Exponentials and logarithms 14B

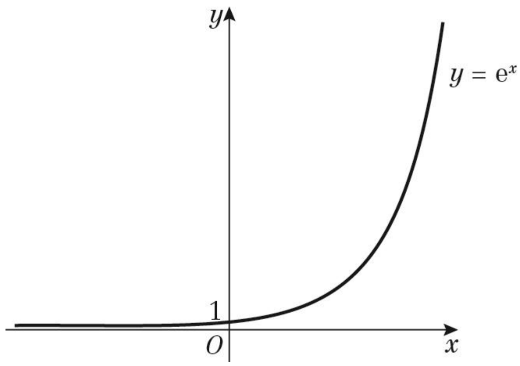
1 **a** 2.71828

b 54.59815

c 0.00005

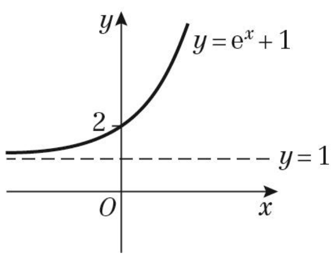
d 1.22140

2 **a**



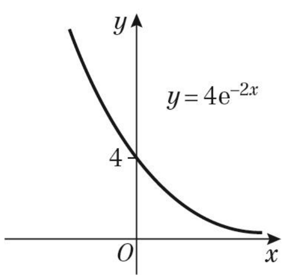
e = 2.71828…
e^3 = 20.08553…

3 **a** $y = e^x + 1$



This is the normal $y = e^x$ ‘moved up’ (translated) 1 unit

b $y = 4e^{-2x}$



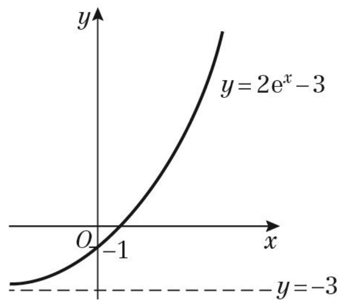
$x = 0 \Rightarrow y = 4$

As $x \to -\infty, y \to \infty$

As $x \to \infty, y \to 0$

This is an exponential decay type of graph.

c $y = 2e^x - 3$

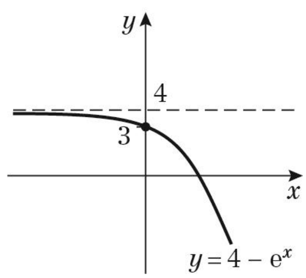


$x = 0 \Rightarrow y = 2\times1 - 3 = -1$

As $x \to \infty,\ y \to \infty$

As $x \to -\infty,\ y \to 2\times0 - 3 = -3$

d $y = 4 - e^x$

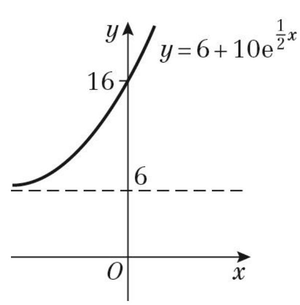


$x = 0 \Rightarrow y = 4 - 1 = 3$

As $x \to \infty,\ y \to 4 - \infty$, i.e. $y \to -\infty$

As $x \to -\infty,\ y \to 4 - 0 = 4$

e $y = 6 + 10e^{\frac{1}{2}x}$

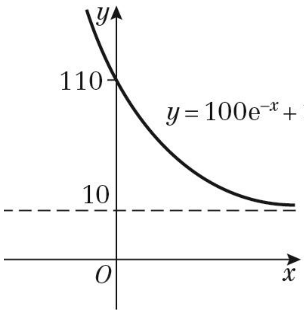


3 e $x = 0 \Rightarrow y = 6 + 10 \times 1 = 16$

As $x \to \infty, y \to \infty$

As $x \to -\infty, y \to 6 + 10 \times 0 = 6$

f $y = 100e^{-x} + 10$



$x = 0 \Rightarrow y = 100 \times 1 + 10 = 110$

As $x \to \infty,\ y \to 100 \times 0 + 10 = 10$

As $x \to -\infty,\ y \to \infty$

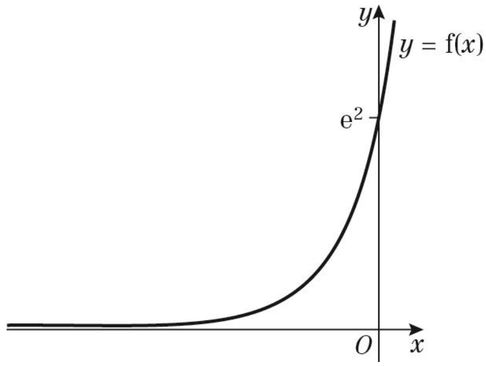
4 a The graph is increasing so b is positive.
The line $y = 5$ is an asymptote, so $C = 5$.
When $x = 0$, $6 = Ae^{b \times 0} + C = A + 5$, so $A = 1$.

b The graph is decreasing so b is negative.
The line $y = 0$ is an asymptote, so $C = 0$.
When $x = 0$, $4 = Ae^{b \times 0} + C = A + 0$, so $A = 4$.

c The graph is increasing so b is positive.
The line $y = 2$ is an asymptote, so $C = 2$.
When $x = 0$, $8 = Ae^{b \times 0} + C = A + 2$, so $A = 6$.

5 $f(x) = e^{3x + 2}$
$= e^{3x} \times e^2$
$= e^2 e^{3x}$
$A = e^2$ and $b = 3$

5



6 a $y = e^{6x}$

$\frac{dy}{dx} = 6e^{6x}$

b $y = e^{-\frac{1}{3}x}$

$\frac{dy}{dx} = -\frac{1}{3}e^{-\frac{1}{3}}$

c $y = 7e^{2x}$

$\frac{dy}{dx} = 2 \times 7e^{2x} = 14e^{2x}$

d $y = 5e^{0.4x}$

$\frac{dy}{dx} = 0.4 \times 5e^{0.4x} = 2e^{0.4x}$

e $y = e^{3x} + 2e^x$

$\frac{dy}{dx} = 3e^{3x} + 2e^x$

f $y = e^x(e^x + 1) = e^{2x} + e^x$

$\frac{dy}{dx} = 2e^{2x} + e^x$

7 a $y = e^{3x}$

$\frac{dy}{dx} = 3e^{3x}$

When $x = 2$,

$\frac{dy}{dx} = 3e^{3 \times 2} = 3e^6$

7 **b** When $x = 0$,

$$\frac{dy}{dx} = 3e^{3 \times 0} = 3$$

c When $x = -0.5$,

$$\frac{dy}{dx} = 3e^{3 \times -0.5} = 3e^{-1.5}$$

8 $f(x) = e^{0.2x}$
$f'(x) = 0.2e^{0.2x}$
The gradient of the tangent when $x = 5$
is $f'(5) = 0.2e^{0.2 \times 5} = 0.2e$
$f(5) = e^{0.2 \times 5} = e$
The equation of the tangent in the form
$y = mx + c$
is $e = 0.2e \times 5 + c$
$e = e + c$
so $c = 0$
Therefore the tangent to the curve at the point $(5, c)$ is in the form $y = mx$.
Thus it so goes through the origin.