^2$

$38 = 4u + \frac{1}{2} \times 3 \times 4^2 = 4u + 24$

$u = \frac{38 - 24}{4} = 3.5$

The initial velocity of the particle is $3.5\,\text{m s}^{-1}$.

b $a = 3, t = 4, u = 3.5, v = ?$

$v = u + at = 3.5 + 3 \times 4 = 15.5$

The final velocity of the particle is $15.5\,\text{m s}^{-1}$.

7 a $u = 18, \; v = 0, \; a = -3, \; s = ?$

$v^2 = u^2 + 2as$

$0^2 = 18^2 + 2 \times (-3) \times s = 324 - 6s$

$s = \frac{324}{6} = 54$

The distance travelled as the car decelerates is 54 m.

7 b $u = 18,\ v = 0,\ a = -3,\ t = ?$

$$v = u + at$$
$$0 = 18 - 3t$$
$$t = \tfrac{18}{3} = 6$$

The time taken for the car to decelerate is 6 s.

8 a $u = 12,\ v = 0,\ a = -0.8,\ s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 + 2 \times (-0.8) \times s = 144 - 1.6s$$
$$s = \tfrac{144}{1.6} = 90$$

The distance moved by the stone is 90 m.

b Half the distance in **a** is 45 m.

$u = 12,\ a = -0.8,\ s = 45,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 12^2 + 2 \times (-0.8) \times 45 = 144 - 72 = 72$$
$$v = \sqrt{72} = 8.49 \text{ (to 3 s.f.)}$$

The speed of the stone is $8.49\,\text{m}\,\text{s}^{-1}$.

9 a $a = 2.5,\ u = 8,\ s = 40,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$40 = 8t + 1.25t^2$$
$$0 = 1.25t^2 + 8t - 40$$
$$t = \frac{-(8) \pm \sqrt{(8)^2 - 4 \times (1.25) \times (-40)}}{2 \times (1.25)}$$
$$t = \frac{-8 + \sqrt{264}}{2.5} = 3.30 \text{ (to 3 s.f.)}$$

The time taken for the particle to move from O to A is 3.30 s.

b $a = 2.5,\ u = 8,\ s = 40,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 8^2 + 2 \times 2.5 \times 40 = 264$$
$$v = \sqrt{264} = 16.2 \text{ (to 3 s.f.)}$$

The speed of the particle at A is $16.2\,\text{m}\,\text{s}^{-1}$.

10 a $a = -2,\ s = 32,\ u = 12,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$32 = 12t - t^2$$
$$t^2 - 12t + 32 = (t-4)(t-8) = 0$$

So $t = 4$ or $t = 8$.

b When $t = 4$,

$$v = u + at = 12 - 2\times 4 = 4$$

The velocity is $4\,\text{m s}^{-1}$ in the direction $\overrightarrow{AB}$.

When $t = 8$,

$$v = u + at = 12 - 2\times 8 = -4$$

The velocity is $4\,\text{m s}^{-1}$ in the direction $\overrightarrow{BA}$.

11 a $a = -5,\ u = 12,\ s = 8,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$8 = 12t - 2.5t^2$$
$$2.5t^2 - 12t + 8 = 0$$
$$5t^2 - 24t + 16 = (5t-4)(t-4) = 0$$

So $t = 0.8$ or $t = 4$.

b $a = -5,\ u = 12,\ s = -8,\ v = ?$

$$\begin{aligned} v^2 &= u^2 + 2as \\ &= 12^2 + 2\times(-5)\times(-8) \\ &= 144 + 80 = 224 \\ v &= \sqrt{224} = -15.0 \text{ (to 3 s.f.)} \end{aligned}$$

The velocity at $x = -8$ is $-15.0\,\text{m s}^{-1}$.

12 a $a = -4,\ u = 14,\ s = 22.5,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$22.5 = 14t - 2t^2$$
$$2t^2 - 14t + 22.5 = 0$$
$$4t^2 - 28t + 45 = (2t-5)(2t-9) = 0$$

The difference between the times is $(4.5 - 2.5)\text{ s} = 2\text{ s}$.

12 b The maximum distance is reached when P reverses direction.
$a = -4,\ u = 14,\ v = 0,\ t = ?$

$v = u + at$

$0 = 14 - 4t \Rightarrow t = \frac{14}{4} = 3.5$

Find the displacement when $t = 3.5$.

$s = ut + \frac{1}{2}at^2$

$= 14 \times 3.5 - 2 \times 3.5^2 = 24.5$

Between $t = 2.5$ and $t = 4.5$ the particle moves back and forward.

Hence total distance travelled $= 2 \times (24.5 - 22.5)$ m $= 4$ m.

13 a From B to C, $u = 14,\ v = 20,\ s = 300,\ a = ?$

$v^2 = u^2 + 2as$

$20^2 = 14^2 + 2 \times a \times 300$

$a = \dfrac{20^2 - 14^2}{600} = 0.34$

The acceleration of the car is 0.34 m s^{-2}.

b From A to C, $v = 20,\ s = 400,\ a = 0.34,\ u = ?$

$v^2 = u^2 + 2as$

$20^2 = u^2 + 2 \times 0.34 \times 400 = u^2 + 272$

$u^2 = 400 - 272 = 128$

$u = \pm\sqrt{128} = \pm 8\sqrt{2}$

Assuming the car is not in reverse at A, $u = +8\sqrt{2}$

$v = u + at$

$20 = 8\sqrt{2} + 0.34t$

$t = \dfrac{20 - 8\sqrt{2}}{0.34} = 25.5$ (to 3 s.f.)

The time taken for the car to travel from A to C is 25.5 s.

14 a For P, $a = 2$, $u = 4$

$$s = ut + \tfrac{1}{2}at^2$$
$$= 4t + \tfrac{1}{2} \times 2t^2 = 4t + t^2$$

The displacement of P is $(4t + t^2)$ m.

For Q, $a = 3.6$, $u = 3$

Q has been moving for $(t - 1)$ seconds since passing through A, so

$$s = u(t-1) + \tfrac{1}{2}a(t-1)^2$$
$$= 3(t-1) + 1.8(t-1)^2 = 1.8t^2 - 0.6t - 1.2$$

The displacement of Q is $(1.8t^2 - 0.6t - 1.2)$ m.

b P and Q meet when $s_P = s_Q$, so, from **a**:

$$4t + t^2 = 1.8t^2 - 0.6t - 1.2$$
$$0.8t^2 - 4.6t - 1.2 = 0$$

Divide throughout by 0.2:

$$4t^2 - 23t - 6 = 0$$
$$(t-6)(4t+1) = 0$$

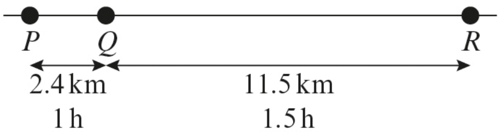
Rejecting a negative solution for time, $t = 6$.

c Substitute $t = 6$ into the equation for one of the displacements (here P):

$$s = 4t + t^2 = 4 \times 6 + 6^2 = 60$$

The distance of A from the point where the particles meet is 60 m.

15



a Let the velocity as the competitor passes point Q be v_Q

For PQ, $s = 2.4$, $t = 1$, $v = v_Q$

$$s = vt - \tfrac{1}{2}at^2$$
$$2.4 = v_Q \times 1 - \tfrac{1}{2}(a \times 1^2) = v_Q - \tfrac{1}{2}a$$
$$v_Q = 2.4 + 0.5a$$

For QR, $s = 11.5$, $t = 1.5$, $u = v_Q$

$$s = ut + \tfrac{1}{2}at^2$$
$$11.5 = v_Q \times 1.5 + \tfrac{1}{2}a \times 1.5^2 = 1.5v_Q + 1.125a$$

Substituting for v_Q:

$$11.5 = 1.5(2.4 + 0.5a) + 1.125a$$
$$= 3.6 + 0.75a + 1.125a$$
$$11.5 - 3.6 = (0.75 + 1.125)a$$
$$a = \frac{11.5 - 3.6}{0.75 + 1.125} = \frac{7.9}{1.875} = 4.21 \text{ (to 3 s.f.)}$$

The acceleration is 4.21 km h^{-2}.

$$4.21 \text{ km h}^{-2} = \frac{4.21 \times 1000}{3600 \times 3600} \text{ m s}^{-2} = 3.25 \times 10^{-4} \text{ m s}^{-2} \text{ (to 3 s.f.)}$$

So her acceleration is 3.25 x 10^{-4} m s^{-2}.

b For PQ, $s = 2.4$, $t = 1$, $a = 4.21$, $u = ?$, using exact figures

$$s = ut + \tfrac{1}{2}at^2$$
$$2.4 = u \times 1 + \tfrac{1}{2} \times \frac{7.9}{1.875} \times 1^2$$
$$u = 0.293 \text{ (to 3 s.f.)}$$

$$0.293 \text{ km h}^{-1} = \frac{0.293 \times 1000}{3600} \text{ m s}^{-1} = 0.0815 \text{ m s}^{-1} \text{ (to 3 s.f.)}$$