Statistical distributions, Mixed Exercise 6

1 a

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$

b $P(2 < X \le 5) = P(X=3)+P(X=4)+P(X=5) = \frac{3}{21}+\frac{4}{21}+\frac{5}{21}=\frac{12}{21}=\frac{4}{7}$

2 a $0.1+0.2+0.3+r+0.1+0.1=1$

$r = 1-0.8$
$= 0.2$

b $P(-1 \le X < 2) = P(-1)+P(0)+P(1) = 0.2+0.3+0.2 = 0.7$

3 a

x	1	2	3	4
$P(X=x)$	$\frac{2}{26}$	$\frac{5}{26}$	$\frac{8}{26}$	$\frac{11}{26}$

b $P(2 < X \le 4) = P(X=3)+P(X=4) = \frac{19}{26}$

4 a For a discrete uniform distribution, the probability of choosing each counter must be equal.

b i $P(X=5) = \frac{1}{16}$

ii The prime numbers are 2, 3, 5, 7, 11 and 13

$P(X \text{ is prime}) = \frac{6}{16} = \frac{3}{8}$

iii $P(3 \le X < 11) = \frac{8}{16} = \frac{1}{2}$

5 a

y	1	2	3	4	5
$P(Y=y)$	$\frac{1}{k}$	$\frac{2}{k}$	$\frac{3}{k}$	$\frac{4}{k}$	$\frac{5}{k}$

$\frac{1}{k}+\frac{2}{k}+\frac{3}{k}+\frac{4}{k}+\frac{5}{k}=1$

$\frac{15}{k}=1, k=15$

5 b

y	1	2	3	4	5
P($Y = y$)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15} = \frac{1}{5}$	$\frac{4}{15}$	$\frac{5}{15} = \frac{1}{3}$

c $P(Y > 3) = P(Y = 4) + P(Y = 5) = \frac{4}{15} + \frac{5}{15} = \frac{9}{15} = \frac{3}{5}$

6 a

t	0	1	2	3	4
P($T = t$)	$\frac{81}{256}$	$\frac{108}{256}$	$\frac{54}{256}$	$\frac{12}{256}$	$\frac{1}{256}$

b $P(T < 3) = P(T = 0) + P(T = 1) + P(T = 2) = \frac{243}{256}$

c

S	1	2	3	4	5
P($S = s$)	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{9}{64}$	$\frac{27}{256}$	$\frac{81}{256}$

d $P(S > 2) = P(S = 3) + P(S = 4) + P(S = 5) = \frac{9}{16}$

7 a $P(X = 20) = \binom{30}{20}(0.73)^{20}(0.27)^{10} = \frac{30!}{20!10!}(0.73)^{20}(0.27)^{10} = 0.114$ (to 3 s.f.)

b Using the binomial cumulative function on a calculator where $x = 13$, $n = 30$ and $p = 0.73$,

$P(X \leq 13) = 0.000580$ (to 3 s.f.)

c Using the binomial cumulative function on a calculator where $x = 11$ and 25, $n = 30$ and p = 0.73,

$P(11 < X \leq 25) = P(X \leq 25) - P(X \leq 11) = 0.937302995 - 0.000033512 = 0.937$ (to 3 s.f.)

8 a Sequence is: H H H H H T

Probability: $\left(\frac{2}{3}\right)^5 \times \frac{1}{3} = \frac{32}{729}$ or 0.0439 (to 3 s.f.)

b Let X = 'number of tails in the first 8 tosses', then

$$P(X = 2) = \binom{8}{2}\left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^6 = 0.273 \text{ (to 3 s.f.)}$$

9 X = number of patients waiting more than $\frac{1}{2}$ hour

$X \sim B(12, 0.3)$

a $P(X = 0) = (0.7)^{12} = 0.01384... = 0.0138$ (to 3 s.f.)

b $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.2528 = 0.7472 = 0.747$ (to 3 d.p.)

10 a **i** There are n independent trials.
ii n is a fixed number.
iii The outcome of each trial is a success or a failure.
iv The probability of success at each trial is constant.
v The outcome of any trial is independent of any other trial.

b X = number of successes

$X \sim B(10, 0.05)$

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.9139 = 0.0861$ (to 3 s.f.)

c Y_n = 'number of successes in n houses'

$Y_n \sim B(n, 0.05)$

Looking for smallest n such that $P(Y_n \geq 1) > 0.99$ or, equivalently, $P(Y_n = 0) < 0.01$.

$P(Y_n = 0) = 0.95^n < 0.01$

So $n = 90$ using logarithms.

11 X = 'number of correctly answered questions' and $X \sim B(10, 0.5)$

a $P(X = 10) = (0.5)^{10} = 0.00097656... = 0.000977$ (to 3 s.f.)

b $P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9453 = 0.0547$ (to 3 s.f. using tables)

12

x	1	2	3	4	5	6
P(X = x)	p	p	p	p	$2p$	p

$7p = 1 \Rightarrow p = \frac{1}{7}$

a Sequence: $\bar{5}\ \bar{5}\ \bar{5}\ \bar{5}\ \bar{5}\ 5$

Probability: $\left(\frac{5}{7}\right)^5\left(\frac{2}{7}\right) = 0.0531$ (to 3 s.f.)

12 b $Y =$ 'number of 5s in 8 throws'

$Y \sim \text{B}(8, \frac{2}{7})$

$\text{P}(Y = 3) = \binom{8}{3} \times \left(\frac{2}{7}\right)^3 \times \left(\frac{5}{7}\right)^5 = 0.24285 = 0.243$ (to 3 s.f.)

13 $X =$ 'number of green chairs in sample of 10'

a $X \sim \text{B}(10, 0.15)$

b $\text{P}(X \geq 5) = 1 - \text{P}(X \leq 4) = 1 - 0.9901 = 0.0099$ (tables)

c $\text{P}(X = 2) = \text{P}(X \leq 2) - \text{P}(X \leq 1) = 0.8202 - 0.5443 = 0.2759$ (tables)

14 $X =$ 'number of yellow beads in sample of 20' and assume $X \sim \text{B}(20, 0.45)$

a $\text{P}(X < 12) = \text{P}(X \leq 11) = 0.8692$ (tables)

b $\text{P}(X = 12) = \text{P}(X \leq 12) - \text{P}(X \leq 11) = 0.9420 - 0.8692 = 0.0728$ (tables)

15 a $\text{P}(X \geq 10) = 1 - \text{P}(X \leq 9) = 1 - 0.1275 = 0.8725$ (tables)

b $\text{P}(X \geq 10 \text{ in 7 out of 12 sets}) = \binom{12}{7}(0.8725)^7(0.1275)^5$

$= 0.0103$ (to 3 s.f.)

c Let $Y =$ 'number of sets out of 12 that she hits the bullseye with at least 50% of her arrows', then

$Y \sim \text{B}(12, 0.8725)$

Using the binomial cumulative function on a calculator where $x = 5$, $n = 12$ and $p = 0.8725$,

$\text{P}(Y < 6) = \text{P}(Y \leq 5) = 0.0002407$

Challenge

$Y \sim \text{B}(18, 0.25)$

$\text{P}(Y \geq 11) = 1 - \text{P}(Y \leq 10) = 1 - 0.9988 = 0.0012$ (tables)