1 a

\[ P(A \cup B) = 0.7 \]

b \[ P(A' \cap B') = 0.3 \]

c \[ P(A') = 0.3 \]

2

\[ P(\text{Sum of } 4) = \frac{3}{36} = \frac{1}{12} \]

\[ P(\text{Same number}) = \frac{6}{36} = \frac{1}{6} \]

\[ P(\text{Sum of } 4) + P(\text{Same number}) = \frac{1}{12} + \frac{1}{6} = \frac{1}{4} \]

\[ P(\text{Sum of } 4 \text{ or same number}) = \frac{6}{36} = \frac{1}{6} \]

\[ P(\text{Sum of } 4) + P(\text{Same number}) \neq P(\text{Sum of } 4 \text{ or same number}), \]
so the events are not mutually exclusive.

Alternatively: A roll of 2 followed by another roll of 2 fits both conditions, so the intersection is not empty, and the events are not mutually exclusive.

3 \[ P(A \text{ and } B) = P(A) \times P(B) = 0.5 \times 0.3 = 0.15 \]

4 \[ P(A \text{ and } B) = P(A) \times P(B) \]

\[ P(B) = P(A \text{ and } B) \div P(A) = 0.045 \div 0.15 = 0.3 \]

5 a The closed curves representing bricks and trains do not overlap and so they are mutually exclusive.

b \[ P(B \text{ and } F) = \frac{1}{3+1+4+6+2+5} = \frac{1}{21} \]

\[ P(B) \times P(F) = \frac{3+1}{21} \times \frac{1+4+6}{21} = \frac{4}{21} \times \frac{11}{21} = \frac{44}{441} \]

As \[ P(B \text{ and } F) \neq P(B) \times P(F), \] 'plays with bricks' and 'plays with action figures' are not independent events.

6 a \[ 0.4 + x + 0.3 + 0.05 = 1 \]

\[ x = 0.25 \]
6  b  \(P(A \text{ and } B) = x = 0.25\)

\[P(A) \times P(B) = 0.65 \times 0.55 = 0.3575\]

As \(P(A \text{ and } B) \neq P(A) \times P(B)\), the two events ‘like pasta’ and ‘like pizza’ are not independent.

7  a  \(P(S \text{ and } T) = P(S) - P(S \text{ but not } T) = 0.3 - 0.18 = 0.12\)

\[P(S) \times P(T) = 0.3 \times 0.4 = 0.12\]

As \(P(S \text{ and } T) = P(S) \times P(T)\), \(S\) and \(T\) are independent events.

b  i  \(P(S \text{ and } T) = 0.12\), as above.

ii  \(P(\text{neither } S \text{ nor } T) = 1 - P(S \text{ or } T) = 1 - (P(S \text{ but not } T) + P(T)) = 1 - (0.18 + 0.4) = 0.42\)

8  \(P(W \text{ and } X) = P(W) - P(W \text{ and not } X) = 0.5 - 0.25 = 0.25\)

\[P(X) = 1 - (P(W \text{ and not } X) + P(\text{neither } W \text{ nor } X)) = 1 - (0.25 + 0.3) = 0.45\]

\[P(W) \times P(X) = 0.5 \times 0.45 = 0.225\]

As \(P(W \text{ and } X) \neq P(W) \times P(X)\), the two events \(W\) and \(X\) are not independent.

9  a  \(P(A \text{ or } R) = P(A) + P(R) = 0.6\) because \(A\) and \(R\) are mutually exclusive.

\[0.2 + 0.25 + x = 0.6, \text{ so } x = 0.15\]

\[y = 1 - (0.2 + 0.25 + 0.15 + 0.1) = 0.3\]

\[(x, y) = (0.15, 0.3)\]

b  \(P(R \text{ and } F) = x = 0.15\)

\[P(R) \times P(F) = 0.4 \times 0.45 = 0.18\]

As \(P(R \text{ and } F) \neq P(R) \times P(F)\), the two events \(R\) and \(F\) are not independent.

10  \(P(A \text{ and } B) = p\)

\[P(A) \times P(B) = (0.42 + p) \times (p + 0.11)\]

\[= (p + 0.42)(p + 0.11)\]

As the events \(A\) and \(B\) are independent, \(P(A \text{ and } B) = P(A) \times P(B)\), so

\[(p + 0.42)(p + 0.11) = p\]

\[p^2 + 0.53p + 0.0462 = p\]

\[p^2 - 0.47p + 0.0462 = 0, \text{ a quadratic in } p, \text{ which we can solve with the quadratic formula}\]
\[ p = \frac{0.47 \pm \sqrt{(-0.47)^2 - 4(1)(0.0462)}}{2(1)} \]

\[ p = \frac{0.47 \pm 0.19}{2} \]

\[ p = 0.33 \text{ or } 0.14 \]

When \( p = 0.14 \), \( q = 1 - (0.42 + 0.14 + 0.11) = 0.33 \)

When \( p = 0.33 \), \( q = 1 - (0.42 + 0.33 + 0.11) = 0.14 \)

\((p, q) = (0.14, 0.33) \text{ or } (0.33, 0.14)\)

**Challenge**

**a**  Set \( P(A) = p \) and \( P(B) = q \)

As \( A \) and \( B \) are independent events, \( P(A \text{ and } B) = P(A) \times P(B) = pq \)

\( P(A \text{ and not } B) = P(A) - P(A \text{ and } B) = p - pq, \) and notice \( P(\text{not } B) = 1 - P(B) = 1 - q \)

Then \( P(A) \times P(\text{not } B) = p(1 - q) = p - pq = P(A \text{ and not } B) \)

As \( P(A \text{ and not } B) = P(A) \times P(\text{not } B), \) the events \( A \) and 'not \( B \)' are independent.

**b**  Still using \( p \) and \( q \) as above,

\( P(\text{not } A \text{ and not } B) = 1 - P(A \text{ or } B) \)

\( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \)

Meaning \( P(\text{not } A \text{ and not } B) = 1 - P(A) - P(B) + P(A \text{ and } B) = 1 - p - q + pq = (1 - p)(1 - q) \)

Remember \( P(\text{not } A) = 1 - p \) and \( P(\text{not } B) = 1 - q \)

So \( P(\text{not } A) \times P(\text{not } B) = (1 - p)(1 - q) = P(\text{not } A \text{ and not } B) \)

As \( P(\text{not } A \text{ and not } B) = P(\text{not } A) \times P(\text{not } B), \) the events 'not \( A \)' and 'not \( B \)' are independent.