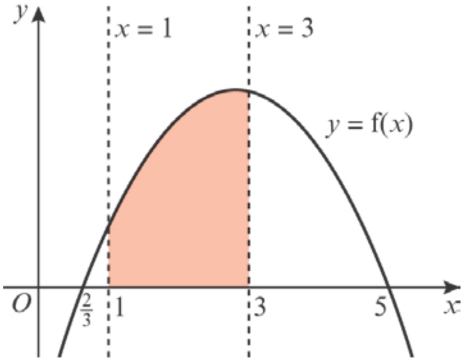
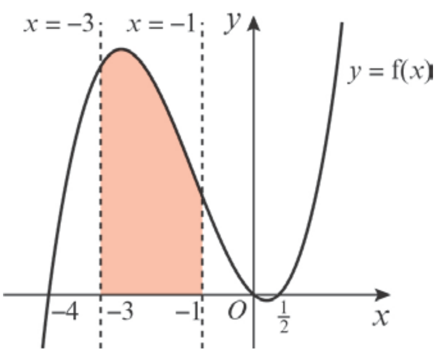
Integration 13E

1 a $-3x^2 + 17x - 10 = 0$
$(-3x + 2)(x - 5) = 0$
$x = \frac{2}{3}$ or $x = 5$



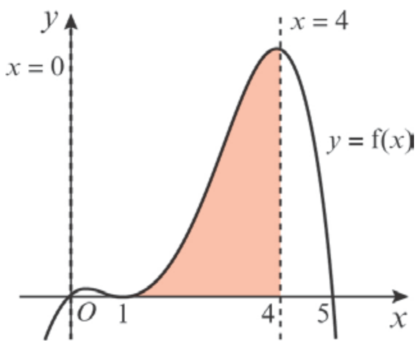
$$\int_1^3 (-3x^2 + 17x - 10)\,dx$$
$$= \left[\frac{-3x^3}{3} + \frac{17x^2}{2} - 10x\right]_1^3$$
$$= \left[-x^3 + \frac{17x^2}{2} - 10x\right]_1^3$$
$$= \left(-(3)^3 + \frac{17(3)^2}{2} - 10(3)\right) - \left(-(1)^3 + \frac{17(1)^2}{2} - 10(1)\right)$$
$$= \left(-27 + \frac{153}{2} - 30\right) - \left(-1 + \frac{17}{2} - 10\right)$$
$$= 22$$

b $2x^3 + 7x^2 - 4x = 0$
$x(2x^2 + 7x - 4) = 0$
$x(2x - 1)(x + 4) = 0$
$x = 0, x = \frac{1}{2}$ or $x = -4$



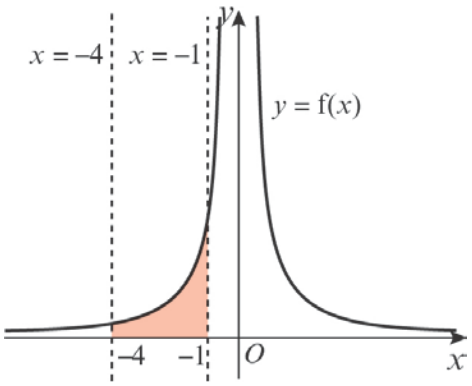
b $$\int_{-3}^{-1} (2x^3 + 7x^2 - 4x)\,dx$$
$$= \left[\frac{2x^4}{4} + \frac{7x^3}{3} - \frac{4x^2}{2}\right]_{-3}^{-1}$$
$$= \left(\frac{(-1)^4}{2} + \frac{7(-1)^3}{3} - 2(-1)^2\right) - \left(\frac{(-3)^4}{2} + \frac{7(-3)^3}{3} - 2(-3)^2\right)$$
$$= \left(\frac{1}{2} - \frac{7}{3} - 2\right) - \left(\frac{81}{2} - \frac{189}{3} - 18\right)$$
$$= 36\tfrac{2}{3}$$

c $-x^4 + 7x^3 - 11x^2 + 5x = 0$
$-x(x - 1)^2(x - 5) = 0$
$x = 0, x = 1$ or $x = 5$



$$\int_0^4 (-x^4 + 7x^3 - 11x^2 + 5x)\,dx$$
$$= \left[-\frac{x^5}{5} + \frac{7x^4}{4} - \frac{11x^3}{3} + \frac{5x^2}{2}\right]_0^4$$
$$= \left(-\frac{4^5}{5} + \frac{7(4)^4}{4} - \frac{11(4)^3}{3} + \frac{5(4)^2}{2}\right) - \left(-\frac{0^5}{5} + \frac{7(0)^4}{4} - \frac{11(0)^3}{3} + \frac{5(0)^2}{2}\right)$$
$$= \left(-\frac{1024}{5} + 448 - \frac{704}{3} + 40\right)$$
$$= 48\tfrac{8}{15}$$

1 d



$$\int_{-4}^{-1}\left(\frac{8}{x^2}\right)\,dx = \int_{-4}^{-1}(8x^{-2})\,dx$$
$$= \left[\frac{8x^{-1}}{-1}\right]_{-4}^{-1}$$
$$= \left[-\frac{8}{x}\right]_{-4}^{-1}$$
$$= \left(-\frac{8}{(-1)}\right) - \left(-\frac{8}{(-4)}\right)$$
$$= (8) - (2)$$
$$= 6$$

2 $A = \int_{-2}^{0} x(x^2 - 4)\,dx = \int_{-2}^{0}(x^3 - 4x)\,dx$

$$= \left(\frac{x^4}{4} - \frac{4x^2}{2}\right)_{-2}^{0}$$
$$= \left(\frac{x^4}{4} - 2x^2\right)_{-2}^{0}$$
$$= (0) - \left(\frac{16}{4} - 2\times 4\right)$$
$$= -4 + 8$$
$$= 4$$

3 $A = \int_{1}^{3}\left(3x + \frac{6}{x^2} - 5\right)dx$

$$= \int_{1}^{3}(3x + 6x^{-2} - 5)\,dx$$
$$= \left(\frac{3x^2}{2} + \frac{6x^{-1}}{-1} - 5x\right)_{1}^{3}$$
$$= \left(\frac{3}{2}x^2 - 6x^{-1} - 5x\right)_{1}^{3}$$

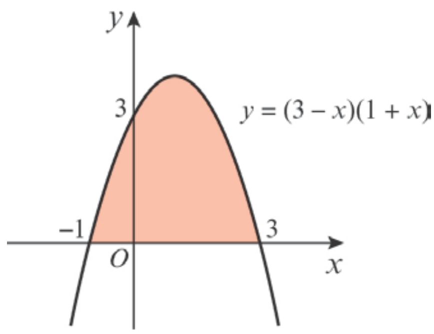
3 $A = \left(\frac{3}{2}\times 9 - \frac{6}{3} - 15\right) - \left(\frac{3}{2} - 6 - 5\right)$

$$= \frac{27}{2} - 17 - \frac{3}{2} + 11$$
$$= \frac{24}{2} - 6$$
$$= 6$$

4 $y = (3 - x)(1 + x)$ is ∩ shaped

$y = 0 \Rightarrow x = 3, -1$

$x = 0 \Rightarrow y = 3$

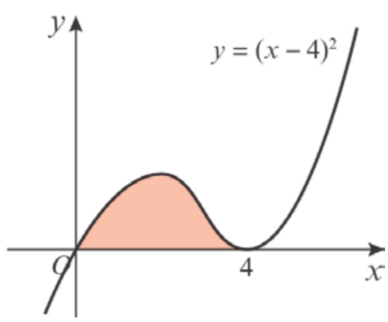


$$A = \int_{-1}^{3}(3 - x)(1 + x)\,dx$$
$$= \int_{-1}^{3}(3 + 2x - x^2)\,dx$$
$$= \left(3x + x^2 - \frac{x^3}{3}\right)_{-1}^{3}$$
$$= \left(9 + 9 - \frac{27}{3}\right) - \left(-3 + 1 + \frac{1}{3}\right)$$
$$= 9 + 1\tfrac{2}{3}$$
$$= 10\tfrac{2}{3}$$

5 $y = x(x - 4)^2$

$y = 0 \Rightarrow x = 0, 4$ (twice)

There is a turning point at $(4, 0)$.



5 Area $= \int_0^4 x(x-4)^2 \, dx$

$$= \int_0^4 x(x^2 - 8x + 16) \, dx$$

$$= \int_0^4 (x^3 - 8x^2 + 16x) \, dx$$

$$= \left(\frac{x^4}{4} - \frac{8x^3}{3} + 8x^2 \right)_0^4$$

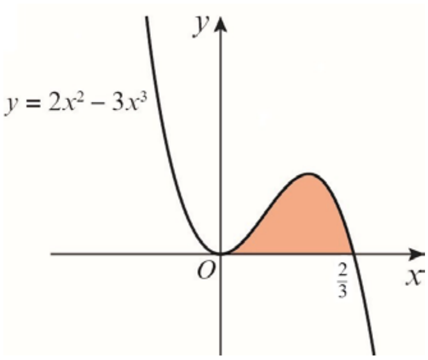
$$= \left(64 - \frac{8}{3} \times 64 + 128 \right) - (0)$$

$$= \tfrac{64}{3} \text{ or } 21\tfrac{1}{3}$$

6 $2x^2 - 3x^3 = 0$

$x^2(2 - 3x) = 0$

$x = 0$ or $x = \frac{2}{3}$



$$\int_0^{\frac{2}{3}} (2x^2 - 3x^3) \, dx = \left[\frac{2x^3}{3} - \frac{3x^4}{4} \right]_0^{\frac{2}{3}}$$

$$= \left(\frac{2(\frac{2}{3})^3}{3} - \frac{3(\frac{2}{3})^4}{4} \right) - \left(\frac{2(0)^3}{3} - \frac{3 \times 0^4}{4} \right)$$

$$= \tfrac{16}{81} - \tfrac{12}{81}$$

$$= \tfrac{4}{81}$$

7

$$\int_0^k (3x^2 - 2x + 2) \, dx = 8$$

$$\left[\frac{3x^3}{3} - \frac{2x^2}{2} + 2x \right]_0^k = 8$$

$$\left[x^3 - x^2 + 2x \right]_0^k = 8$$

$$(k^3 - k^2 + 2k) - (0^3 - 0^2 + 2(0)) = 8$$

$$k^3 - k^2 + 2k - 8 = 0$$

7 Using the factor theorem, $k = 2$ as

$2^3 - 2^2 + 2(2) - 8 = 0$

Therefore, $k = 2$

8 a $-x^2 + 2x + 3 = 0$

$(-x + 3)(x + 1) = 0$

$x = 3$ or $x = -1$

$A(-1, 0)$ and $B(3, 0)$

b $\int_{-1}^3 (-x^2 + 2x + 3) \, dx$

$$= \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 3x \right]_{-1}^3$$

$$= \left[-\frac{x^3}{3} + x^2 + 3x \right]_{-1}^3$$

$$= \left(-\frac{3^3}{3} + 3^2 + 3(3) \right) - \left(-\frac{(-1)^3}{3} + (-1)^2 + 3(-1) \right)$$

$$= (-9 + 9 + 9) - \left(\frac{1}{3} + 1 - 3 \right)$$

$$= 10\tfrac{2}{3}$$

9 $\int_0^2 x^2(2 - x) \, dx = \int_0^2 2x^2 - x^3 \, dx$

$$= \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2$$

$$= \left(\frac{2(2)^3}{3} - \frac{2^4}{4} \right) - \left(\frac{2(0)^3}{3} - \frac{0^4}{4} \right)$$

$$= \left(\frac{16}{3} - \frac{16}{4} \right)$$

$$= 1\tfrac{1}{3}$$

