Exponentials and logarithms 14F

1 a
$$2^x = 75$$
$$\log 2^x = \log 75$$
$$x \log 2 = \log 75$$
$$x = \frac{\log 75}{\log 2}$$
$$= 6.23 \ (3 \text{ s.f.})$$

b
$$3^x = 10$$
$$\log 3^x = \log 10$$
$$x \log 3 = \log 10$$
$$x = \frac{\log 10}{\log 3}$$
$$= 2.10 \ (3 \text{ s.f.})$$

c
$$5^x = 2$$
$$\log 5^x = \log 2$$
$$x \log 5 = \log 2$$
$$x = \frac{\log 2}{\log 5}$$
$$= 0.431 \ (3 \text{ s.f.})$$

d
$$4^{2x} = 100$$
$$\log 4^{2x} = \log 100$$
$$2x \log 4 = \log 100$$
$$x = \frac{\log 100}{2 \log 4}$$
$$= 1.66 \ (3 \text{ s.f.})$$

e
$$9^{x+5} = 50$$
$$\log 9^{x+5} = \log 50$$
$$(x+5) \log 9 = \log 50$$
$$x \log 9 + 5 \log 9 = \log 50$$
$$x \log 9 = \log 50 - 5 \log 9$$
$$x = \frac{\log 50 - 5 \log 9}{\log 9}$$
$$= -3.22 \ (3 \text{ s.f.})$$

f
$$7^{2x-1} = 23$$
$$\log 7^{2x-1} = \log 23$$
$$(2x-1) \log 7 = \log 23$$
$$2x \log 7 - \log 7 = \log 23$$
$$2x \log 7 = \log 23 + \log 7$$
$$x = \frac{\log 23 + \log 7}{2 \log 7}$$
$$= 1.31 \ (3 \text{ s.f.})$$

g
$$11^{3x-2} = 65$$
$$\log 11^{3x-2} = \log 65$$
$$(3x-2) \log 11 = \log 65$$
$$3x - 2 = \frac{\log 65}{\log 11}$$
$$= 1.740855$$
$$x = 1.25 \ (3 \text{ s.f.})$$

h
$$2^{3-2x} = 88$$
$$\log 2^{3-2x} = \log 88$$
$$(3-2x) \log 2 = \log 88$$
$$\log_2 88 = 3 - 2x$$
$$3 - 2x = 6.45943$$
$$x = -1.73 \ (3 \text{ s.f.})$$

2 a Let $y = 2^x$
$$y^2 - 6y + 5 = 0$$
$$(y-1)(y-5) = 0$$
So $y = 1$ or $y = 5$
If $y = 1$, $2^x = 1$, $x = 0$
If $y = 5$, $2^x = 5$
$$\log 2^x = \log 5$$
$$x \log 2 = \log 5$$
$$x = \frac{\log 5}{\log 2}$$
$$x = 2.32 \ (3 \text{ s.f.})$$
So $x = 0$ or $x = 2.32$

2 b Let $y = 3^x$

$y^2 - 15y + 44 = 0$

$(y-4)(y-11) = 0$

So $y = 4$ or $y = 11$

If $y = 4$, $3^x = 4$

$\log 3^x = \log 4$

$x \log 3 = \log 4$

$x = \dfrac{\log 4}{\log 3}$

$x = 1.26$ (3 s.f.)

If $y = 11$, $3^x = 11$

$\log 3^x = \log 11$

$x \log 3 = \log 11$

$x = \dfrac{\log 11}{\log 3}$

$x = 2.18$ (3 s.f.)

So $x = 1.26$ or $x = 2.18$

c Let $y = 5^x$

$y^2 - 6y - 7 = 0$

$(y+1)(y-7) = 0$

So $y = -1$ or $y = 7$

If $y = -1$, $5^x = -1$. No Solution.

If $y = 7$, $5^x = 7$

$\log 5^x = \log 7$

$x \log 5 = \log 7$

$x = \dfrac{\log 7}{\log 5}$

$x = 1.21$ (3 s.f.)

d Let $y = 3^x$

$(3^x)^2 + (3^x \times 3) - 10 = 0$

$y^2 + 3y - 10 = 0$

$(y+5)(y-2) = 0$

So $y = -5$ or $y = 2$

If $y = -5$, $3^x = -5$. No Solution.

If $y = 2$, $3^x = 2$

$\log 3^x = \log 2$

$x \log 3 = \log 2$

d $x = \dfrac{\log 2}{\log 3}$

$x = 0.631$ (3 s.f.)

e Let $y = 7^x$

$(7^x)^2 + 12 = 7^x \times 7$

$y^2 + 12 = 7y$

$y^2 - 7y + 12 = 0$

$(y-3)(y-4) = 0$

So $y = 3$ or $y = 4$

If $y = 3$, $7^x = 3$

$x \log 7 = \log 3$

$x = \dfrac{\log 3}{\log 7}$

$x = 0.565$ (3 s.f.)

If $y = 4$, $7^x = 4$

$x \log 7 = \log 4$

$x = \dfrac{\log 4}{\log 7}$

$x = 0.712$ (3 s.f.)

So $x = 0.565$ or $x = 0.712$

f Let $y = 2^x$

Then $y^2 + 3y - 4 = 0$

So $(y+4)(y-1) = 0$

So $y = -4$ or $y = 1$

$2^x = -4$ has no solution.

Therefore $2^x = 1$

So $x = 0$ is the only solution.

g Let $y = 3^x$

Then $3y^2 - 26y - 9 = 0$

So $(3y+1)(y-9) = 0$

So $y = -\frac{1}{3}$ or $y = 9$

$3^x = -\frac{1}{3}$ has no solution.

Therefore $3^x = 9$

So $x = 2$ is the only solution.

2 h Let $y = 3^x$

Then $12y^2 + 17y - 7 = 0$

So $(3y - 1)(4y + 7) = 0$

So $y = \frac{1}{3}$ or $y = -\frac{7}{4}$

$3^x = -\frac{7}{4}$ has no solution.

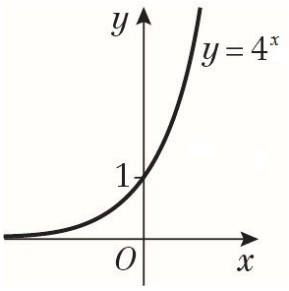
Therefore $3^x = \frac{1}{3}$

So $x = -1$ is the only solution.

3 a
$$3^{x+1} = 2000$$
$$\log_3 2000 = x + 1$$
$$x + 1 = 6.9186$$
$$x = 5.92 \text{ (3 s.f.)}$$

b
$$5^{-1} = x - 3$$
$$x - 3 = \tfrac{1}{5}$$
$$x = 3.2$$

4 a (0, 1)



b Let $y = 4^x$

$4^{2x} - 10(4^x) + 16 = 0$

$y^2 - 10y + 16 = 0$

$(y - 2)(y - 8) = 0$

$y = 2$ or $y = 8$

Therefore, $4^x = 2$ or $4^x = 8$

$\log_4 2 = x$ or $\log_4 8 = x$

$x = \frac{1}{2}$ or $x = \frac{3}{2}$

5 a
$$5^x = 2^{x+1}$$
$$\log 5^x = \log 2^{x+1}$$
$$x\log 5 = (x + 1)\log 2$$
$$x\log 5 = x\log 2 + \log 2$$
$$x\log 5 - x\log 2 = \log 2$$
$$x(\log 5 - \log 2) = \log 2$$
$$x = \frac{\log 2}{\log 5 - \log 2}$$
$$x = 0.7565 \text{ (4 d.p.)}$$

b
$$3^{x+5} = 6^x$$
$$\log 3^{x+5} = \log 6^x$$
$$(x + 5)\log 3 = x\log 6$$
$$x\log 3 + 5\log 3 = x\log 6$$
$$5\log 3 = x\log 6 - x\log 3$$
$$5\log 3 = x(\log 6 - \log 3)$$
$$x = \frac{5\log 3}{\log 6 - \log 3}$$
$$x = 7.9248 \text{ (4 d.p.)}$$

c
$$7^{x+1} = 3^{x+2}$$
$$\log 7^{x+1} = \log 3^{x+2}$$
$$(x + 1)\log 7 = (x + 2)\log 3$$
$$x\log 7 + \log 7 = x\log 3 + 2\log 3$$
$$x\log 7 - x\log 3 = 2\log 3 - \log 7$$
$$x(\log 7 - \log 3) = 2\log 3 - \log 7$$
$$x = \frac{2\log 3 - \log 7}{\log 7 - \log 3}$$
$$x = 0.2966 \text{ (4 d.p.)}$$