Differentiation 12K

1 $\theta = t^2 - 3t$

$\frac{d\theta}{dt} = 2t - 3$

2 $A = 2\pi r$

$\frac{dA}{dr} = 2\pi$

3 $r = \frac{12}{t} = 12t^{-1}$

$\frac{dr}{dt} = -12t^{-2} = -\frac{12}{t^2}$

When $t = 3$,

$\frac{dr}{dt} = -\frac{12}{3^2} = -\frac{12}{9} = -\frac{4}{3}$

4 $A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

When $r = 6$,

$\frac{dA}{dr} = 8\pi \times 6$

$= 48\pi \text{ cm}^2$ per cm

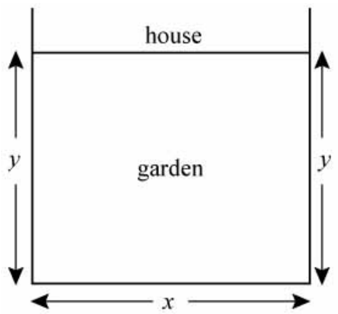
5 $s = t^2 + 8t$

$\frac{ds}{dt} = 2t + 8$

When $t = 5$,

$\frac{ds}{dt} = 2 \times 5 + 8 = 18 \text{ m s}^{-1}$

6 a



Let the width of the garden be x m.

Then $x + 2y = 80$

$x = 80 - 2y$ (1)

Area $A = xy$

$= y(80 - 2y)$

$= 80y - 2y^2$

6 b $\frac{dA}{dy} = 80 - 4y$

Putting $\frac{dA}{dy} = 0$ for maximum area:

$80 - 4y = 0$

$y = 20$

Substituting in (1): $x = 40$

So area $= 40 \text{ m} \times 20 \text{ m} = 800 \text{ m}^2$

7 a Total surface area $= 2\pi rh + 2\pi r^2$

$2\pi rh + 2\pi r^2 = 600\pi$

$rh = 300 - r^2$

Volume $= \pi r^2 h = \pi r(rh) = \pi r(300 - r^2)$

So $V = 300\pi r - \pi r^3$

b For maximum volume, $\frac{dV}{dr} = 0$

$\frac{dV}{dr} = 300\pi - 3\pi r^2$

$300\pi - 3\pi r^2 = 0$

$r^2 = 100$

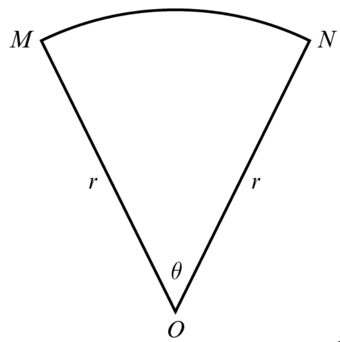
$r = 10$

Substituting $r = 10$ into V gives:

$V = 300\pi \times 10 - \pi \times 10^3 = 2000\pi$

Maximum volume $= 2000\pi \text{ cm}^3$

8 a



Let angle $MON = \theta$ radians

Then perimeter $P = 2r + r\theta$ **(1)**

and area $A = \frac{1}{2}r^2\theta$

Area $= 100 \text{ cm}^2$

$\frac{1}{2}r^2\theta = 100$

$r\theta = \frac{200}{r}$

Substituting into **(1)** gives:

$P = 2r + \frac{200}{r}$ **(2)**

8 a Since area of circle > area of sector

$$\pi r^2 > 100$$

$$r > \sqrt{\frac{100}{\pi}}$$

b For minimum perimeter, $\frac{dP}{dr} = 0$

$$\frac{dP}{dr} = 2 - \frac{200}{r^2}$$

$$2 - \frac{200}{r^2} = 0$$

$$r^2 = 100$$

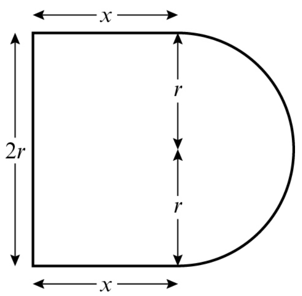
$$r = 10$$

Substituting into (2) gives:

$$P = 20 + \frac{200}{10} = 40$$

Minimum perimeter = 40 cm

9 a



Let the rectangle have dimensions $2r$ by x cm.

Perimeter of figure = $(2r + 2x + \pi r)$ cm

Perimeter = 40 cm, so

$$2r + 2x + \pi r = 40$$

$$x = \frac{40 - \pi r - 2r}{2}$$

Area = rectangle + semicircle

$$= 2rx + \frac{1}{2}\pi r^2$$

Substituting $x = \frac{40 - \pi r - 2r}{2}$:

$$A = r(40 - \pi r - 2r) + \frac{1}{2}\pi r^2$$

$$= 40r - 2r^2 - \frac{1}{2}\pi r^2$$

b For maximum area, $\frac{dA}{dr} = 0$:

$$\frac{dA}{dr} = 40 - 4r - \pi r$$

$$40 - 4r - \pi r = 0$$

9 b $r = \frac{40}{4 + \pi}$

When $r = \frac{40}{4 + \pi}$,

$$A = 40 \times \frac{40}{4 + \pi} - 2\left(\frac{40}{4 + \pi}\right)^2 - \frac{1}{2}\pi\left(\frac{40}{4 + \pi}\right)^2$$

$$= \frac{1600}{4 + \pi} - \left(2 + \frac{1}{2}\pi\right)\left(\frac{40}{4 + \pi}\right)^2$$

$$= \frac{1600}{4 + \pi} - \frac{4 + \pi}{2} \times \frac{1600}{(4 + \pi)^2}$$

$$= \frac{1600}{4 + \pi} - \frac{800}{4 + \pi}$$

$$= \frac{800}{4 + \pi}$$

So maximum area = $\frac{800}{4 + \pi}$ cm^2

10 a Total length of wire = $(18x + 14y)$ mm

Length = 1512 mm, so

$$18x + 14y = 1512$$

$$y = \frac{1512 - 18x}{14}$$

Total area A mm^2 is given by:

$$A = 2y \times 6x$$

Substituting $y = \frac{1512 - 18x}{14}$:

$$A = 12x\left(\frac{1512 - 18x}{14}\right)$$

$$= 1296x - \frac{108}{7}x^2$$

b For maximum area $\frac{dA}{dx} = 0$:

$$\frac{dA}{dx} = 1296 - \frac{216}{7}x$$

$$1296 - \frac{216}{7}x = 0$$

$$x = \frac{7 \times 1296}{216} = 42$$

When $x = 42$,

$$A = 1296 \times 42 - \frac{108}{7} \times 42^2$$

$$= 27\,216$$

Maximum area = 27 216 mm^2

(Check: $\frac{d^2A}{dx^2} = -\frac{216}{7} < 0 \quad \therefore$ maximum)