Hypothesis testing 7B

1  a  The critical value is the first value to fall inside of the critical region.

b  A critical region is a region of the probability distribution which, if the test statistic falls within it, would cause you to reject the null hypothesis.

c  The acceptance region is the area in which we accept the null hypothesis.

2  B (10, 0.2)

\[ P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.8791 = 0.1209 > 0.05 \]
\[ P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9672 = 0.0328 < 0.05 \]

The critical value is \( x = 5 \) and the critical region is \( X \geq 5 \) since \( P(X \geq 5) = 0.0328 < 0.05 \)

3  B (20, 0.15)

\[ P(X \leq 1) = 0.1756 > 0.05 \]
\[ P(X = 0) = 0.0388 < 0.05 \]

The critical value is \( x = 0 \) and the critical region is \( X = 0 \)

4  a  B (20, 0.4)

\[ P(X \leq 4) = 0.0510 > 0.025 \]
\[ P(X \leq 3) = 0.0160 < 0.025 \]

The critical value is \( x = 3 \)

\[ P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9790 = 0.0210 < 0.025 \]
\[ P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9435 = 0.0565 > 0.025 \]

The critical value is \( x = 13 \)
The critical region is \( X \geq 13 \) and \( X \leq 3 \)

b  The actual significance level is \( 0.021 + 0.016 = 0.037 = 3.7\% \)
5. B (20, 0.18)

\[ B(X = 0) = 0.0189 < 0.05 \]
\[ B(X \leq 1) = 0.1018 > 0.05 \]

The critical value is \( x = 0 \)
The critical region is \( X = 0 \)

6. a. B (10, 0.63)

\[ P(X = 10) = 0.63^{10} = 0.0098 < 0.05 \]
\[ P(X \geq 9) = 0.0098 + 10(0.63)^9(0.37) = 0.0675 > 0.05 \]

The critical value is \( x = 10 \)
The critical region is \( X = 10 \)

b. The actual significance level is 0.0098 = 0.98%

7. a. The test statistic is the number of components in the sample that fail.

b. \( H_0: p = 0.3 \)
\( H_1: p < 0.3 \)

c. Assume \( H_0 \) is true then \( X \sim B(20, 0.3) \)

\[ P(X \leq 2) = 0.0355 \text{ (closer to 0.05)} \]
\[ P(X \leq 3) = 0.1071 \]

The critical region is \( X \leq 2 \)

d. 0.0355 = 3.55%

8. a. The test statistic is the number of seedlings that survive.

\[ H_0: p = \frac{1}{3}, \]
\[ H_1: p > \frac{1}{3} \]

b. Assume \( H_0 \) is true then \( X \sim B(36, \frac{1}{3}) \)

Using a calculator

\[ P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.8906 = 0.1094 > 0.1 \]
\[ P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9416 = 0.0584 < 0.1 \]

The critical region is \( X \geq 17 \)

c. 0.0584 = 5.84%
9 a In a given time, the number of customers choosing lasagne out of the total number.

\[ H_0: p = 0.2 \]
\[ H_1: p \neq 0.2 \]

b Assume \( H_0 \) is true then \( X \sim B(25, 0.2) \)

Consider the lower tail:

\[ P(X \leq 0) = 0.0038 \]
\[ P(X \leq 1) = 0.0274 \text{ (closer to 0.025)} \]

Consider the upper tail:

\[ P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9532 = 0.0468 \]
\[ P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9827 = 0.0173 \text{ (closer to 0.025)} \]

The critical region is \( X \leq 1 \) and \( X \geq 10 \).

c The probability of incorrectly rejecting \( H_0 \) is \( 0.0274 + 0.0173 = 0.0447 = 4.47\% \)

Challenge

a Assume \( H_0 \) is true then \( X \sim B(50, 0.7) \)

Consider the lower tail:

\[ P(X \leq 29) = 0.0478 \text{ (closer to 0.05)} \]
\[ P(X \leq 30) = 0.0848 \]

Consider the upper tail:

\[ P(X \geq 41) = 1 - P(X \leq 40) = 1 - 0.9598 = 0.0402 \text{ (closer to 0.05)} \]
\[ P(X \geq 40) = 1 - P(X \leq 39) = 1 - 0.9211 = 0.0789 \]

The critical region is \( X \leq 29 \) and \( X \geq 41 \)

b The probability of one observation falling within the critical region is \( 0.0478 + 0.0402 = 8.8\% \)

The probability of two observations falling within the critical region is \( 0.088^2 = 0.007744 = 0.77\% \)

The probability that Chloe has incorrectly rejected \( H_0 \) is 0.77\%