Statistical distributions 6C

1. $X \sim B(9, 0.2)$
   
   a. $P(X \leq 4) = 0.9804$ (tables)
   
   b. $P(X < 3) = P(X \leq 2) = 0.7382$ (tables)
   
   c. $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.4362 = 0.5638$ (tables)
   
   d. $P(X = 1) = P(X \leq 1) - P(X = 0) = 0.4362 - 0.1342 = 0.3020$ (tables)

2. $X \sim B(20, 0.35)$
   
   a. $P(X \leq 10) = 0.9468$ (tables)
   
   b. $P(X > 6) = 1 - P(X \leq 6) = 1 - 0.4166 = 0.5834$ (tables)
   
   c. $P(X = 5) = P(X \leq 5) - P(X \leq 4) = 0.2454 - 0.1182 = 0.1272$ (tables)
   
   d. $P(2 \leq X \leq 7) = P(X \leq 7) - (X \leq 1) = 0.6010 - 0.0021 = 0.5989$ (tables)

3. a. Using the binomial cumulative function on a calculator where $x = 19$, $n = 40$ and $p = 0.47$,
   
   $P(X < 20) = P(X \leq 19) = 0.5888$
   
   b. Using the binomial cumulative function on a calculator where $x = 16$, $n = 40$ and $p = 0.47$,
   
   $P(X > 16) = 1 - P(X \leq 16) = 0.7662$
   
   c. Using the binomial cumulative function on a calculator where $x = 10$ and $15$, $n = 40$ and $p = 0.47$,
   
   $P(11 \leq X \leq 15) = P(X \leq 15) - P(X \leq 10) = 0.1478 - 0.0036 = 0.1442$
   
   d. Using the binomial cumulative function on a calculator where $x = 10$ and $16$, $n = 40$ and $p = 0.47$,
   
   $P(10 < X < 17) = P(X \leq 16) - P(X \leq 10) = 0.2338 - 0.0036 = 0.2302$

4. a. Using the binomial cumulative function on a calculator where $x = 20$, $n = 37$ and $p = 0.65$,
   
   $P(X > 20) = 1 - P(X \leq 20) = 0.8882$
   
   b. Using the binomial cumulative function on a calculator where $x = 26$, $n = 37$ and $p = 0.65$,
   
   $P(X \leq 26) = 0.7992$
   
   c. Using the binomial cumulative function on a calculator where $x = 19$ and $14$, $n = 37$ and $p = 0.65$,
   
   $P(15 \leq X < 20) = P(X \leq 19) - P(X \leq 14) = 0.06061 - 0.00068 = 0.05993$
4 d Using the binomial cumulative function on a calculator where $x = 23$ and 22, $n = 37$ and $p = 0.65$,

\[ P(X = 23) = P(X \leq 23) - P(X \leq 22) = 0.4184 - 0.2926 = 0.1258 \]

5 \( X = \) 'number of heads'
\( X \sim B(8, 0.5) \) (coins are fair so $p = 0.5$)

\begin{align*}
\text{a} & \quad P(X = 0) = (0.5)^8 = 0.0039 \quad \text{(tables)} \\
\text{b} & \quad P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.0352 = 0.9648 \quad \text{(tables)} \\
\text{c} & \quad P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.3637 = 0.6363 \quad \text{(tables)}
\end{align*}

6 \( X = \) 'number of plants with blue flowers on tray of 15'

\begin{align*}
\text{a} & \quad P(X = 4) = P(X \leq 4) - P(X \leq 3) = 0.6865 - 0.4613 = 0.2252 \quad \text{(tables)} \\
\text{b} & \quad P(X \leq 3) = 0.4613 \quad \text{(tables)} \\
\text{c} & \quad P(3 \leq X \leq 6) = P(X \leq 6) - P(X \leq 2) = 0.9434 - 0.2361 = 0.7073 \quad \text{(tables)}
\end{align*}

7 \( X \sim B(50, 0.40) \)

\begin{align*}
\text{a} & \quad P(X \leq 13) = 0.0280 \\
& \quad P(X \leq 14) = 0.0540 \quad \text{(tables)} \\
& \quad \therefore k = 13 \\
\text{b} & \quad P(X \leq 27) = 0.9840 \\
& \quad \Rightarrow P(X > 27) = 0.0160 > 0.01 \\
& \quad P(X \leq 28) = 0.9924 \\
& \quad \Rightarrow P(X > 28) = 0.0076 < 0.01 \\
& \quad \therefore r = 28
\end{align*}

8 \( X \sim B(40, 0.10) \)

\begin{align*}
\text{a} & \quad P(X = 0) = 0.0148 < 0.02 \\
& \quad P(X \leq 1) = 0.0805 > 0.02 \quad \text{(tables)} \\
& \quad P(X < 1) = 0.0148 < 0.02 \\
& \quad \therefore k = 1
\end{align*}
8 b \( P(X \leq 8) = 0.9845 \) (tables)
\[ \Rightarrow P(X > 8) = 0.0155 > 0.01 \]
\( P(X \leq 9) = 0.9949 \)
\[ \Rightarrow P(X > 9) = 0.0051 < 0.01 \]

\( r = 9 \)

c \( P(k \leq X \leq r) = P(X \leq r) - P(X \leq k - 1) \)
\[ = P(X \leq 9) - P(X = 0) \]
\[ = 0.9949 - 0.0148 \]
\[ = 0.9801 \]

9 a A suitable distribution is \( X \sim B(10, 0.30) \). Assumptions: There are two possible outcomes of each trial listen or don’t listen. There is a fixed number of trials, 10, and fixed probability of success: 0.3. Each member in the sample is assumed to listen independently.

b \( P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8497 = 0.1503 \) (tables)

c \( P(X \leq 6) = 0.9894 \) so \( P(X \geq 7) = 1 - 0.9894 = 0.0106 > 0.01 \)
\( P(X \leq 7) = 0.9984 \) so \( P(X \geq 8) = 1 - 0.9984 = 0.0016 < 0.01 \) (tables)

Therefore \( s = 8 \) is the smallest such value.

10 \( X \) = number of defects in 50 components
\( X \sim B(50, 0.05) \)

a \( P(X < 2) = P(X \leq 1) = 0.2794 \) (tables)

b \( P(X > 5) = 1 - P(X \leq 5) = 1 - 0.9622 = 0.0378 \) (tables)

c Seek smallest \( d \) such that \( P(X > d) < 0.05 \)
\( P(X \leq 4) = 0.8964 \) so \( P(X > 4) = 0.1036 > 0.05 \)
\( P(X \leq 5) = 0.9622 \) so \( P(X > 5) = 0.0378 < 0.05 \)

\( \therefore d = 5 \)