Constant acceleration 9D

1 \[ a = 2.5, \ u = 3, \ s = 8, \ v = ? \]

\[ v^2 = u^2 + 2as = 3^2 + 2 \times 2.5 \times 8 = 9 + 40 = 49 \]
\[ v = \sqrt{49} = 7 \]

The velocity of the particle as it passes through \( B \) is 7 m s\(^{-1}\).

2 \[ u = 8, \ t = 6, \ s = 60, \ a = ? \]

\[ s = ut + \frac{1}{2}at^2 \]
\[ 60 = 8 \times 6 + \frac{1}{2} \times a \times 6^2 = 48 + 18a \]
\[ a = \frac{60 - 48}{18} = \frac{2}{3} \]

The acceleration of the car is 0.667 m s\(^{-2}\) (to 3 s.f.)

3 \[ u = 12, \ v = 0, \ s = 36, \ a = ? \]

\[ v^2 = u^2 + 2as \]
\[ 0^2 = 12^2 + 2 \times a \times 36 = 144 + 72a \]
\[ a = -\frac{144}{72} = -2 \]

The deceleration is 2 m s\(^{-2}\).

4 \[ u = 15, \ v = 20, \ s = 500, \ a = ? \]

54 km h\(^{-1}\) = \[ \frac{54 \times 1000}{3600} \] ms\(^{-1}\) = 15 m s\(^{-1}\)

72 km h\(^{-1}\) = \[ \frac{72 \times 1000}{3600} \] ms\(^{-1}\) = 20 m s\(^{-1}\)

\[ v^2 = u^2 + 2as \]
\[ 20^2 = 15^2 + 2 \times a \times 500 \]
\[ 400 = 225 + 1000a \]
\[ a = \frac{400 - 225}{1000} = 0.175 \]

The acceleration of the train is 0.175 m s\(^{-2}\).

5a \[ s = 48, \ u = 4, \ v = 16, \ a = ? \]

\[ v^2 = u^2 + 2as \]
\[ 16^2 = 4^2 + 2 \times a \times 48 \]
5 a \[256 = 16 + 96a\]
\[a = \frac{256 - 16}{96} = 2.5\]

The acceleration of the particle is 2.5 m/s\(^2\).

b \[u = 4, \ v = 16, \ a = 2.5, \ t = ?\]

\[v = u + at\]
\[16 = 4 + 2.5t\]
\[t = \frac{16 - 4}{2.5} = 4.8\]

The time taken to move from A to B is 4.8 s.

6 a \[a = 3, \ s = 38, \ t = 4, \ u = ?\]

\[s = ut + \frac{1}{2}at^2\]
\[38 = 4u + \frac{1}{2} \times 3 \times 4^2 = 4u + 24\]
\[u = \frac{38 - 24}{4} = 3.5\]

The initial velocity of the particle is 3.5 m/s\(^{-1}\).

b \[a = 3, \ t = 4, \ u = 3.5, \ v = ?\]

\[v = u + at = 3.5 + 3 \times 4 = 15.5\]

The final velocity of the particle is 15.5 m/s\(^{-1}\).

7 a \[u = 18, \ v = 0, \ a = -3, \ s = ?\]

\[v^2 = u^2 + 2as\]
\[0^2 = 18^2 + 2 \times (-3) \times s = 324 - 6s\]
\[s = \frac{324}{6} = 54\]

The distance travelled as the car decelerates is 54 m.

b \[u = 18, \ v = 0, \ a = -3, \ t = ?\]

\[v = u + at\]
\[0 = 18 - 3t\]
\[t = \frac{18}{3} = 6\]

The time taken for the car to decelerate is 6 s.
8 a \( u = 12, \ v = 0, \ a = -0.8, \ s = ? \)

\[
v^2 = u^2 + 2as \\
0^2 = 12^2 + 2 \times (-0.8) \times s = 144 - 1.6s \\
s = \frac{144}{1.6} = 90
\]

The distance moved by the stone is 90 m.

b Half the distance in a is 45 m.

\( u = 12, \ a = -0.8, \ s = 45, \ v = ? \)

\[
v^2 = u^2 + 2as \\
= 12^2 + 2 \times (-0.8) \times 45 = 144 - 72 = 72 \\
v = \sqrt{72} = 8.49 \text{ (to 3 s.f.)}
\]

The speed of the stone is 8.49 ms\(^{-1}\).

9 a \( a = 2.5, \ u = 8, \ s = 40, \ t = ? \)

\[
s = ut + \frac{1}{2}at^2 \\
40 = 8t + 1.25t^2 \\
0 = 1.25t^2 + 8t - 40 \\
t = \frac{-8 \pm \sqrt{(8)^2 - 4 \times (1.25) \times (-40)}}{2 \times (1.25)} \\
t = \frac{-8 + \sqrt{264}}{2.5} = 3.30 \text{ (to 3 s.f.)}
\]

The time taken for the particle to move from O to A is 3.30 s.

b \( a = 2.5, \ u = 8, \ s = 40, \ v = ? \)

\[
v^2 = u^2 + 2as \\
= 8^2 + 2 \times 2.5 \times 40 = 264 \\
v = \sqrt{264} = 16.2 \text{ (to 3 s.f.)}
\]

The speed of the particle at A is 16.2 ms\(^{-1}\).

10 a \( a = -2, \ s = 32, \ u = 12, \ t = ? \)

\[
s = ut + \frac{1}{2}at^2 \\
32 = 12t - t^2 \\
t^2 - 12t + 32 = (t - 4)(t - 8) = 0 \\
\]

So \( t = 4 \) or \( t = 8 \).
10 b When $t = 4$,

$$v = u + at = 12 - 2 \times 4 = 4$$

The velocity is $4 \text{ ms}^{-1}$ in the direction $\overrightarrow{AB}$.

When $t = 8$,

$$v = u + at = 12 - 2 \times 8 = -4$$

The velocity is $4 \text{ ms}^{-1}$ in the direction $\overrightarrow{BA}$.

11 a $a = -5, \ u = 12, \ s = 8, \ t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$8 = 12t - 2.5t^2$$

$$2.5t^2 - 12t + 8 = 0$$

$$5t^2 - 24t + 16 = (5t - 4)(t - 4) = 0$$

So $t = 0.8$ or $t = 4$.

b $a = -5, \ u = 12, \ s = -8, \ v = ?$

$$v^2 = u^2 + 2as$$

$$= 12^2 + 2 \times (-5) \times (-8)$$

$$= 144 + 80 = 224$$

$$v = \sqrt{224} = 15.0 \text{ (to } 3 \text{ s.f.)}$$

The velocity at $x = -8$ is $15.0 \text{ m s}^{-1}$.

12 a $a = -4, \ u = 14, \ s = 22.5, \ t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$22.5 = 14t - 2t^2$$

$$2t^2 - 14t + 22.5 = 0$$

$$4t^2 - 28t + 45 = (2t - 5)(2t - 9) = 0$$

The difference between the times is $(4.5 - 2.5) \text{ s} = 2 \text{ s}$.

b The maximum distance is reached when $P$ reverses direction.

$a = -4, \ u = 14, \ v = 0, \ t = ?$

$$v = u + at$$

$$0 = 14 - 4t \Rightarrow t = \frac{14}{4} = 3.5$$
12 b  Find the displacement when $t = 3.5$.

\[ s = ut + \frac{1}{2}at^2 \]

\[ = 14 \times 3.5 - 2 \times 3.5^2 = 24.5 \]

Between $t = 2.5$ and $t = 4.5$ the particle moves back and forward.

Hence total distance travelled $= 2 \times (24.5 - 22.5) \text{ m} = 4 \text{ m}$.

13 a  From $B$ to $C$, $u = 14$, $v = 20$, $s = 300$, $a =$?

\[ v^2 = u^2 + 2as \]

\[ 20^2 = 14^2 + 2 \times a \times 300 \]

\[ a = \frac{20^2 - 14^2}{600} = 0.34 \]

The acceleration of the car is $0.34 \text{ m s}^{-2}$.

b From $A$ to $C$, $v = 20$, $s = 400$, $a = 0.34$, $u =$?

\[ v^2 = u^2 + 2as \]

\[ 20^2 = u^2 + 2 \times 0.34 \times 400 = u^2 + 272 \]

\[ u^2 = 400 - 272 = 128 \]

\[ u = \pm \sqrt{128} = \pm 8\sqrt{2} \]

Assuming the car is not in reverse at $A$, $u = +8\sqrt{2}$

\[ v = u + at \]

\[ 20 = 8\sqrt{2} + 0.34t \]

\[ t = \frac{20 - 8\sqrt{2}}{0.34} = 25.5 \text{ (to 3 s.f.)} \]

The time taken for the car to travel from $A$ to $C$ is $25.5 \text{ s}$.

14 a  For $P$, $a = 2$, $u = 4$

\[ s = ut + \frac{1}{2}at^2 \]

\[ = 4t + \frac{1}{2} \times 2t^2 = 4t + t^2 \]

The displacement of $P$ is $(4t + t^2) \text{ m}$.

For $Q$, $a = 3.6$, $u = 3$
14 a  $Q$ has been moving for $(t - 1)$ seconds since passing through $A$, so

$$s = ut - \frac{1}{2} at^2$$

$$3(t - 1) + 1.8(t - 1)^2 = 1.8t^2 - 0.6t - 1.2$$

The displacement of $Q$ is $(1.8t^2 - 0.6t - 1.2)$ m.

b  $P$ and $Q$ meet when $s_P = s_Q$, so, from a:

$$4t + t^2 = 1.8t^2 - 0.6t - 1.2$$

$$0.8t^2 - 4.6t - 1.2 = 0$$

Divide throughout by 0.2:

$$4t^2 - 23t - 6 = 0$$

$$(t - 6)(4t + 1) = 0$$

Rejecting a negative solution for time, $t = 6$.

c  Substitute $t = 6$ into the equation for one of the displacements (here $P$):

$$s = 4t + t^2 = 4 \times 6 + 6^2 = 60$$

The distance of $A$ from the point where the particles meet is 60 m.

15

- **a** Let the velocity as the competitor passes point $Q$ be $v_Q$

  For $PQ$, $s = 2.4$, $t = 1$, $v = v_Q$

  $$s = vt - \frac{1}{2} at^2$$

  $$2.4 = v_Q \times 1 - \frac{1}{2} (a \times 1^2) = v_Q - \frac{1}{2} a$$

  $$v_Q = 2.4 + 0.5a$$

  For $QR$, $s = 11.5$, $t = 1.5$, $u = v_Q$

  $$s = ut + \frac{1}{2} at^2$$

  $$11.5 = v_Q \times 1.5 + \frac{1}{2} a \times 1.5^2 = 1.5v_Q + 1.125a$$
15a Substituting for $v_Q$:

\[ 11.5 = 1.5(2.4 + 0.5a) + 1.125a \]
\[ = 3.6 + 0.75a + 1.125a \]
\[ 11.5 - 3.6 = (0.75 + 1.125)a \]
\[ a = \frac{11.5 - 3.6}{0.75 + 1.125} = \frac{7.9}{1.875} = 4.21 \text{ (to 3 s.f.)} \]

The acceleration is $4.21 \text{ km h}^{-2}$.

\[ 4.21 \text{ km h}^{-2} = \frac{4.21 \times 1000}{3600 \times 3600} \text{ m s}^{-2} = 3.25 \times 10^{-4} \text{ m s}^{-2} \text{ (to 3 s.f.)} \]

So her acceleration is $3.25 \times 10^{-4} \text{ m s}^{-2}$.

b For $PQ$, $s = 2.4$, $t = 1$, $a = 4.21$, $u =$ ?, using exact figures

\[ s = ut + \frac{1}{2}at^2 \]
\[ 2.4 = u \times 1 + \frac{1}{2} \times \frac{7.9}{1.875} \times 1^2 \]
\[ u = 0.293 \text{ (to 3 s.f.)} \]

\[ 0.293 \text{ km h}^{-1} = \frac{0.293 \times 1000}{3600} \text{ m s}^{-1} = 0.0815 \text{ m s}^{-1} \text{ (to 3 s.f.)} \]