Practice paper

1 a $4 = \sqrt[3]{64} = 4^{\frac{1}{3}}$

so $n = \frac{1}{3}$

b $\sqrt{50} = \sqrt{25\times 2}$

$= \sqrt{25}\times\sqrt{2}$

$= 5\sqrt{2}$

2 $2x - 3y + 4 = 0$

$3y = 2x + 4$

$y = \frac{2}{3}x + \frac{4}{3}$

The gradient of this line is $\frac{2}{3}$.

The equation of the parallel line is:

$y - y_1 = m(x - x_1)$

$y - 6 = \frac{2}{3}(x - 5)$

$y - 6 = \frac{2}{3}x - \frac{10}{3}$

$y = \frac{2}{3}x + \frac{8}{3}$

3 a Error 1: $-\frac{3}{\sqrt{x}} = -3x^{-\frac{1}{2}}$, not $-3x^{\frac{1}{2}}$

Error 2: $\left[\frac{x^5}{5} - 2x^{\frac{3}{2}} + 2x\right]_1^2$

$= \left(\frac{32}{5} - 2\sqrt{8} + 4\right) - \left(\frac{1}{5} - 2 + 2\right)$

not $\left(\frac{1}{5} - 2 + 2\right) - \left(\frac{32}{5} - 2\sqrt{8} + 4\right)$

b $\int_1^2 \left(x^4 - \frac{3}{\sqrt{x}} + 2\right) dx$

$= \int_1^2 \left(x^4 - 3x^{-\frac{1}{2}} + 2\right) dx$

$= \left[\frac{x^5}{5} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + 2x\right]_1^2$

$= \left[\frac{x^5}{5} - 6x^{\frac{1}{2}} + 2x\right]_1^2$

$= \left(\frac{32}{5} - 6\sqrt{2} + 4\right) - \left(\frac{1}{5} - 6 + 2\right)$

$= 5.71$ (3 s.f.)

4 $2\sin^2(2x) - \cos(2x) - 1 = 0$

$2(1 - \cos^2(2x)) - \cos(2x) - 1 = 0$

$2 - 2\cos^2(2x) - \cos(2x) - 1 = 0$

$2\cos^2(2x) + \cos(2x) - 1 = 0$

$(2\cos(2x) - 1)(\cos(2x) + 1) = 0$

So $\cos(2x) = \frac{1}{2}$ or $\cos(2x) = -1$

As $0 \leq x \leq 180°$, $0 \leq 2x \leq 360°$

When $\cos(2x) = \frac{1}{2}$, $2x = 60°$

or $2x = 360° - 60° = 300°$

giving $x = 30°$ or $150°$

When $\cos(2x) = -1$, $2x = 180°$

giving $x = 90°$

$x = 30°, 90°$ or $150°$

5 a Volume $= x \times (x + 3) \times 2x$

$= 2x^2(x + 3)$

b $2x^2(x + 3) = 980$

$2x^3 + 6x^2 = 980$

$2x^3 + 6x^2 - 980 = 0$

$x^3 + 3x^2 - 490 = 0$ (as required)

c $f(x) = x^3 + 3x^2 - 490$

$f(7) = (7)^3 + 3(7)^2 - 490$

$= 343 + 147 - 490$

$= 0$

So $x = 7$ is a solution to $x^3 + 3x^2 - 490 = 0$.

d

$$\begin{array}{r}
x^2 + 10x + 70 \\
x - 7 \overline{\big)\, x^3 + 3x^2 + 0x - 490} \\
\underline{x^3 - 7x^2} \qquad\qquad \\
10x^2 + 0x \qquad \\
\underline{10x^2 - 70x} \qquad \\
70x - 490 \\
\underline{70x - 490} \\
0
\end{array}$$

$x^3 + 3x^2 - 490 = (x - 7)(x^2 + 10x + 70)$

Using the discriminant for $x^2 + 10x + 70$:

$b^2 - 4ac = (10)^2 - 4(1)(70)$

$= 100 - 280$

$= -180$

As $-180 < 0$, there are no real solutions to $x^2 + 10x + 70$.

Therefore, there are no other real solutions to the equation $x^3 + 3x^2 - 490 = 0$.

6 $f(x) = x^3 - 5x^2 - 2 + x^{-2}$
$f'(x) = 3x^2 - 10x - 2x^{-3}$
When $x = -1$, gradient of curve
$= f'(-1)$
$= 3(-1)^2 - 10(-1) - \frac{2}{(-1)^3}$
$= 3 + 10 + 2$
$= 15$
So gradient of normal is $-\frac{1}{15}$.
When $x = -1$,
$y = (-1)^3 - 5(-1)^2 - 2 + \frac{1}{(-1)^2}$
$= -7$
The equation of the normal is:
$y - y_1 = m(x - x_1)$
$y + 7 = -\frac{1}{15}(x + 1)$
$15y + 105 = -x - 1$
$x + 15y + 106 = 0$

7 a $P = ab^t$
$\log_{10} P = \log_{10}(ab^t)$
$= \log_{10} a + \log_{10} b^t$
$= \log_{10} a + t\log_{10} b$
Gradient of line $= \log_{10} b$
$= \frac{2.2 - 2}{20 - 0}$
$= \frac{0.2}{20}$
$= 0.01$
Intercept $= \log_{10} a = 2$
Equation of line l is $\log_{10} P = 0.01t + 2$.

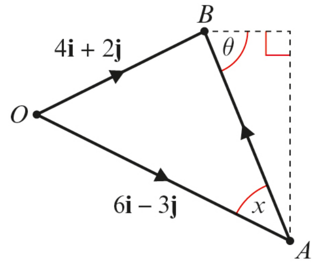
b $\log_{10} a = 2$
$a = 10^2$
$= 100$
100 is the initial population of Caledonian owlet-nightjars.

c $\log_{10} b = 0.01$
$b = 10^{0.01}$
$= 1.023$ (3 d.p.)

d $P = 100 \times 1.023^{30}$
$= 197.8\ldots$
The population when $t = 30$ is 198.

8 LHS
$= 1 + \cos^4 x - \sin^4 x$
$= 1 + (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$
$= 1 + \cos^2 x - \sin^2 x$
$= 1 - \sin^2 x + \cos^2 x$
$= \cos^2 x + \cos^2 x$
$= 2\cos^2 x$
$=$ RHS

9



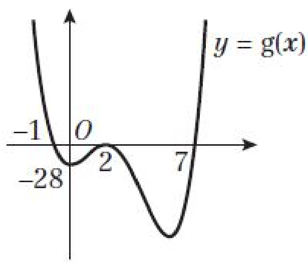
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= 4\mathbf{i} + 2\mathbf{j} - (6\mathbf{i} - 3\mathbf{j})$
$= -2\mathbf{i} + 5\mathbf{j}$
$|\overrightarrow{AB}| = \sqrt{2^2 + 5^2} = \sqrt{29}$
$\tan x = \frac{2}{5}$
$x = 21.8°$
$\theta = 21.8° + 90° = 111.8°$
$= 112°$ (3 s.f.)

10 a Using the cosine rule
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
$= \frac{12.7^2 + 7.5^2 - 10.6^2}{2(12.7)(7.5)}$
$= \frac{105.18}{190.5}$
$= 0.55212\ldots$
Angle $BAC = 56.5°$ (3 s.f.)

b Cost
$=$ area of lawn $\times 1.25$
$= \frac{1}{2}bc\sin A \times 1.25$
$= \frac{1}{2} \times 12.7 \times 7.5 \times \sin 56.4870... \times 1.25$
$= 49.6348...$
$=$ £49.63

11 a $y = g(x) = (x - 2)^2(x + 1)(x - 7)$
$0 = (x - 2)^2(x + 1)(x - 7)$
So $x = 2$, $x = -1$ or $x = 7$

11 a The curve touches the x-axis at (2, 0) and crosses it at (−1, 0) and (7, 0).
When $x = 0$, $y = (-2)^2 \times 1 \times (-7) = -28$
So the curve crosses the y-axis at (0, −28).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$



b $g(x+3) = (x+3-2)^2(x+3+1)(x+3-7)$
$= (x+1)^2(x+4)(x-4)$
$0 = (x+1)^2(x+4)(x-4)$
The roots of $g(x+3)$ are
$x = -1$, $x = -4$ and $x = 4$.

12 $9^{2x} = 27^{x^2-5}$
$(3^2)^{2x} = (3^3)^{x^2-5}$
So $2(2x) = 3(x^2-5)$
$4x = 3x^2 - 15$
$3x^2 - 4x - 15 = 0$
$(3x+5)(x-3) = 0$
So $x = -\frac{5}{3}$ or $x = 3$

13 a $f(x)$
$= (1-3x)^5$
$= 1^5 + \binom{5}{1}1^4(-3x) + \binom{5}{2}1^3(-3x)^2 + \ldots$
$= 1 - 15x + 90x^2$

b $1 - 3x = 0.97$
$3x = 0.03$
$x = 0.01$
Substituting $x = 0.01$ into the expansion for $(1-3x)^5$:
$0.97^5 \approx 1 - 15(0.01) + 90(0.01)^2$
$= 0.859$

c This approximation is greater than the true value as the next term will be negative and the subsequent positive terms will be smaller.

14 a $f'(x) = \dfrac{\sqrt{x} - x^2 - 1}{x^2}$
$= x^{-\frac{3}{2}} - 1 - x^{-2}$
$f(x) = \int (x^{-\frac{3}{2}} - 1 - x^{-2})\,dx$
$= \dfrac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - x - \dfrac{x^{-1}}{-1} + c$
$= -\dfrac{2}{\sqrt{x}} - x + \dfrac{1}{x} + c$
$= -\dfrac{2\sqrt{x} + x^2 - 1}{x} + c$
$= -\dfrac{x^2 + 2\sqrt{x} - 1}{x} + c$

b $f(3) = -1$
$-1 = -\dfrac{3^2 + 2\sqrt{3} - 1}{3} + c$
$c = -1 + \dfrac{8 + 2\sqrt{3}}{3}$
$= \dfrac{5 + 2\sqrt{3}}{3}$
$= \dfrac{5}{3} + \dfrac{2}{3}\sqrt{3}$
$p = \frac{5}{3}$, $q = \frac{2}{3}$, $r = 3$

15 a Substituting $x = 5$ and $y = 1$ into the equation for C:
$5^2 + 1^2 - 4(5) + 6(1) = 25 + 1 - 20 + 6$
$= 12$
Therefore, A lies on C.
Rearranging the equation:
$x^2 - 4x + y^2 + 6y = 12$
Completing the square:
$(x-2)^2 - 4 + (y+3)^2 - 9 = 12$
$(x-2)^2 + (y+3)^2 = 25$
The circle has centre (2, −3) and radius 5.

b Gradient of the radius at $A(5, 1)$
$= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1+3}{5-2} = \dfrac{4}{3}$
Gradient of the tangent $= -\frac{3}{4}$
Equation of the tangent at A:
$y - y_1 = m(x - x_1)$
$y - 1 = -\frac{3}{4}(x - 5)$
$y = -\frac{3}{4}x + \frac{19}{4}$

15 c Solving the simultaneous equations
$y = x^2 - 2$ and $y = -\frac{3}{4}x + \frac{19}{4}$ to find P and Q:

$x^2 - 2 = -\frac{3}{4}x + \frac{19}{4}$

$4x^2 - 8 = -3x + 19$

$4x^2 + 3x - 27 = 0$

$(4x - 9)(x + 3) = 0$

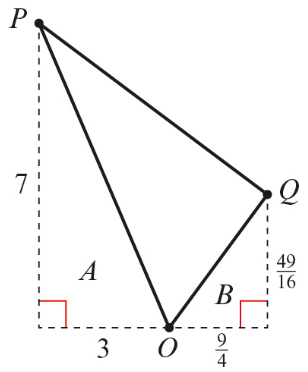
$x = \frac{9}{4}$ or $x = -3$

When $x = \frac{9}{4}$, $y = -\frac{3}{4}\left(\frac{9}{4}\right) + \frac{19}{4} = \frac{49}{16}$

When $x = -3$, $y = -\frac{3}{4}\left(-3\right) + \frac{19}{4} = 7$

The point P is $(-3, 7)$ and
the point Q is $(\frac{9}{4}, \frac{49}{16})$.

Draw a diagram



Area of triangle POQ
= area of trapezium − area of triangle a
− area of triangle b

Area of trapezium $= \frac{1}{2} \times \frac{21}{4} \times \left(7 + \frac{49}{16}\right)$
$= 26\frac{53}{128}$

Area of triangle $a = \frac{1}{2} \times 3 \times 7$
$= 10\frac{1}{2}$

Area of triangle $b = \frac{1}{2} \times \frac{9}{4} \times \frac{49}{16}$
$= 3\frac{57}{128}$

Area of triangle $POQ = 26\frac{53}{128} - 10\frac{1}{2} - 3\frac{57}{128}$
$= 12\frac{15}{32}$