Hypothesis testing 7C

1 Distribution, B(10, 0.25)

$H_0 : p = 0.25 \qquad H_1 : p > 0.25$

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - 0.9219 \\ &= 0.0781 > 0.05 \end{aligned}$$

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.25$

2 Distribution, B(10, 0.40)

$H_0 : p = 0.40 \qquad H_1 : p < 0.40$

$P(X \leq 1) = 0.0464 < 0.05$

There is sufficient evidence to reject H_0 so $p < 0.04$

3 Distribution, B(20, 0.30)

$H_0 : p = 0.30 \qquad H_1 : p > 0.30$

$$\begin{aligned} P(X \geq 10) &= 1 - P(X \leq 9) \\ &= 1 - 0.9520 \\ &= 0.0480 < 0.05 \end{aligned}$$

There is sufficient evidence to reject H_0 so $p > 0.3$

4 Distribution, B(20, 0.45)

$H_0 : p = 0.45 \qquad H_1 : p < 0.45$

$P(X \leq 3) = 0.0049 < 0.01$

There is sufficient evidence to reject H_0 so $p < 0.45$

5 Distribution, B(20, 0.28)

$H_0 : p = 0.28 \qquad H_1 : p < 0.28$

p-value = $P(X \leq 2) = 0.0526 > 0.05$

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.28$

6 Distribution, B(8, 0.32)

$H_0 : p = 0.32 \qquad H_1 : p < 0.32$

$$\begin{aligned} P(X \geq 7) &= 1 - P(X \leq 6) \\ &= 1 - 0.9980 \\ &= 0.002 < 0.05 \end{aligned}$$

There is sufficient evidence to reject H_0 so $p < 0.32$

7 Distribution, B(12, $\frac{1}{6}$)

$H_0 : p = \frac{1}{6} \qquad H_1 : p < \frac{1}{6}$

$P(X \leq 1) = 0.3813 > 0.05$

There is insufficient evidence to reject H_0 so there is no evidence that the probability of a 6 on this dice is less than $\frac{1}{6}$

8 a Distribution, B(n, 0.68)

Reasons:
- Fixed number of trials.
- utcomes of the trials are independent.
- There are two outcomes, success and failure.
- The probabil ty of success is constant.

b Distribution, B(10, 0.68)

$H_0 : p = 0.68 \qquad H_1 : p < 0.68$

$P(X \leq 3) = 0.0155 < 0.05$

There is sufficient evidence to reject H_0 so $p < 0.68$. The treatment is not as effective as claimed.

9 a X is the number of seeds in the trial for which the germination method was successful.
p is the probability of success for each seed.

$X \sim B(20, p)$

H_0: $p = 0.4 \qquad H_1$: $p > 0.4$

$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9435 = 0.0565 \geq 0.05$
$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9790 = 0.021 \leq 0.05$

The critical region is $X \geq 13$

9 b 14 lies within the critical region, so we can reject the null hypothesis. There is evidence that the new technique has improved the number of plants that germinate.

10 a The test statistic is the number of people who support the candidate.

$H_0: p = 0.35$ $H_1: p > 0.35$

b $X \sim B(50, 0.35)$

$P(X \geq 23) = 1 - P(X \leq 22) = 1 - 0.9290 = 0.071 > 0.05$
$P(X \geq 24) = 1 - P(X \leq 23) = 1 - 0.9604 = 0.0396 < 0.05$

The critical region is $X \geq 24$

c 28 lies in the critical region, so we can reject the null hypothesis. There is evidence that the candidate's level of popularity has increased.