Differentiation 12G

1 a $f(x) = 3x^2 + 8x + 2$
$f'(x) = 6x + 8$
If $f'(x) \geq 0$ then
$6x + 8 \quad 0$
$6x \geq -8$
$x \quad -\frac{4}{3}$
So $f(x)$ is increasing for $x \geq -\frac{4}{3}$.

b $f(x) = 4x - 3x^2$
$f'(x) = 4 - 6x$
If $f'(x) \geq 0$ then
$4 - 6x \geq 0$
$4 \geq 6x$
$x \leq \frac{4}{6}$
$x \leq \frac{2}{3}$
So $f(x)$ is increasing for $x \leq \frac{2}{3}$.

c $f(x) = 5 - 8x - 2x^2$
$f'(x) = -8 - 4x$
If $f'(x) \geq 0$ then
$-8 - 4x \geq 0$
$-8 \geq 4x$
$x \leq -2$
So $f(x)$ is increasing for $x \leq -2$.

d $f(x) = 2x^3 - 15x^2 + 36x$
$f'(x) = 6x^2 - 30x + 36$
If $f'(x) \geq 0$ then
$6x^2 - 30x + 36 \geq 0$
$6(x^2 - 5x + 6) \geq 0$
$6(x - 2)(x - 3) \geq 0$
Considering the 3 regions:

	$x \leq 2$	$2 \leq x \leq 3$	$x \geq 3$
$6(x - 2)(x - 3)$	+ve	−ve	+ve

So $x \leq 2$ or $x \geq 3$
So $f(x)$ is increasing for $x \leq 2$ and $x \geq 3$.

e $f(x) = 3 + 3x - 3x^2 + x^3$
$f'(x) = 3 - 6x + 3x^2$
If $f'(x) \geq 0$ then
$3 - 6x + 3x^2 \geq 0$
$3(1 - 2x + x^2) \geq 0$
$3(1 - x)^2 \geq 0$
So $f(x)$ is increasing for $x \in \mathbb{R}$.

f $f(x) = 5x^3 + 12x$
$f'(x) = 15x^2 + 12$
If $f'(x) \geq 0$ then
$15x^2 + 12 \geq 0$
This is true for all real values of x.
So $f(x)$ is increasing for $x \in \mathbb{R}$.

$\geq$

g $f(x) = x^4 + 2x^2$
$f'(x) = 4x^3 + 4x$
If $f'(x) \geq 0$ then
$4x^3 + 4x \geq 0$
$4x(x^2 + 1) \geq 0$
$x \geq 0$
So $f(x)$ is increasing for $x \geq 0$.

h $f(x) = x^4 - 8x^3 \geq$
$f'(x) = 4x^3 - 24x^2$
If $f'(x) \geq 0$ then
$4x^3 - 24x^2 \geq 0$
$4x^2(x - 6) \geq 0$
$x \geq 6$
So $f(x)$ is increasing for $x \geq 6$.

2 a $f(x) = x^2 - 9x$
$f'(x) = 2x - 9$
If $f'(x) \leq 0$ then
$2x - 9 \leq 0$
$2x \leq 9$
$x \leq \frac{9}{2}$
So $f(x)$ is decreasing for $x \leq \frac{9}{2}$.

b $f(x) = 5x - x^2$
$f'(x) = 5 - 2x$
If $f'(x) \leq 0$ then
$5 - 2x \leq 0$
$2x \geq 5$
$x \geq \frac{5}{2}$
So $f(x)$ is decreasing for $x \geq \frac{5}{2}$.

c $f(x) = 4 - 2x - x^2$
$f'(x) = -2 - 2x$
If $f'(x) \leq 0$ then
$-2 - 2x \leq 0$
$2x \geq -2$
$x \geq -1$
So $f(x)$ is decreasing for $x \geq -1$.

2 d $f(x) = 2x^3 - 3x^2 - 12x$
$f'(x) = 6x^2 - 6x - 12$
If $f'(x) \leq 0$ then
$6x^2 - 6x - 12 \leq 0$
$6(x^2 - x - 2) \leq 0$
$6(x-2)(x+1) \leq 0$
Considering the 3 regions:

	$x \leq -1$	$-1 \leq x \leq 2$	$x \geq 2$
$6(x-2)(x+1)$	+ve	−ve	+ve

So $-1 \leq x \leq 2$
So f(x) is decreasing on the interval $[-1, 2]$.

e $f(x) = 1 - 27x + x^3$
$f'(x) = -27 + 3x^2$
If $f'(x) \leq 0$ then
$-27 + 3x^2 \leq 0$
$3x^2 \leq 27$
$x^2 \leq 9$
$-3 \leq x \leq 3$
So f(x) is decreasing on the interval $[-3, 3]$.

f $f(x) = x + 25x^{-1}$
$f'(x) = 1 - \frac{25}{x^2}$
If $f'(x) \leq 0$ then
$1 - \frac{25}{x^2} \leq 0$
$1 \leq \frac{25}{x^2}$
$x^2 \leq 25$
$-5 \leq x \leq 5$
f(x) is not defined for $x = 0$.
So f(x) is decreasing on the intervals $[-5, 0)$ and $(0, 5]$.

g $f(x) = x^{\frac{1}{2}} + 9x^{-\frac{1}{2}}$
$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 9 \times \frac{1}{2}x^{-\frac{3}{2}}$
If $f'(x) \leq 0$ then
$\frac{1}{2}x^{-\frac{1}{2}} - 9 \times \frac{1}{2}x^{-\frac{3}{2}} \leq 0$
$\frac{1}{2}x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} \leq 0$
$\frac{x^{-\frac{3}{2}}}{2}(x-9) \leq 0$
f(x) is defined for $x > 0$.
$f'(x) \leq 0$ for $x \leq 9$, so f(x) is decreasing on the interval $(0, 9]$.

2 h $f(x) = x^2(x+3)$
$= x^3 + 3x^2$
$f'(x) = 3x^2 + 6x$
If $f'(x) \leq 0$ then
$3x^2 + 6x \leq 0$
$3x(x+2) \leq 0$
Considering the 3 regions:

	$x \leq -2$	$-2 \leq x \leq 0$	$x \geq 0$
$3x(x+2)$	+ve	−ve	+ve

So f(x) is decreasing on the interval $[-2, 0]$.

3 $f(x) = 4 - x(2x^2 + 3) = 4 - 2x^3 - 3x$
$f'(x) = -6x^2 - 3$
$x^2 \geq 0$ for all $x \in \mathbb{R}$, so $-6x^2 - 3 \leq 0$ for all $x \in \mathbb{R}$.
Therefore, f(x) is decreasing for all $x \in \mathbb{R}$.

4 a $f(x) = x^2 + px$
$f'(x) = 2x + p \geq 0$ when $-1 \leq x \leq 1$
When $x = -1$, $f'(x) = -2 + p \geq 0$, so $p \geq 2$
So for $f'(x) \geq 0$, $p \geq 2$ e.g. $p = 3$
When $x = 1$, $f'(x) = 2 + p \geq 0$, so $p \geq -2$
However, $p \geq 2$ to work with $x = -1$.

b Using the proof from part **a**, any value $p \geq 2$ will work.