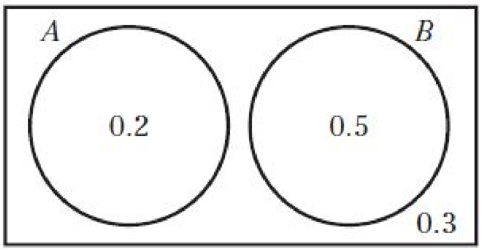
Probability 5C

1 a



b $P(A \cup B) = 0.7$

c $P(A' \cap B') = 0.3$

2 $P(\text{Sum of } 4) = \frac{3}{36} = \frac{1}{12}$

$P(\text{Same number}) = \frac{6}{36} = \frac{1}{6}$

$P(\text{Sum of } 4) + P(\text{Same number}) = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}$

$P(\text{Sum of } 4 \text{ or same number}) = \frac{8}{36} = \frac{2}{9}$

$P(\text{Sum of } 4) + P(\text{Same number}) \neq P(\text{Sum of 4 or same number})$,
so the events are not mutually exclusive.

Alternatively: A roll of 2 followed by another roll of 2 fits both conditions, so the intersection is not empty, and the events are not mutually exclusive.

3 P(A and B) = P(A) × P(B) = 0.5 × 0.3 = 0.15

4 P(A and B) = P(A) × P(B)

P(B) = P(A and B) ÷ P(A) = 0.045 ÷ 0.15 = 0.3

5 a The closed curves representing bricks and trains do not overlap and so they are mutually exclusive.

b P(B and F) = $\frac{1}{3+1+4+6+2+5} = \frac{1}{21}$

P(B) × P(F) = $\frac{3+1}{21} \times \frac{1+4+6}{21} = \frac{4}{21} \times \frac{11}{21} = \frac{44}{441}$

As P(B and F) ≠ P(B) × P(F), 'plays with bricks' and 'plays with action figures' are not independent events.

6 a 0.4 + x + 0.3 + 0.05 = 1

x = 0.25

6 b P(A and B) = x = 0.25

P(A) × P(B) = 0.65 × 0.55 = 0.3575

As P(A and B) ≠ P(A) × P(B), the two events 'like pasta' and 'like pizza' are not independent.

7 a P(S and T) = P(S) – P(S but not T) = 0.3 − 0.18 = 0.12

P(S) × P(T) = 0.3 × 0.4 = 0.12

As P(S and T) = P(S) × P(T), S and T are independent events.

b i P(S and T) = 0.12, as above.

ii P(neither S nor T) = 1 – P(S or T) = 1 – (P(S but not T) + P(T)) = 1 – (0.18 + 0.4) = 0.42

8 P(W and X) = P(W) – P(W and not X) = 0.5 − 0.25 = 0.25

P(X) = 1 – (P(W and not X) + P(neither W nor X)) = 1 – (0.25 + 0.3) = 0.45

P(W) × P(X) = 0.5 × 0.45 = 0.225

As P(W and X) ≠ P(W) × P(X), the two events W and X are not independent.

9 a P(A or R) = P(A) + P(R) = 0.6 because A and R are mutually exclusive.

0.2 + 0.25 + x = 0.6, so x = 0.15

$y = 1 - (0.2 + 0.25 + 0.15 + 0.1) = 0.3$

$(x, y) = (0.15, 0.3)$

b P(R and F) = x = 0.15

P(R) × P(F) = 0.4 × 0.45 = 0.18

As P(R and F) ≠ P(R) × P(F), the two events R and F are not independent.

10 P(A and B) = p

P(A) × P(B) = (0.42 + p) × (p + 0.11)

$= (p + 0.42)(p + 0.11)$

As the events A and B are independent, P(A and B) = P(A) × P(B), so

$(p + 0.42)(p + 0.11) = p$

$p^2 + 0.53p + 0.0462 = p$

$p^2 - 0.47p + 0.0462 = 0$, a quadratic in p, which we can solve with the quadratic formula

10 $p = \dfrac{0.47 \pm \sqrt{(-0.47)^2 - 4(1)(0.0462)}}{2(1)}$

$p = \dfrac{0.47 \pm 0.19}{2}$

$p = 0.33$ or 0.14

When $p = 0.14$, $q = 1 - (0.42 + 0.14 + 0.11) = 0.33$

When $p = 0.33$, $q = 1 - (0.42 + 0.33 + 0.11) = 0.14$

$(p, q) = (0.14, 0.33)$ or $(0.33, 0.14)$

Challenge

a Set P(A) = p and P(B) = q

As A and B are independent events, P(A and B) = P(A) × P(B) = pq

P(A and not B) = P(A) − P(A and B) = $p - pq$, and notice P(not B) = 1 – P(B) = 1 - q

Then P(A) × P(not B) = $p(1 - q) = p - pq$ = P(A and not B)

As P(A and not B) = P(A) × P(not B), the events A and 'not B' are independent.

b Still using p and q as above,

P(not A and not B) = 1 − P(A or B)

P(A or B) = P(A) + P(B) − P(A and B)

Meaning P(not A and not B) = 1 − P(A) − P(B) + P(A and B) = $1 - p - q + pq = (1 - p)(1 - q)$

Remember P(not A) = $1 - p$ and P(not B) = $1 - q$

So P(not A) × P(not B) = $(1 - p)(1 - q)$ = P(not A and not B)

As P(not A and not B) = P(not A) × P(not B), the events 'not A' and 'not B' are independent.