Hypothesis testing, Mixed Exercise 7

1 $X \sim \text{B}(10, 0.20)$

$\text{H}_0 : p = 0.20 \qquad \text{H}_1 : p > 0.20$

$$\begin{aligned} \text{P}(X \geq 3) &= 1 - \text{P}(X \leq 2) \\ &= 1 - 0.6778 \\ &= 0.3222 > 0.05 \end{aligned}$$

There is insufficient evidence to reject H_0
There is no evidence that the trains are late more often.

2 $X \sim \text{B}(5, 0.5)$

$\text{H}_0 : p = 0.50 \qquad \text{H}_1 : p > 0.50$

$$\begin{aligned} \text{P}(X \geq 4) &= 1 - \text{P}(X \leq 3) \\ &= 1 - 0.8125 \\ &= 0.1875 > 0.05 \end{aligned}$$

There is insufficient evidence to reject H_0
There is insufficient evidence that the company's claims are true.

3 a Fixed number; independent trials; two outcomes (pass or fail); p constant for each car.

b $X \sim \text{B}(5, 0.30)$

$\text{P(all pass)} = 0.70^5 = 0.16807$

c $X \sim \text{B}(10, 0.30)$

$\text{H}_0 : p = 0.30 \qquad \text{H}_1 : p < 0.30$

$\text{P}(X \leq 2) = 0.3828 > 0.05$

There is insufficient evidence to reject H_0.
There is no evidence that the garage fails fewer than the national average.

4 a $X \sim B(50, 0.1)$

$H_0 : p = 0.10 \qquad H_1 : p \neq 0.10$

$P(X \leq 1) = 0.0338$ (closer to 0.025)
$P(X = 0) = 0.0052$
critical value = 1

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9421 = 0.0579$
$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9755 = 0.0245$ (closer to 0.025)

critical value = 10

Critical region $X \leq 1$ and $X \geq 10$

b Actual significance level = 0.0338 + 0.0245 = 0.0583 = 5.83%

c $X \sim B(20, 0.1)$

$H_0 : p = 0.1 \qquad H_1 : p > 0.1$

$$P(X \geq 4) = 1 - P(X \leq 3)$$
$$= 1 - 0.8670$$
$$= 0.133 > 0.1$$

p-value $= 0.133$

Accept H_0. There is no evidence that the proportion of defective articles has increased.

5 $X \sim B(20, 0.5)$
$H_0 : p = 0.50 \qquad H_1 : p \neq 0.50$

8 used Oriels powder.

$P(X \leq 8) = 0.2517 > 0.025$

There is insufficient evidence to reject H_0.
There is no evidence that the claim is wrong.

6 $X \sim B(50, 0.2)$

a $P(X \leq 4) = 0.0185$ (closer to 0.025)
$P(X \leq 5) = 0.0480$

$c_1 = 4$

$P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.9692 = 0.0308$ (closer to 0.025)
$P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9856 = 0.0144$

$c_2 = 16$

Critical region $X \leq 4$ and $X \geq 16$

b Actual significance level = 0.0185 + 0.0308 = 0.0493 = 4.93%

c This is not in the critical region. Therefore, there is insufficient evidence to reject H_0. There is no evidence to suggest that the proportion of people buying that certain make of computer differs from 0.2.

7 a i A hypothesis is a statement made about the value of a population parameter. A hypothesis test uses a sample or an experiment to determine whether or not to reject the hypothesis.

ii The critical value is the first value to fall inside of the critical region.

iii The acceptance region is the region where we accept the null hypothesis.

b $H_0: p = 0.2$ $H_1: p \neq 0.2$

If H_0 is true $X \sim B(20, 0.2)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.05$ and $P(X \geq c_2) \leq 0.05$

For the lower tail:

$P(X = 0) = 0.0115 < 0.05$
$P(X \leq 1) = 0.0692 > 0.05$

So $c_1 = 0$

For the upper tail:

$P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.9133 = 0.0978 > 0.05$
$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9679 = 0.0321 < 0.05$

So $c_2 = 8$

So the critical region is $X = 0$ and $X \geq 8$

c Actual significance level = 0.0115 + 0.0321 = 0.0436 = 4.36%

7 d As 7 does not lie in the critical region, H_0 is not rejected. Therefore, the proportion of times that Johan is late for school has not changed.

8 X is the number of days with zero or a trace of rain.

$X \sim B(30, 0.5)$

$H_0: p = 0.5$ $H_1: p > 0.5$

$P(X \geq 19) = 1 - P(X \leq 18) = 1 - 0.8998 = 0.1002 > 0.05$
$P(X \geq 20) = 1 - P(X \leq 19) = 1 - 0.9506 = 0.0494 < 0.05$

The critical region is $X \geq 20$

21 lies in the critical region, so we can reject the null hypothesis. There is evidence that the likelihood of a rain-free day in 2015 has increased.

9 a $H_0: p = 0.35$ $H_1: p \neq 0.35$

If H_0 is true $X \sim B(30, 0.35)$

Let c_1 and c_2 be the two critical values so $P(X \leq c_1) \leq 0.025$ and $P(X \geq c_2) \leq 0.025$

For the lower tail:

$P(X \leq 5) = 0.0233 < 0.025$
$P(X \leq 6) = 0.0586 > 0.025$

So $c_1 = 5$

For the upper tail:

$P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.9699 = 0.0301$, $0.0301 - 0.025 = 0.0051$
$P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9876 = 0.0124$, $0.025 - 0.0124 = 0.0126$

So $c_2 = 16$

So the critical region is $X \leq 5$ and $X \geq 16$

b Actual significance test is $0.0233 + 0.0301 = 0.0534 = 5.34\%$

c $X = 4$ lies in the critical region so there is enough evidence to reject H_0.

10 a $X \sim B(20, 0.85)$

b $P(X = 16) = \binom{20}{16} 0.85^{16} 0.15^{4} = 0.18$

10 c X is the number of patients that recover
p is the probability that a patient recovers

$H_0 : p = 0.85 \qquad H_1 : p < 0.85$

If H_0 is true $X \sim B(30, 0.85)$

Expected value would be $30 \times 0.85 = 25.5$
The observed value, 20, is less than this so consider $P(X \leq 20)$

p-value = $P(X \leq 20) = 0.009657... < 0.05$ (one-tailed)

There is evidence to reject H_0. The percentage of patients who recover after treatment with the new ointment is lower than 85%.

Large Data Set

1 a X is the number of days with a recorded daily mean temperature greater than 15 °C.

$X \sim (10, 0.163)$

H_0: $p = 0.163 \qquad H_1$: $p > 0.163$

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.935 = 0.065 > 0.05$
$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9959 = 0.0141 < 0.05$

The critical region is $X \geq 5$

b A random sample of temperatures, 15.6, 17.3, 12.5, 14.1, 11.9, 14.5, 13.0, 9.1, 14.1, 10.0, 9.3 has 2 days with a daily mean temperature greater than 15 °C.

c 2 does not lie in the critical region so H_0 is not rejected. Therefore, there is no reason to suggest that $p \neq 0.163$.

2 X is the number of days with daily mean temperature greater than 25 °C.

$X \sim (10, 0.23)$

H_0: $p = 0.23 \qquad H_1$: $p > 0.23$

$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9431 = 0.0569 > 0.05$
$P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9870 = 0.0130 < 0.05$

The critical region is $X \geq 6$

A random sample of temperatures 17.5, 18.9, 25.9, 27.7, 30.4, 26.6, 27.4, 27.0, 19.4 and 13.9 has 6 days with a mean temperature greater than 25 °C.

6 does lie in the critical region so H_0 is rejected. Therefore, there is reason to suggest that $p > 0.23$.