Modelling in mechanics 8D

1  a  2.1 m s\(^{-1}\)
   b  500 m
   c  \(-1.8\) m s\(^{-1}\)
   d  \(-2.7\) m s\(^{-1}\)
   e  \(-750\) m
   f  2.5 m s\(^{-1}\)

2  a  speed \(|v| = \sqrt{12^2 + 10^2} = \sqrt{244}\)
    The speed of the car is 15.6 m s\(^{-1}\) (to 3 s.f.)

   b  Let the acute angle made with \(\textbf{i}\) be \(\theta\), then

   \[\tan \theta = \frac{10}{12} = 0.8333\] so \(\theta = 39.8^\circ\) (to 3 s.f.)
    The direction of motion of the car is 39.8° from the \(\textbf{i}\) vector.

3  a  \(|\mathbf{a}| = \sqrt{3^2 + 4^2} = \sqrt{25}\)
    The magnitude of the acceleration is 5 m s\(^{-2}\).

   b  Let the acute angle made with \(\mathbf{j}\) be \(\theta\)

   \[\tan \theta = \frac{3}{4} = 0.75\] so \(\theta = 36.9^\circ\) (to 3 s.f.)
    Angle required = 180° - \(\theta = 180^\circ - 36.9^\circ = 143.1^\circ\)
    The direction of the acceleration is 143° from the \(\mathbf{j}\) vector.
4  a  \[ \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} \]
\[
\overrightarrow{AC} = \begin{pmatrix} 10 \\ 3 \end{pmatrix} + \begin{pmatrix} -7 \\ 12 \end{pmatrix} = \begin{pmatrix} 3 \\ 15 \end{pmatrix}
\]
\[|\overrightarrow{AC}| = \sqrt{3^2 + 15^2} = \sqrt{234}\]

The magnitude of the displacement is 15.3 m (to 3 s.f.)

b  \[ |\overrightarrow{AB}| = \sqrt{10^2 + 3^2} = \sqrt{109} \]
\[|\overrightarrow{BC}| = \sqrt{7^2 + 12^2} = \sqrt{193}\]
\[|\overrightarrow{AB}| + |\overrightarrow{BC}| = \sqrt{109} + \sqrt{193} = 24.3\]

The girl cycles 24.3 km (to 3 s.f.)

c  Let the acute angle made with \(\mathbf{i}\) be \(\theta\)

\[\overrightarrow{AC} = 3\mathbf{i} + 15\mathbf{j}\]

\[\tan \theta = \frac{15}{3} = 5 \quad \text{so} \quad \theta = 78.7^\circ \text{ (to 3 s.f.)}\]

The direction of motion of the car is 78.7° from the \(\mathbf{i}\) vector.