Hypothesis testing 7B

1 a The critical value is the first value to fall inside of the critical region.

b A critical region is a region of the probability distribution which, if the test statistic falls within it, would cause you to reject the null hypothesis.

c The acceptance region is the area in which we accept the null hypothesis.

2 B (10, 0.2)

$$\begin{aligned}P(X \geq 4) &= 1 - P(X \leq 3)\\ &= 1 - 0.8791\\ &= 0.1209 > 0.05\end{aligned}$$

$$\begin{aligned}P(X \geq 5) &= 1 - P(X \leq 4)\\ &= 1 - 0.9672\\ &= 0.0328 < 0.05\end{aligned}$$

The critical value is $x = 5$ and the critical region is $X \geq 5$ since $P(X \geq 5) = 0.0328 < 0.05$

3 B (20, 0.15)

$$P(X \leq 1) = 0.1756 > 0.05$$
$$P(X = 0) = 0.0388 < 0.05$$

The critical value is $x = 0$ and the critical region is $X = 0$

4 a B (20, 0.4)

$$P(X \leq 4) = 0.0510 > 0.025$$
$$P(X \leq 3) = 0.0160 < 0.025$$

The critical value is $x = 3$

$$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9790 = 0.0210 < 0.025$$
$$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9435 = 0.0565 > 0.025$$

The critical value is $x = 13$
The critical region is $X \geq 13$ and $X \leq 3$

b The actual significance level is 0.021 + 0.016 = 0.037 = 3.7%

5 B (20, 0.18)

$\mathrm{B}(X = 0) = 0.0189 < 0.05$
$\mathrm{B}(X \leq 1) = 0.1018 > 0.05$

The critical value is $x = 0$
The critical region is $X = 0$

6 **a** B (10, 0.22)

$\mathrm{P}(X < 5) = 0.952$
$\mathrm{P}(X \geq 5) = 0.0478 > 0.005$

The critical value is $x = 5$
The critical region is $X \geq 5$

b The actual significance level is 0.0478 = 4.78%

7 **a** The test statistic is the number of components in the sample that fail.

b H_0: $p = 0.3$
H_1: $p < 0.3$

c Assume H_0 is true then $X \sim \mathrm{B}(20, 0.3)$

$\mathrm{P}(X \leq 2) = 0.0355$ (closer to 0.05)
$\mathrm{P}(X \leq 3) = 0.1071$

The critical region is $X \leq 2$

d 0.0355 = 3.55%

8 **a** The test statistic is the number of seedlings that survive.

H_0: $p = \frac{1}{3}$,
H_1: $p > \frac{1}{3}$

b Assume H_0 is true then $X \sim \mathrm{B}(36, \frac{1}{3})$

Using a calculator

$\mathrm{P}(X \geq 16) = 1 - \mathrm{P}(X \leq 15) = 1 - 0.8906 = 0.1094 > 0.1$

$\mathrm{P}(X \geq 17) = 1 - \mathrm{P}(X \leq 16) = 1 - 0.9416 = 0.0584 < 0.1$

The critical region is $X \geq 17$

c 0.0584 = 5.84%

9 **a** In a given time, the number of customers choosing lasagne out of the total number.

H_0: $p = 0.2$
H_1: $p \neq 0.2$

b Assume H_0 is true then $X \sim B(25, 0.2)$

Consider the lower tail:

$P(X \leq 0) = 0.0038$
$P(X \leq 1) = 0.0274$ (closer to 0.025)

Consider the upper tail:

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9532 = 0.0468$
$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.9827 = 0.0173$ (closer to 0.025)

The critical region is $X \leq 1$ and $X \geq 10$.

c The probability of incorrectly rejecting H_o is 0.0274 + 0.0173 = 0.0447 = 4.47%

Challenge

a Assume H_0 is true then $X \sim B(50, 0.7)$

Consider the lower tail:

$P(X \leq 29) = 0.0478$ (closer to 0.05)
$P(X \leq 30) = 0.0848$

Consider the upper tail:

$P(X \geq 41) = 1 - P(X \leq 40) = 1 - 0.9598 = 0.0402$ (closer to 0.05)
$P(X \geq 40) = 1 - P(X \leq 39) = 1 - 0.9211 = 0.0789$

The critical region is $X \leq 29$ and $X \geq 41$

b The probability of one observation falling within the critical region is 0.0478 + 0.0402 = 8.8%

The probability of two observations falling within the critical region is $0.088^2 = 0.007744 = 0.77\%$

The probability that Chloe has incorrectly rejected H_0 is 0.77%