Probability, Mixed Exercise 5

1 a \[ P(\text{RRB or RRG}) = \left( \frac{7}{15} \times \frac{7}{15} \times \frac{3}{15} \right) + \left( \frac{7}{15} \times \frac{7}{15} \times \frac{5}{15} \right) = \frac{392}{3375} \]

b \[ P(\text{RBG}) + P(\text{RGB}) + P(\text{BGR}) + P(\text{BRG}) + P(\text{GRB}) + P(\text{GBR}) = \left( \frac{7}{15} \times \frac{3}{15} \times \frac{5}{15} \right) + \left( \frac{7}{15} \times \frac{5}{15} \times \frac{3}{15} \right) + \left( \frac{5}{15} \times \frac{3}{15} \times \frac{7}{15} \right) + \left( \frac{5}{15} \times \frac{7}{15} \times \frac{3}{15} \right) = \frac{14}{75} \]

2 a \[ P(\text{HHH}) = 0.341 \times 0.341 \times 0.341 = 0.0397 \] (to 3 s.f.)

b \[ P(\text{NNN}) = 0.659 \times 0.659 \times 0.659 = 0.286 \] (to 3 s.f.)

c \[ P(\text{at least one } H) = 1 - P(\text{NNN}) = 1 - 0.28619118 = 0.714 \] (to 3 s.f.)

3 a \[ P(\text{female}) = \frac{8 + 13 + 19 + 30 + 26 + 32}{250} = \frac{128}{250} = \frac{64}{125} \]

b \[ P(s < 35) = \frac{7 + 8 + 15 + 13 + 18 + 19}{250} = \frac{80}{250} = \frac{8}{25} \]

c \[ P(\text{male with score between 25 and 34}) = \frac{15 + 18}{250} = \frac{33}{250} \]

d Using interpolation:

Number of students passing = \[ \frac{40 - 37}{40 - 35} \times (25 + 30) + 30 + 26 + 27 + 32 \]

\[ = \left( \frac{3}{5} \times 55 \right) + 30 + 26 + 27 + 32 = 148 \]

P(pass) = \[ \frac{148}{250} = \frac{74}{125} \]

The assumption is that the marks between 35 and 40 are uniformly distributed.

4 a \[ P(> 3) = \frac{0.5 \times 50 + 0.5 \times 30 + 2 \times 2}{1 \times 6 + 0.5 \times 50 + 0.5 \times 30 + 2 \times 2} = \frac{44}{50} = \frac{22}{25} \]

b \[ P(< 3.75) = \frac{(1 \times 6) + (0.5 \times 50) + 0.5 \times (0.5 \times 30)}{50} = \frac{38.5}{50} = 0.77 \]
5 a

\[ T \]
\[ \begin{array}{ccc}
18 & 5 & 35 \\
8 & 4 & 10 \\
40 & 30 & \\
\end{array} \]

b i \( P(\text{None}) = \frac{30}{150} = \frac{1}{5} \)

ii \( P(\text{No more than one}) = \frac{30 + 40 + 18 + 35}{150} = \frac{123}{150} = \frac{41}{50} \)

6 a \( P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B \text{ or both}) = \frac{1}{3} + \frac{1}{4} - \frac{1}{2} = \frac{1}{12} \)

\( P(A \text{ and not } B) = \frac{1}{3} - \frac{1}{12} = \frac{1}{4} \)

\( P(B \text{ and not } A) = \frac{1}{4} - \frac{1}{12} = \frac{1}{6} \)

b \( P(A \text{ and } B) = \frac{1}{12} \)

\( P(A) \times P(B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12} \)

As \( P(A \text{ and } B) = P(A) \times P(B) \), \( A \) and \( B \) are independent events.

7 a Cricket and swimming do not overlap so are mutually exclusive.

b \( P(C \text{ and } F) = \frac{13}{38} \)

\( P(C) \times P(F) = \frac{21}{38} \times \frac{22}{38} = \frac{462}{1444} = \frac{231}{722} \)

As \( P(C \text{ and } F) \neq P(C) \times P(F) \), the events 'likes cricket' and 'likes football' are not independent.
8 a

\[ P(J \text{ and } K) = 0.05 \]

\[ P(J) \times P(K) = 0.3 \times 0.25 = 0.075 \]

As \( P(J \text{ and } K) \neq P(J) \times P(K) \), the events \( J \) and \( K \) are not independent.

b 

\[ P(J \text{ and } K) = 0.05 \]

\[ P(J) \times P(K) = 0.3 \times 0.25 = 0.075 \]

As \( P(J \text{ and } K) \neq P(J) \times P(K) \), the events \( J \) and \( K \) are not independent.

9 a  \( P(\text{Phone and MP3}) = 0.85 + 0.6 - (1 - 0.05) = 0.5 = 50\% \)

\[ P(\text{only P}) = 0.35 \]

d  \( P(\text{P and M}) = 0.5 \)

\[ P(\text{P}) \times P(\text{M}) = 0.85 \times 0.6 = 0.51 \]

As \( P(\text{P and M}) \neq P(\text{P}) \times P(\text{M}) \), the events \( \text{P} \) and \( \text{M} \) are not independent.

10  \( x = 1 - (0.3 + 0.4 + 0.15) = 0.15 \)

\[ P(A \text{ and } B) = x = 0.15 \]

\[ P(A) \times P(B) = 0.45 \times 0.55 = 0.2475 \]

As \( P(A \text{ and } B) \neq P(A) \times P(B) \), the events \( A \) and \( B \) are not independent.
11 a

\[ P(D_1D_2D_3) = \frac{4}{5} \times \frac{2}{3} \times \frac{1}{2} = \frac{4}{15} \]

ii Where \( D \) means a diamond and \( D' \) means no diamond,

\[
P(\text{exactly one diamond}) = P(D, D', D') + P(D', D, D') + P(D', D', D)
\]

\[
= \left( \frac{4}{5} \times \frac{1}{3} \times \frac{1}{2} \right) + \left( \frac{1}{5} \times \frac{2}{3} \times \frac{1}{2} \right) + \left( \frac{1}{5} \times \frac{1}{3} \times \frac{1}{2} \right) = \frac{7}{30}
\]

11 c \( P(\text{at least two diamonds}) = 1 - P(\text{at most one diamond}) = 1 - (P(\text{none}) + P(\text{exactly one diamond})) \)

\[
= 1 - \left( \frac{4}{5} \times \frac{1}{3} \times \frac{1}{2} + \frac{7}{30} \right) = 1 - \frac{4}{15} = \frac{11}{15}
\]

12 a

b i \( P(\text{B and defective}) = 0.5 \times 0.03 = 0.015 \)

ii \( P(\text{defective}) = 0.16 \times 0.04 + 0.5 \times 0.03 + 0.34 \times 0.07 = 0.0452 \)
Challenge

The probability that a wife is retired is 0.4.

Given that she is retired, the probability that her husband is also retired is 0.8.

Hence the probability that both are retired is $0.4 \times 0.8 = 0.32$.

The probability that a husband is retired is 0.7.

From this data you can deduce the following Venn diagram of the probabilities:

![Venn Diagram]

Let $H = \text{husband retired}$, $H' = \text{husband not retired}$, $W = \text{wife retired}$, $W' = \text{wife not retired}$.

The permutations where only one husband and only one wife is retired are:

<table>
<thead>
<tr>
<th>Couple 1</th>
<th>Probability</th>
<th>Couple 2</th>
<th>Probability</th>
<th>Combined probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>H W'</td>
<td>0.38</td>
<td>H' W</td>
<td>0.08</td>
<td>$0.38 \times 0.08$</td>
</tr>
<tr>
<td>H' W</td>
<td>0.08</td>
<td>H W'</td>
<td>0.38</td>
<td>$0.08 \times 0.38$</td>
</tr>
<tr>
<td>H W</td>
<td>0.32</td>
<td>H' W'</td>
<td>0.22</td>
<td>$0.32 \times 0.22$</td>
</tr>
<tr>
<td>H' W'</td>
<td>0.22</td>
<td>H W</td>
<td>0.32</td>
<td>$0.22 \times 0.32$</td>
</tr>
</tbody>
</table>

$P(\text{only one husband and only one wife is retired}) = (0.38 \times 0.08 + 0.32 \times 0.22) \times 2 = 0.2016$