Differentiation 12A

1 a Examples of estimates of gradients:
Gradient of tangent at $x = -1$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{(-1) - (-0.5)} = -4$$

Gradient of tangent at $x = 0$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{(-0.5) - (0.5)} = -2$$

Gradient of tangent at $x = 1$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (-1)}{2 - 0} = 0$$

Gradient of tangent at $x = 2$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - 1}{1.5 - 2.5} = 2$$

Gradient of tangent at $x = 3$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{3 - 2.5} = 4$$

***x*-coordinate**	−1	0	1	2	3
Estimate for gradient of curve	−4	−2	0	2	4

b The gradient of the curve at the point where $x = p$ is $2p - 2$.

c Gradient of tangent at $x = 1.5$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1.7) - 0.3}{0.5 - 2.5} = 1$$

$2p - 2 = 2(1.5) - 2 = 1$

2 a Substituting $x = 0.6$ into $y = \sqrt{1 - x^2}$:
$y = \sqrt{1 - 0.6^2} = \sqrt{0.64} = 0.8$, therefore the point A (0.6, 0.8) lies on the curve.

b Gradient of tangent at $x = 0.6$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1.1 - 0.8}{0.2 - 0.6} = -0.75$$

2 c i Gradient of AD $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0.8 - \sqrt{0.19}}{0.6 - 0.9} = -1.21$ (3 s.f.)

ii Gradient of AC $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0.8 - 0.6}{0.6 - 0.8} = -1$

iii Gradient of AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0.8 - \sqrt{0.51}}{0.6 - 0.7} = -0.859$ (3 s.f.)

d As the points move closer to A, the gradient tends to −0.75.

3 a i Gradient $= \dfrac{16 - 9}{4 - 3} = \dfrac{7}{1} = 7$

ii Gradient $= \dfrac{12.25 - 9}{3.5 - 3} = \dfrac{3.25}{0.5} = 6.5$

iii Gradient $= \dfrac{9.61 - 9}{3.1 - 3} = \dfrac{0.61}{0.1} = 6.1$

iv Gradient $= \dfrac{9.0601 - 9}{3.01 - 3} = \dfrac{0.0601}{0.01} = 6.01$

v Gradient $= \dfrac{(3 + h)^2 - 9}{(3 + h) - 3} = \dfrac{6h + h^2}{h} = \dfrac{h(6 + h)}{h} = 6 + h$

3 b When h is small, the gradient of the chord is close to the gradient of the tangent, and $6 + h$ is close to the value 6.
So the gradient of the tangent at (3, 9) is 6.

4 a i Gradient = $\frac{25-16}{5-4}=\frac{9}{1}=9$

ii Gradient = $\frac{20.25-16}{4.5-4}=\frac{4.25}{0.5}=8.5$

iii Gradient = $\frac{16.81-16}{4.1-4}=\frac{0.81}{0.1}=8.1$

iv Gradient = $\frac{16.0801-16}{4.01-4}$

$$=\frac{0.0801}{0.01}=8.01$$

v Gradient $=\frac{(4+h)^2-16}{4+h-4}$

$$=\frac{16+8h+h^2-16}{h}$$

$$=\frac{8h+h^2}{h}$$

$$=\frac{h(8+h)}{h}$$

$$=8+h$$

b When h is small, the gradient of the chord is close to the gradient of the tangent, and $8 + h$ is close to the value 8.
So the gradient of the tangent at (4, 16) is 8.