Differentiation 12D

1 a $y = 2x^2 - 6x + 3$

$\frac{dy}{dx} = 2(2x) - 6(1) + 0 = 4x - 6$

b $y = \frac{1}{2}x^2 + 12x$

$\frac{dy}{dx} = \frac{1}{2}(2x) + 12(1) = x + 12$

c $y = 4x^2 - 6$

$\frac{dy}{dx} = 4(2x) - 0 = 8x$

d $y = 8x^2 + 7x + 12$

$\frac{dy}{dx} = 8(2x) + 7(1) + 0 = 16x + 7$

e $y = 5 + 4x - 5x^2$

$\frac{dy}{dx} = 0 + 4(1) - 5(2x) = 4 - 10x$

2 a $y = 3x^2$

$\frac{dy}{dx} = 6x$

At the point (2, 12), $x = 2$

Substituting $x = 2$ into $\frac{dy}{dx} = 6x$ gives:

Gradient = 6 × 2 = 12

b $y = x^2 + 4x$

$\frac{dy}{dx} = 2x + 4$

At the point (1, 5), $x = 1$

Substituting $x = 1$ into $\frac{dy}{dx} = 2x + 4$ gives:

Gradient = 2 × 1 + 4 = 6

c $y = 2x^2 - x - 1$

$\frac{dy}{dx} = 4x - 1$

At the point (2, 5), $x = 2$

Substituting $x = 2$ into $\frac{dy}{dx} = 4x - 1$ gives:

Gradient = 4 × 2 − 1 = 7

2 d $y = \frac{1}{2}x^2 + \frac{3}{2}x$

$\frac{dy}{dx} = x + \frac{3}{2}$

At the point (1, 2), $x = 1$

Substituting $x = 1$ into $\frac{dy}{dx} = x + \frac{3}{2}$ gives:

Gradient $= 1 + \frac{3}{2} = 2\frac{1}{2}$

e $y = 3 - x^2$

$\frac{dy}{dx} = -2x$

At the point (1, 2), $x = 1$

Substituting $x = 1$ into $\frac{dy}{dx} = -2x$ gives:

Gradient = −2 × 1 = −2

f $y = 4 - 2x^2$

$\frac{dy}{dx} = -4x$

At the point (−1, 2), $x = -1$

Substituting $x = -1$ into $\frac{dy}{dx} = -4x$ gives:

Gradient = −4 × −1 = 4

3 $y = 3 + 2x - x^2$

When $x = 1$, $y = 3 + 2 - 1$

$\Rightarrow y = 4$ when $x = 1$

$\frac{dy}{dx} = 2 - 2x$

When $x = 1, \frac{dy}{dx} = 2 - 2$

$\Rightarrow \frac{dy}{dx} = 0$ when $x = 1$

Therefore, the y-coordinate is 4 and the gradient is 0 when the x-coordinate is 1 on the given curve.

4 $y = x^2 + 5x - 4$

$\frac{dy}{dx} = 2x + 5$

$2x + 5 = 3$

$2x = -2$

$x = -1$

4 Substituting $x = -1$ into $y = x^2 + 5x - 4$:
$y = (-1)^2 + 5(-1) - 4 = 1 - 5 - 4 = -8$
So $(-1, -8)$ is the point where the gradient is 3.

5 The curve $y = x^2 - 5x + 10$ meets the line $y = 4$ when:
$x^2 - 5x + 10 = 4$
$x^2 - 5x + 6 = 0$
$(x - 3)(x - 2) = 0$
$x = 3$ or $x = 2$

Gradient of curve $= \frac{dy}{dx} = 2x - 5$

When $x = 3$, $\frac{dy}{dx} = 2 \times 3 - 5 = 1$

When $x = 2$, $\frac{dy}{dx} = 2 \times 2 - 5 = -1$

So the gradient is -1 at $(2, 4)$ and 1 at $(3, 4)$.

6 The curve $y = 2x^2$ meets the line $y = x + 3$ when
$2x^2 = x + 3$
$2x^2 - x - 3 = 0$
$(2x - 3)(x + 1) = 0$
$x = 1.5$ or -1

Gradient of curve $= \frac{dy}{dx} = 4x$

When $x = -1$, $\frac{dy}{dx} = 4 \times -1 = -4$

When $x = 1.5$, $\frac{dy}{dx} = 4 \times 1.5 = 6$

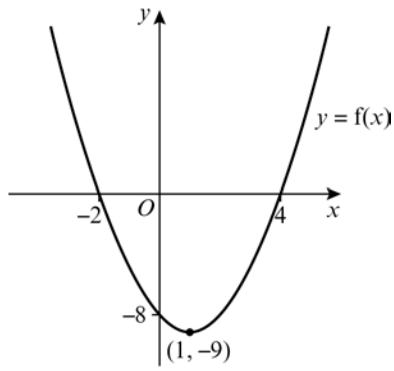
So the gradient is -4 at $(-1, 2)$ and 6 at $(1.5, 4.5)$.

7 a $y = f(x) = x^2 - 2x - 8$
As $a = 1$ is positive, the graph has a $\cup$ shape and a minimum point.

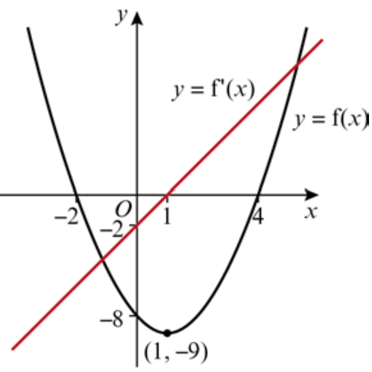
When $x = 0$, $y = -8$, so the graph crosses the y-axis at $(0, -8)$.
When $y = 0$,
$x^2 - 2x - 8 = 0$
$(x + 2)(x - 4) = 0$
$x = -2$ or $x = 4$, so the graph crosses the x-axis at $(-2, 0)$ and $(4, 0)$.

Completing the square:
$x^2 - 2x - 8 = (x - 1)^2 - 1 - 8$
$= (x - 1)^2 - 9$
So the minimum point has coordinates $(1, -9)$.

7 a The sketch of the graph is:



b $f'(x) = 2x - 2 + 0 = 2x - 2$



c At the turning point the gradient of $y = f(x)$ is zero, i.e. $f'(x) = 0$.