

Integration 13B

1 a $\int x^3 dx = \frac{x^4}{4} + c$

b $\int x^7 dx = \frac{x^8}{8} + c$

c $\int 3x^{-4} dx = \frac{3x^{-3}}{-3} + c = -x^{-3} + c$

d $\int 5x^2 dx = \frac{5x^3}{3} + c$

2 a $\int (x^4 + 2x^3) dx = \frac{5x^5}{5} + \frac{2x^4}{4} + c$
 $= x^5 + \frac{x^4}{2} + c$

b $\int (2x^3 - x^2 + 5x) dx = \frac{2x^4}{4} - \frac{x^3}{3} + \frac{5x^2}{2} + c$
 $= \frac{x^4}{2} - \frac{x^3}{3} + \frac{5x^2}{2} + c$

c $\int (5x^{\frac{3}{2}} - 3x^2) dx = \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{3x^3}{3} + c$
 $= 2x^{\frac{5}{2}} - x^3 + c$

3 a $\int (4x^{-2} + 3x^{\frac{1}{2}}) dx = \frac{4x^{-1}}{-1} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c$
 $= -4x^{-1} + 6x^{\frac{3}{2}} + c$

b $\int (6x^{-1} - x^{\frac{1}{2}}) dx = \frac{6x^{-1}}{-1} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$
 $= -6x^{-1} - 2x^{\frac{3}{2}} + c$

c $\int (2x^{\frac{3}{2}} + x^2 - x^{\frac{1}{2}}) dx$
 $= \frac{2x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^3}{3} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$
 $= -4x^{\frac{5}{2}} + \frac{x^3}{3} - 2x^{\frac{3}{2}} + c$

4 a $\int (4x^3 - 3x^{-4} + r) dx = \frac{4x^4}{4} - \frac{3x^{-3}}{-3} + rx + c$
 $= x^4 + x^{-3} + rx + c$

b $\int (x + x^{\frac{1}{2}} + x^{-\frac{3}{2}}) dx = \frac{x^2}{2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$
 $= \frac{x^2}{2} + 2x^{\frac{3}{2}} - 2x^{-\frac{1}{2}} + c$

c $\int (px^4 + 2q + 3x^{-2}) dx$
 $= \frac{px^5}{5} + \frac{4qx^2}{2} + \frac{3x^{-1}}{-1} + c$
 $= \frac{px^5}{5} + 2qx^2 - 3x^{-1} + c$

5 a $\int (3t^2 - t^{-2}) dt = \frac{3t^3}{3} - \frac{t^{-1}}{-1} + c$
 $= t^3 + t^{-1} + c$

b $\int (2t^2 - 3t^{-\frac{3}{2}} + 1) dt = \frac{2t^3}{3} - \frac{3t^{-\frac{1}{2}}}{-\frac{1}{2}} + t + c$
 $= \frac{2t^3}{3} + 6t^{-\frac{1}{2}} + t + c$

c $\int (pt^3 + q^2 + pr^3) dt$
 $= \frac{pt^4}{4} + q^2t + pr^3t + c$

6 a $\int \frac{(2x^2 + 3)}{x^2} dx = \int \left(\frac{2x^2}{x^2} + \frac{3}{x^2} \right) dx$
 $= \int (2 + 3x^{-2}) dx$
 $= 2x + \frac{3x^{-1}}{-1} + c$
 $= 2x - \frac{3}{x} + c$

b $\int (2x + 3)^2 dx = \int (4x^2 + 12x + 9) dx$
 $= \frac{4x^3}{3} + 6x^2 + 9x + c$

$$\begin{aligned}
 6 \text{ c } \int (2x+3)\sqrt{x} \, dx &= \int (2x+3)x^{\frac{1}{2}} \, dx \\
 &= \int \left(2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) \, dx \\
 &= \frac{2x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c \\
 &= \frac{4x^{\frac{5}{2}}}{5} + 2x^{\frac{3}{2}} + c
 \end{aligned}$$

$$\begin{aligned}
 7 \text{ a } \int \left(x + \frac{1}{x} \right)^2 \, dx &= \int \left(x^2 + 2 + \frac{1}{x^2} \right) \, dx \\
 &= \int \left(x^2 + 2 + x^{-2} \right) \, dx \\
 &= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + c \\
 &= \frac{x^3}{3} + 2x - \frac{1}{x} + c
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \int (\sqrt{x} + 2)^2 \, dx &= \int (x + 4\sqrt{x} + 4) \, dx \\
 &= \int \left(x + 4x^{\frac{1}{2}} + 4 \right) \, dx \\
 &= \frac{x^2}{2} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + 4x + c \\
 &= \frac{x^2}{2} + \frac{8x^{\frac{3}{2}}}{3} + 4x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \int \left(\frac{1}{\sqrt{x}} + 2\sqrt{x} \right) \, dx &= \int \left(x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} \right) \, dx \\
 &= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + c \\
 &= 2x^{\frac{1}{2}} + \frac{4x^{\frac{3}{2}}}{3} + c
 \end{aligned}$$

$$\begin{aligned}
 8 \text{ a } \int \left(x^{\frac{2}{3}} + \frac{4}{x^3} \right) \, dx &= \int \left(x^{\frac{2}{3}} + 4x^{-3} \right) \, dx \\
 &= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{4x^{-2}}{-2} + c \\
 &= \frac{3x^{\frac{5}{3}}}{5} - \frac{2}{x^2} + c
 \end{aligned}$$

$$\begin{aligned}
 8 \text{ b } \int \left(\frac{2+x}{x^3} + 3 \right) \, dx &= \int (2x^{-3} + x^{-2} + 3) \, dx \\
 &= \frac{x^{-2}}{-2} + \frac{x^{-1}}{-1} + 3x + c \\
 &= -\frac{1}{x^2} - \frac{1}{x} + 3x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \int (x^2 + 3)(x-1) \, dx &= \int (x^3 - x^2 + 3x - 3) \, dx \\
 &= \frac{x^4}{4} - \frac{x^3}{3} + \frac{3x^2}{2} - 3x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{d } \int \frac{(2x+1)^2}{\sqrt{x}} \, dx &= \int \left(\frac{4x^2 + 4x + 1}{x^{\frac{1}{2}}} \right) \, dx \\
 &= \int (4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{-\frac{1}{2}}) \, dx \\
 &= \frac{8x^{\frac{5}{2}}}{5} + \frac{8x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}} + c
 \end{aligned}$$

$$\begin{aligned}
 \text{e } \int \left(3 + \frac{\sqrt{x} + 6x^3}{x} \right) \, dx &= \int \left(3 + x^{\frac{-1}{2}} + 6x^2 \right) \, dx \\
 &= 3x + 2x^{\frac{1}{2}} + 2x^3 + c
 \end{aligned}$$

$$\begin{aligned}
 \text{f } \int \sqrt{x}(\sqrt{x} + 3)^2 \, dx &= \int x^{\frac{1}{2}} \left(x + 6x^{\frac{1}{2}} + 9 \right) \, dx \\
 &= \int (x^{\frac{3}{2}} + 6x + 9x^{\frac{1}{2}}) \, dx \\
 &= \frac{2x^{\frac{5}{2}}}{5} + 3x^2 + 6x^{\frac{3}{2}} + c
 \end{aligned}$$

$$\begin{aligned}
 9 \text{ a } \int \left(\frac{A}{x^2} - 3 \right) \, dx &= \int (Ax^{-2} - 3) \, dx \\
 &= \frac{Ax^{-1}}{-1} - 3x + c \\
 &= -\frac{A}{x} - 3x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{b } \int \left(\sqrt{Px} + \frac{2}{x^3} \right) \, dx &= \int \left(\sqrt{Px}^{\frac{1}{2}} + 2x^{-3} \right) \, dx \\
 &= \frac{\sqrt{Px}^{\frac{3}{2}}}{\frac{3}{2}} + \frac{2x^{-2}}{-2} + c \\
 &= \frac{2\sqrt{Px}^{\frac{3}{2}}}{3} - \frac{1}{x^2} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{9 \ c} \quad & \int \left(\frac{p}{x^2} + q\sqrt{x} + r \right) dx \\
 &= \frac{px^{-1}}{-1} + \frac{qx^{\frac{3}{2}}}{\frac{3}{2}} + rx + c \\
 &= -\frac{p}{x} + \frac{2qx^{\frac{3}{2}}}{3} + rx + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{10} \quad & \int \left(\frac{6}{x^2} + 4\sqrt{x} - 3x + 2 \right) dx \\
 &= \int (6x^{-2} + 4x^{\frac{1}{2}} - 3x + 2) dx \\
 &= \frac{6x^{-1}}{-1} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3x^2}{2} + 2x + c \\
 &= -\frac{6}{x} + \frac{8x^{\frac{3}{2}}}{3} - \frac{3x^2}{2} + 2x + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{11} \quad & \int \left(8x^3 + 6x - \frac{3}{\sqrt{x}} \right) dx \\
 &= \int (8x^3 + 6x - 3x^{-\frac{1}{2}}) dx \\
 &= \frac{8x^4}{4} + \frac{6x^2}{2} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c \\
 &= 2x^4 + 3x^2 - 6\sqrt{x} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{12 \ a} \quad & (2 + 5\sqrt{x})^2 = (2 + 5\sqrt{x})(2 + 5\sqrt{x}) \\
 &= 4 + 10\sqrt{x} + 10\sqrt{x} + 25x \\
 &= 4 + 20\sqrt{x} + 25x
 \end{aligned}$$

So $k = 20$

$$\begin{aligned}
 \mathbf{b} \quad & \int (4 + 20\sqrt{x} + 25x) dx \\
 &= \int (4 + 20x^{\frac{1}{2}} + 25x) dx \\
 &= 4x + \frac{20x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{25x^2}{2} + c \\
 &= 4x + \frac{40x^{\frac{3}{2}}}{3} + \frac{25x^2}{2} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{13} \quad & \int \left(3x^5 - \frac{4}{\sqrt{x}} \right) dx = \int (3x^5 - 4x^{-\frac{1}{2}}) dx \\
 &= \frac{3x^6}{6} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + c \\
 &= \frac{x^6}{2} - 8\sqrt{x} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{14} \quad & \int \left(\frac{p}{2x^2} + pq \right) dx = \int \left(\frac{px^{-2}}{2} + pq \right) dx \\
 &= \frac{px^{-1}}{2 \times -1} + pqx + c \\
 &= -\frac{p}{2x} + pqx + c \\
 &= \frac{2}{x} + 10x + c \\
 &-\frac{p}{2} = 2 \text{ and } pq = 10 \\
 &p = -4 \text{ and } q = -2.5
 \end{aligned}$$

15 a Using the binomial expansion:

$$\begin{aligned}
 f(x) &= 2^{10} + \binom{10}{1} 2^9 (-x)^1 + \binom{10}{2} 2^8 (-x)^2 \\
 &= 1024 - 5120x + 11520x^2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad & \int (1024 - 5120x + 11520x^2) dx \\
 &= 1024x - \frac{5120x^2}{2} + \frac{11520x^3}{3} + c \\
 &= 1024x - 2560x^2 + 3840x^3 + c \\
 \text{So } A &= 1024, B = -2560 \text{ and } C = 3840
 \end{aligned}$$