

Integration 13A

$$\mathbf{1\ a} \quad \frac{dy}{dx} = x^5$$

$$y = \frac{x^6}{6} + c$$

$$\mathbf{b} \quad \frac{dy}{dx} = 10x^4$$

$$y = \frac{10x^5}{5} + c$$

$$y = 2x^5 + c$$

$$\mathbf{c} \quad \frac{dy}{dx} = -x^{-2}$$

$$y = -\frac{x^{-1}}{-1} + c$$

$$y = x^{-1} + c \text{ or}$$

$$y = \frac{1}{x} + c$$

$$\mathbf{d} \quad \frac{dy}{dx} = -4x^{-3}$$

$$y = \frac{-4x^{-2}}{-2} + c$$

$$y = 2x^{-2} + c \text{ or}$$

$$y = \frac{2}{x^2} + c$$

$$\mathbf{e} \quad \frac{dy}{dx} = x^{\frac{2}{3}}$$

$$y = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$$

$$y = \frac{3}{5}x^{\frac{5}{3}} + c$$

$$\mathbf{f} \quad \frac{dy}{dx} = 4x^{\frac{1}{2}}$$

$$y = 4\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$y = \frac{8}{3}x^{\frac{3}{2}} + c$$

$$\mathbf{g} \quad \frac{dy}{dx} = -2x^6$$

$$y = -\frac{x^7}{7} + c$$

$$y = -\frac{2}{7}x^7 + c$$

$$\mathbf{h} \quad \frac{dy}{dx} = x^{\frac{1}{2}}$$

$$y = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$y = 2x^{\frac{3}{2}} + c \text{ or}$$

$$y = 2\sqrt{x} + c$$

$$\mathbf{i} \quad \frac{dy}{dx} = 5x^{\frac{3}{2}}$$

$$y = \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$y = -10x^{\frac{1}{2}} + c \text{ or}$$

$$y = -\frac{10}{\sqrt{x}} + c$$

$$\mathbf{j} \quad \frac{dy}{dx} = 6x^{\frac{1}{3}}$$

$$y = \frac{6x^{\frac{4}{3}}}{\frac{4}{3}} + c$$

$$y = \frac{18}{4}x^{\frac{4}{3}} + c$$

$$y = \frac{9}{2}x^{\frac{4}{3}} + c$$

$$\mathbf{k} \quad \frac{dy}{dx} = 36x^{11}$$

$$y = \frac{36x^{12}}{12} + c$$

$$y = \frac{18}{6}x^{12} + c$$

$$y = 3x^{12} + c$$

$$\begin{aligned}
 \mathbf{1 \quad l} \quad \frac{dy}{dx} &= -14x^{-8} \\
 y &= \frac{-14x^{-7}}{-7} + c \\
 y &= 2x^{-7} + c \text{ or} \\
 y &= \frac{2}{x^7} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{m} \quad \frac{dy}{dx} &= -3x^{-\frac{2}{3}} \\
 y &= \frac{-3x^{\frac{1}{3}}}{\frac{1}{3}} + c \\
 y &= -9x^{\frac{1}{3}} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{n} \quad \frac{dy}{dx} &= -5 \\
 &= -5x^0 \\
 y &= \frac{-5x^1}{1} + c \\
 y &= -5x + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{o} \quad \frac{dy}{dx} &= 6x \\
 y &= \frac{6x^2}{2} + c \\
 y &= 3x^2 + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{p} \quad \frac{dy}{dx} &= 2x^{-0.4} \\
 y &= \frac{2x^{0.6}}{0.6} + c \\
 y &= \frac{20}{6}x^{0.6} + c \\
 y &= \frac{10}{3}x^{0.6} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{2 \quad a} \quad \frac{dy}{dx} &= x^3 - \frac{3}{2}x^{\frac{1}{2}} - 6x^{-2} \\
 y &= \frac{x^4}{4} - \frac{3}{2} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 6 \frac{x^{-1}}{-1} + c \\
 y &= \frac{1}{4}x^4 - 3x^{\frac{1}{2}} + 6x^{-1} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{2 \quad b} \quad \frac{dy}{dx} &= 4x^3 + x^{-\frac{2}{3}} - x^{-2} \\
 y &= \frac{4x^4}{4} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{x^{-1}}{-1} + c \\
 y &= x^4 + 3x^{\frac{1}{3}} + x^{-1} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad \frac{dy}{dx} &= 4 - 12x^{-4} + 2x^{-\frac{1}{2}} \\
 y &= 4x - \frac{12x^{-3}}{-3} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c \\
 y &= 4x + 4x^{-3} + 4x^{\frac{1}{2}} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad \frac{dy}{dx} &= 5x^{\frac{2}{3}} - 10x^4 + x^{-3} \\
 y &= \frac{5x^{\frac{5}{3}}}{\frac{5}{3}} - \frac{10x^5}{5} + \frac{x^{-2}}{-2} + c \\
 y &= 3x^{\frac{5}{3}} - 2x^5 - \frac{1}{2}x^{-2} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} \quad \frac{dy}{dx} &= -\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x \\
 y &= \frac{-\frac{4}{3}x^{-\frac{1}{3}}}{-\frac{1}{3}} - 3x + \frac{8x^2}{2} + c \\
 y &= 4x^{\frac{1}{3}} - 3x + 4x^2 + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{f} \quad \frac{dy}{dx} &= 5x^4 - x^{-\frac{3}{2}} - 12x^{-5} \\
 y &= \frac{5x^5}{5} - \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - \frac{12x^{-4}}{-4} + c \\
 y &= x^5 + 2x^{-\frac{1}{2}} + 3x^{-4} + c
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{3 \quad a} \quad f'(x) &= 12x + \frac{3}{2}x^{-\frac{3}{2}} + 5 \\
 f(x) &= \frac{12x^2}{2} + \frac{\frac{3}{2}x^{-\frac{1}{2}}}{-\frac{1}{2}} + 5x + c \\
 f(x) &= 6x^2 - 3x^{-\frac{1}{2}} + 5x + c
 \end{aligned}$$

$$3 \text{ b } f'(x) = 6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$$

$$f(x) = \frac{6x^6}{6} + \frac{6x^{-6}}{-6} - \frac{1}{6} \frac{x^{-\frac{1}{6}}}{-\frac{1}{6}} + c$$

$$f(x) = x^6 - x^{-6} + x^{\frac{1}{6}} + c$$

$$c \text{ } f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$$

$$f(x) = \frac{1}{2} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2} \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$f(x) = x^{\frac{1}{2}} + x^{\frac{1}{2}} + c$$

$$d \text{ } f'(x) = 10x^4 + 8x^{-3}$$

$$f(x) = \frac{10x^5}{5} + \frac{8x^{-2}}{-2} + c$$

$$f(x) = 2x^5 - 4x^{-2} + c$$

$$e \text{ } f'(x) = 2x^{\frac{1}{3}} + 4x^{\frac{5}{3}}$$

$$f(x) = \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} + \frac{4x^{\frac{2}{3}}}{-\frac{2}{3}} + c$$

$$f(x) = 3x^{\frac{2}{3}} - 6x^{\frac{2}{3}} + c$$

$$f \text{ } f'(x) = 9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$$

$$f(x) = \frac{9x^3}{3} + \frac{4x^{-2}}{-2} + \frac{1}{4} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$f(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^{\frac{1}{2}} + c$$

$$4 \text{ } \frac{dy}{dx} = (2x + 3)^2$$

$$= 4x^2 + 12x + 9$$

$$y = \frac{4x^3}{3} + 6x^2 + 9x + c$$

$$5 \text{ } f'(x) = 3x^{-2} + 6x^{\frac{1}{2}} + x - 4$$

$$f(x) = \frac{3x^{-1}}{-1} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^2}{2} - 4x + c$$

$$= -3x^{-1} + 4x^{\frac{3}{2}} + \frac{1}{2}x^2 - 4x + c$$

Challenge

$$\frac{dy}{dx} = (2\sqrt{x} - x^2) \left(\frac{3+x}{x^5} \right)$$

$$= (2x^{\frac{1}{2}} - x^2) (3x^{-5} + x^{-4})$$

$$= 6x^{\frac{9}{2}} + 2x^{\frac{7}{2}} - 3x^{-3} - x^{-2}$$

$$y = \frac{6x^{\frac{7}{2}}}{-\frac{7}{2}} + \frac{2x^{\frac{5}{2}}}{-\frac{5}{2}} - \frac{3x^{-2}}{-2} - \frac{x^{-1}}{-1} + c$$

$$= -\frac{12}{7x^{\frac{7}{2}}} - \frac{4}{5x^{\frac{5}{2}}} + \frac{3}{2x^2} + \frac{1}{x} + c$$