1 a

\[
\begin{array}{c|cccccc}
    x & 1 & 2 & 3 & 4 & 5 & 6 \\
    \hline
    P(X = x) & \frac{1}{21} & \frac{2}{21} & \frac{3}{21} & \frac{4}{21} & \frac{5}{21} & \frac{6}{21} \\
\end{array}
\]

b  \( P(2 < X \leq 5) = P(X = 3) + P(X = 4) + P(X = 5) = \frac{3}{21} + \frac{4}{21} + \frac{5}{21} = \frac{12}{21} = \frac{4}{7} \)

2 a  \( 0.1 + 0.2 + 0.3 + r + 0.1 + 0.1 = 1 \)

\( r = 1 - 0.8 \)

\( = 0.2 \)

b  \( P(-1 \leq X < 2) = P(-1) + P(0) + P(1) = 0.2 + 0.3 + 0.2 = 0.7 \)

3 a

\[
\begin{array}{c|cccc}
    x & 1 & 2 & 3 & 4 \\
    \hline
    P(X = x) & \frac{2}{26} & \frac{5}{26} & \frac{8}{26} & \frac{11}{26} \\
\end{array}
\]

b  \( P(2 < X \leq 4) = P(X = 3) + P(X = 4) = \frac{10}{26} \)

4 a  For a discrete uniform distribution, the probability of choosing each counter must be equal.

b i  \( P(X = 5) = \frac{1}{16} \)

ii  The prime numbers are 2, 3, 5, 7, 11 and 13

\( P(X \text{ is prime}) = \frac{6}{16} = \frac{3}{8} \)

iii  \( P(3 \leq X < 11) = \frac{8}{16} = \frac{1}{2} \)

5 a

\[
\begin{array}{c|cccc}
    y & 1 & 2 & 3 & 4 & 5 \\
    \hline
    P(Y = y) & \frac{1}{k} & \frac{2}{k} & \frac{3}{k} & \frac{4}{k} & \frac{5}{k} \\
\end{array}
\]

\[
\frac{1}{k} + \frac{2}{k} + \frac{3}{k} + \frac{4}{k} + \frac{5}{k} = 1
\]

\[
\frac{15}{k} = 1, \ k = 15
\]
5  b

<table>
<thead>
<tr>
<th>y</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(Y = y))</td>
<td>(\frac{1}{15})</td>
<td>(\frac{2}{15})</td>
<td>(\frac{3}{15} = \frac{1}{5})</td>
<td>(\frac{4}{15})</td>
<td>(\frac{5}{15} = \frac{1}{3})</td>
</tr>
</tbody>
</table>

c \(P(Y > 3) = P(Y = 4) + P(Y = 5) = \frac{4}{15} + \frac{5}{15} = \frac{9}{15} = \frac{3}{5}\)

6  a

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(T = t))</td>
<td>(0.75^4 = 0.316)</td>
<td>(0.25 \times 0.75^3 \times 4 = 0.422)</td>
<td>(0.25^2 \times 0.75^2 \times 6 = 0.211)</td>
<td>(0.25^3 \times 0.75 \times 4 = 0.0469)</td>
<td>(0.25^4 = 0.00391)</td>
</tr>
</tbody>
</table>

d \(P(T < 3) = P(T = 0) + P(T = 1) + P(T = 2) = 0.949\) (to 3 s.f.)

c

<table>
<thead>
<tr>
<th>S</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(S = s))</td>
<td>(0.25)</td>
<td>(0.25 \times 0.75 = 0.188)</td>
<td>(0.25 \times 0.75^2 = 0.141)</td>
<td>(0.25 \times 0.75^3 = 0.105)</td>
<td>(0.25 \times 0.75^4 + 0.75^5 = 0.316)</td>
</tr>
</tbody>
</table>

d \(P(S > 2) = P(S = 3) + P(S = 4) + P(S = 5) = 0.563\) (to 3 s.f. using exact figures)

7  a \(P(X = 20) = \binom{30}{20}(0.73)^{20}(0.27)^{10} = \frac{30!}{20!10!}(0.73)^{20}(0.27)^{10} = 0.114\) (to 3 s.f.)

b Using the binomial cumulative function on a calculator where \(x = 13\), \(n = 30\) and \(p = 0.73\),
\(P(X \leq 13) = 0.000580\) (to 3 s.f.)

c Using the binomial cumulative function on a calculator where \(x = 11\) and \(25\), \(n = 30\) and \(p = 0.73\),
\(P(11 < X \leq 25) = P(X \leq 25) − P(X \leq 11) = 0.937302995 − 0.000033512 = 0.937\) (to 3 s.f.)

8  a Sequence is: H H H H H T

Probability: \(\left(\frac{2}{3}\right)^5 \times \frac{1}{3} = \frac{32}{729}\) or \(0.0439\) (to 3 s.f.)

b Let \(X = \)‘number of tails in the first 8 tosses’, then
\(P(X = 2) = \binom{8}{2}\left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^6 = 0.273\) (to 3 s.f.)
9  \(X = \text{number of patients waiting more than } \frac{1}{2} \text{ hour}\)

\(X \sim B(12, 0.3)\)

a  \(P(X = 0) = (0.7)^{12} = 0.01384... = 0.0138 \text{ (to 3 s.f.)}\)

b  \(P(X > 2) = 1 - P(X \leq 2) = 1 - 0.2528 = 0.7472 = 0.747 \text{ (to 3 d.p.)}\)

10 a  i  There are \(n\) independent trials.

ii  \(n\) is a fixed number.

iii  The outcome of each trial is a success or a failure.

iv  The probability of success at each trial is constant.

v  The outcome of any trial is independent of any other trial.

b  \(X = \text{number of successes}\)

\(X \sim B(10, 0.05)\)

\(P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.9139 = 0.0861 \text{ (to 3 s.f.)}\)

c  \(Y_n = \text{number of successes in } n \text{ houses}\)

\(Y_n \sim B(n, 0.05)\)

Looking for smallest \(n\) such that \(P(Y_n \geq 1) > 0.99\) or, equivalently, \(P(Y_n = 0) < 0.01\).

\(P(Y_n = 0) = 0.95^n < 0.01\)

So \(n = 90\) using logarithms.

11  \(X = \text{number of correctly answered questions'} \text{ and } X \sim B(10, 0.5)\)

a  \(P(X = 10) = (0.5)^{10} = 0.00097656... = 0.000977 \text{ (to 3 s.f.)}\)

b  \(P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9453 = 0.0547 \text{ (to 3 s.f. using tables)}\)

12

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(X = x))</td>
<td>(p)</td>
<td>(p)</td>
<td>(p)</td>
<td>(p)</td>
<td>(2p)</td>
<td>(p)</td>
</tr>
</tbody>
</table>

\(7p = 1 \Rightarrow p = \frac{1}{7}\)

a  Sequence: \(\frac{3}{7}, \frac{5}{7}, \frac{5}{7}, \frac{5}{7}, \frac{5}{7}, 5\)

Probability: \(\left(\frac{2}{7}\right)^5 \left(\frac{5}{7}\right) = 0.0531 \text{ (to 3 s.f.)}\)
12 b  \( Y = \) 'number of 5s in 8 throws'

\[
Y \sim B(8, \frac{2}{7})
\]

\[
P(Y = 3) = \binom{8}{3} \times \left( \frac{2}{7} \right)^3 \times \left( \frac{5}{7} \right)^5 = 0.24285 = 0.243 \text{ (to 3 s.f.)}
\]

13  \( X = \) 'number of green chairs in sample of 10'

a  \( X \sim B(10, 0.15) \)

b  \( P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9901 = 0.0099 \) (tables)

c  \( P(X = 2) = P(X \leq 2) - P(X \leq 1) = 0.8202 - 0.5443 = 0.2759 \) (tables)

14  \( X = \) 'number of yellow beads in sample of 20' and assume \( X \sim B(20, 0.45) \)

a  \( P(X < 12) = P(X \leq 11) = 0.8692 \) (tables)

b  \( P(X = 12) = P(X \leq 12) - P(X \leq 11) = 0.9420 - 0.8692 = 0.0728 \) (tables)

15 a  \( P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.1275 = 0.8725 \) (tables)

b  \( P(X \geq 10 \text{ in 7 out of 12 sets}) = \binom{12}{7}(0.8725)^7(0.1275)^5 \)

\[
= 0.0103 \text{ (to 3 s.f.)}
\]

c  Let \( Y = \) 'number of sets out of 12 that she hits the bullseye with at least 50% of her arrows', then

\( Y \sim B(12, 0.8725) \)

Using the binomial cumulative function on a calculator where \( x = 5, n = 12 \) and \( p = 0.8725 \),

\( P(Y < 6) = P(Y \leq 5) = 0.0002407 \)

**Challenge**

\( Y \sim B(18, 0.25) \)

\( P(Y \geq 11) = 1 - P(Y \leq 10) = 1 - 0.9988 = 0.0012 \) (tables)