Probability, Mixed Exercise 5

1 a P(*RRB* or *RRG*) = $\left(\frac{7}{15}\times\frac{7}{15}\times\frac{3}{15}\right)+\left(\frac{7}{15}\times\frac{7}{15}\times\frac{5}{15}\right)$

$= \frac{392}{3375}$

b P(*RBG*) + P(*RGB*) + P(*BGR*) + P(*BRG*) + P(*GBR*) + P(*GRB*)

$=\left(\frac{7}{15}\times\frac{3}{15}\times\frac{5}{15}\right)+\left(\frac{7}{15}\times\frac{5}{15}\times\frac{3}{15}\right)+\left(\frac{3}{15}\times\frac{5}{15}\times\frac{7}{15}\right)+\left(\frac{3}{15}\times\frac{7}{15}\times\frac{5}{15}\right)+\left(\frac{5}{15}\times\frac{3}{15}\times\frac{7}{15}\right)+\left(\frac{5}{15}\times\frac{7}{15}\times\frac{3}{15}\right)$

$= 6\times\left(\frac{7\times3\times5}{15^3}\right)=\frac{630}{3375} = \frac{14}{75}$

2 a P(*HHH*) = 0.341 × 0.341 × 0.341 = 0.0397 (to 3 s.f.)

b P(*NNN*) = 0.659 × 0.659 × 0.659 = 0.286 (to 3 s.f.)

c P(at least one *H*) = 1 − P(*NNN*) = 1 − 0.28619118 = 0.714 (to 3 s.f.)

3 a P(female) = $\frac{8+13+19+30+26+32}{250} = \frac{128}{250} = \frac{64}{125}$

b P(s < 35) = $\frac{7+8+15+13+18+19}{250} = \frac{80}{250} = \frac{8}{25}$

c P(male with score between 25 and 34) = $\frac{15+18}{250} = \frac{33}{250}$

d Using interpolation:

Number of students passing $=\frac{40-37}{40-35}\times(25+30)+30+26+27+32$

$=\left(\frac{3}{5}\times55\right)+30+26+27+32=148$

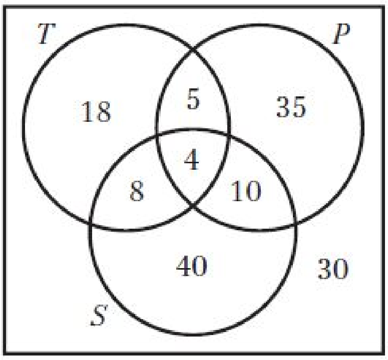
P(pass) = $\frac{148}{250} = \frac{74}{125}$

The assumption is that the marks between 35 and 40 are uniformly distributed.

4 a P(> 3) = $\frac{0.5\times50+0.5\times30+2\times2}{1\times6+0.5\times50+0.5\times30+2\times2} = \frac{44}{50} = \frac{22}{25}$

b P(< 3.75) = $\frac{(1\times6)+(0.5\times50)+0.5\times(0.5\times30)}{50} = \frac{38.5}{50}$ = 0.77

5 a



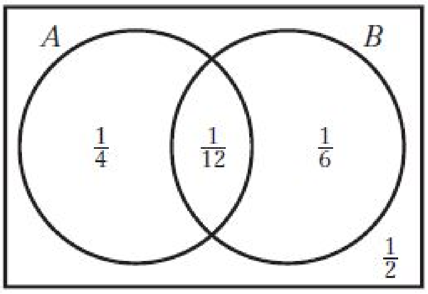
b i $\text{P(None)} = \frac{30}{150} = \frac{1}{5}$

ii $\text{P(No more than one)} = \frac{30+40+18+35}{150} = \frac{123}{150} = \frac{41}{50}$

6 a P(A and B) = P(A) + P(B) – P(A or B or both) = $\frac{1}{3} + \frac{1}{4} - \frac{1}{2} = \frac{1}{12}$

P(A and not B) = $\frac{1}{3} - \frac{1}{12} = \frac{1}{4}$

P(B and not A) = $\frac{1}{4} - \frac{1}{12} = \frac{1}{6}$



b P(A and B) = $\frac{1}{12}$

P(A) × P(B) = $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

As P(A and B) = P(A) × P(B), A and B are independent events.

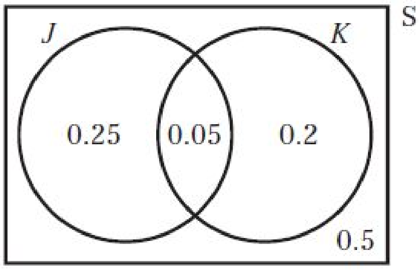
7 a Cricket and swimming do not overlap so are mutually exclusive.

b P(C and F) = $\frac{13}{38}$

P(C) × P(F) = $\frac{21}{38} \times \frac{22}{38} = \frac{462}{1444} = \frac{231}{722}$

As P(C and F) ≠ P(C) × P(F), the events 'likes cricket' and 'likes football' are not independent.

8 a



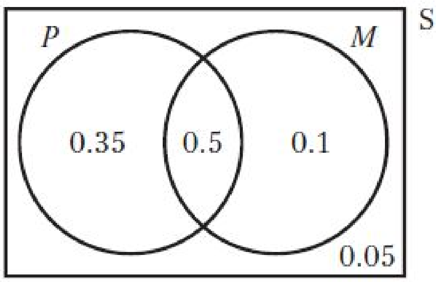
b P(J and K) = 0.05

P(J) × P(K) = 0.3 × 0.25 = 0.075

As P(J and K) ≠ P(J) × P(K), the events J and K are not independent.

9 a $\text{P}(\text{Phone and MP3}) = 0.85 + 0.6 - (1 - 0.05) = 0.5 = 50\%$

b



c P(only P) = 0.35

d P(P and M) = 0.5

P(P) × P(M) = 0.85 × 0.6 = 0.51

As P(P and M) ≠ P(P) × P(M), the events P and M are not independent.

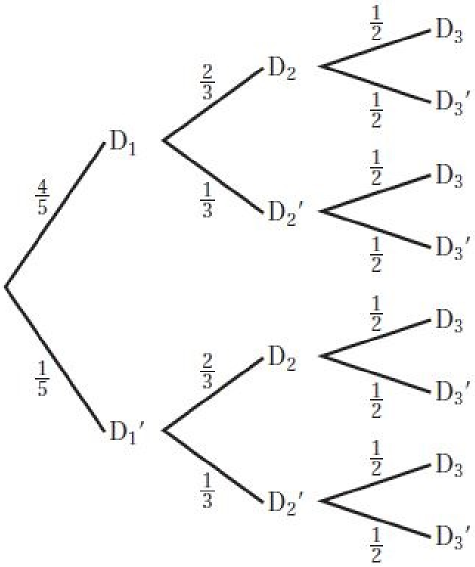
10 $x = 1 - (0.3 + 0.4 + 0.15) = 0.15$

P(A and B) = x = 0.15

P(A) × P(B) = 0.45 × 0.55 = 0.2475

As P(A and B) ≠ P(A) × P(B), the events A and B are not independent.

11 a



11 b i $P(D_1D_2D_3) = \frac{4}{5}\times\frac{2}{3}\times\frac{1}{2}=\frac{4}{15}$

ii Where D means a diamond and D' means no diamond,

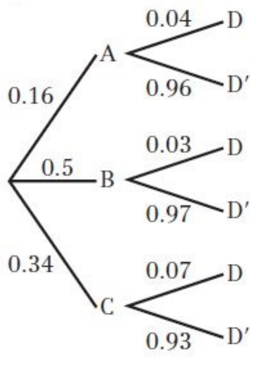
P (exactly one diamond) = P(D, D', D') + P(D', D, D') + P(D', D', D)

$$=\left(\frac{4}{5}\times\frac{1}{3}\times\frac{1}{2}\right)+\left(\frac{1}{5}\times\frac{2}{3}\times\frac{1}{2}\right)+\left(\frac{1}{5}\times\frac{1}{3}\times\frac{1}{2}\right)=\frac{7}{30}$$

11 c P(at least two diamonds) = 1 – P(at most one diamond) = 1 – (P(none) + P(exactly one diamond))

$$= 1 - \left(\frac{1}{5}\times\frac{1}{3}\times\frac{1}{2}+\frac{7}{30}\right) = 1-\frac{4}{15} = \frac{11}{15}$$

12 a



b i $P(B \text{ and defective}) = 0.5\times0.03=0.015$

ii $P(\text{defective}) = 0.16\times0.04+0.5\times0.03+0.34\times0.07=0.0452$

Challenge

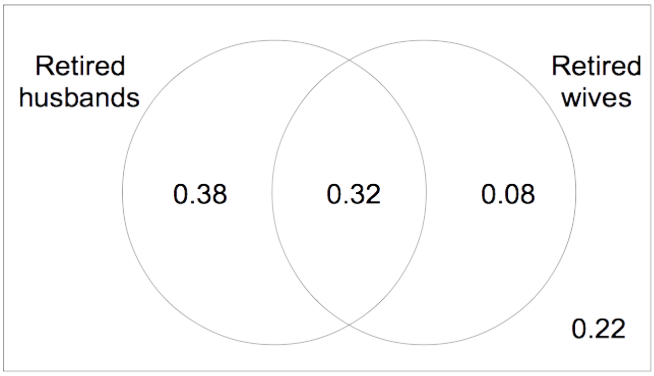
The probability that a wife is retired is 0.4.

Given that she is retired, the probability that her husband is also retired is 0.8.

Hence the probability that both are retired is $0.4 \times 0.8 = 0.32$.

The probability that a husband is retired is 0.7.

From this data you can deduce the following Venn diagram of the probabilities:



Let H = husband retired, H′ = husband not retired, W = wife retired, W′ = wife not retired.

The permutations where only one husband and only one wife is retired are:

Couple 1	Probability	Couple 2	Probability	Combined probability
H W′	0.38	H′ W	0.08	0.38×0.08
H′ W	0.08	H W′	0.38	0.08×0.38
H W	0.32	H′ W′	0.22	0.32×0.22
H′ W′	0.22	H W	0.32	0.22×0.32

$$\text{P(only one husband and only one wife is retired)} = (0.38 \times 0.08 + 0.32 \times 0.22) \times 2 = 0.2016$$