1 a \( P(X = 2) = \binom{8}{2} \times \left( \frac{1}{3} \right)^2 \times \left( \frac{2}{3} \right)^6 \)

= 0.273 (to 3 s.f.)

b \( P(X = 5) = \binom{8}{5} \times \left( \frac{1}{3} \right)^5 \times \left( \frac{2}{3} \right)^3 \)

= 0.0683 (to 3 s.f.)

c \( P(X \leq 1) = P(X = 1) + P(X = 0) \)

= 8 \left( \frac{1}{3} \right)^7 \left( \frac{2}{3} \right)^8 + \left( \frac{2}{3} \right)^7 \left( \frac{8}{3} + \frac{2}{3} \right)

= \left( \frac{2}{3} \right)^7 \times 10 \times \frac{3}{3}

= 0.195 (to 3 s.f.)

2 a \( P(T = 5) = \binom{15}{5} \times \left( \frac{2}{3} \right)^5 \times \left( \frac{1}{3} \right)^{10} = 0.00670 \) (to 3 s.f.)

b \( P(T = 10) = \binom{15}{10} \times \left( \frac{2}{3} \right)^{10} \times \left( \frac{1}{3} \right)^5 = 0.214 \) (to 3 s.f.)

c \( P(3 \leq T \leq 4) = P(T = 3) + P(T = 4) = \left( \frac{15}{3} \right) \times \left( \frac{2}{3} \right)^3 \times \left( \frac{1}{3} \right)^{12} + \left( \frac{15}{4} \right) \times \left( \frac{2}{3} \right)^4 \times \left( \frac{1}{3} \right)^{11} \)

= 0.00025367... + 0.00152206...

= 0.00178 (to 3 s.f.)

3 a \( X = 'number of defective bolts in a sample of 20' \)

\( X \sim \text{B}(20, 0.01) \)

\( n = 20 \)
\( p = 0.01 \)

Assume bolts are defective independently of one another.
3  b  \( X = \text{'number of times wait or stop in 6 lights'} \)

\[ X \sim B(6, 0.52) \]

\( n = 6 \)
\( p = 0.52 \)

Assume the lights operate independently and the time lights are on/off is constant.

c  \( X = \text{'number of aces in Stephanie's next 30 serves'} \)

\[ X \sim B(30, \frac{1}{8}) \]

\( n = 30 \)
\( p = \frac{1}{8} \)

Assume serving an ace occurs independently and the probability of an ace is constant.

4  a  \( X = \text{'number of people in class of 14 who are Rh–'} \)

\[ X \sim B(14, 0.15) \] is a reasonable model if we assume that being Rh– is independent from pupil to pupil - so no siblings.

b  This is not binomial since the number of trials or tosses is not known and fixed. The probability of a head at each toss is constant \( (p = 0.5) \) but there is no value for \( n \).

c  Assuming, reasonably, that the colours of the cars are independent,

\( X = \text{'number of red cars out of 15'} \)

\[ X \sim B(15, 0.12) \]

5  a  Let \( X = \text{'number of balloons that do not burst'} \)

\[ P(X = 0) = (0.95)^{20} \]
\[ = 0.358 \quad \text{(to 3 s.f.)} \]

b  Let \( Y = \text{'number of balloons that do burst'} \)

\[ P(Y = 2) = \binom{20}{2} (0.95)^{18} (0.05)^2 \]
\[ = 0.189 \quad \text{(to 3 s.f.)} \]
6 a There are two possible outcomes of each trial: faulty or not faulty. There are a fixed number of trials, 10, and fixed probability of success: 0.08. Assuming each member in the sample is independent, a suitable model is $X \sim B(10, 0.08)$.

b $P(X = 4) = \binom{10}{4} (0.08)^4 (0.92)^6 = \frac{10!}{4!6!} (0.08)^4 (0.92)^6 = 0.00522$ (to 3 s.f.)

7 a Assumptions are that there is a fixed sample size, that there are only two outcomes for the genetic marker (present or not present), and that there is a fixed probability of people having the marker.

b $X \sim B(50, 0.04)$

$P(X = 6) = \binom{50}{6} (0.04)^6 (0.96)^{44} = \frac{50!}{6!44!} (0.04)^6 (0.96)^{44} = 0.0108$ (to 3 s.f.)

8 a We are assuming that each roll of the dice is independent. A suitable model is $X \sim B(15, 0.3)$.

b $X \sim B(15, 0.3)$

$P(X = 4) = \binom{15}{4} (0.3)^4 (0.7)^{11} = \frac{15!}{4!11!} (0.3)^4 (0.7)^{11} = 0.219$ (to 3 s.f.)

c $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= (0.7)^5 + \binom{15}{1} (0.3)^1 (0.7)^{14} + \binom{15}{2} (0.3)^2 (0.7)^{13}$

$= (0.7)^5 + \frac{15!}{1!14!} (0.3)^1 (0.7)^{14} + \frac{15!}{2!13!} (0.3)^2 (0.7)^{13}$

$= 0.127$ (to 3 s.f.)